# Pion Single-Charge Exchange in the Isobar Doorway Model

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The isobar doorway model of Kisslinger and Wang is extended to include the pion single-charge-exchange channel in addition to the elastic one. By use of this model, the integrated charge-exchange cross section to the isospin analog state is calculated and compared with the $^{13}C(\pi^+, \pi^0)^{13}N$ measurements. Good agreement with experiment is achieved. The connection between elastic scattering and charge exchange is discussed.

Recently, the $^{13}C(\pi^+, \pi^0)^{13}N$ pion single-charge-exchange (CE) integrated cross section was measured[1] with use of the $\beta$-radioactivation method. In $^{13}N$, all states except the ground state (g.s.) are particle-unstable and therefore the measurement includes only CE to the $^{13}N$ g.s., which is the isospin analog of the $^{13}C$ g.s. The excitation function measured around the $\Delta_{33}$ resonance energy (80–200 MeV) is shown in Fig. 1(a). The shape of the excitation function is flat and the value of the cross section is about $\sigma_{CE} \approx 1$ mb. Comparing the CE excitation function with the elastic one[2] [see Fig. 1(b)], I note that the shapes are similar. The total elastic cross section has the form of a very broad peak centered around $E = 150$ MeV with a width of about 300 MeV.

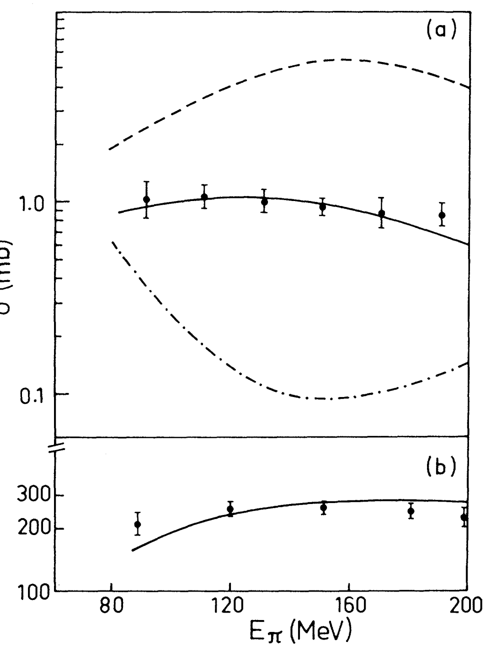


FIG. 1. (a) The experimental and calculated $^{13}C(\pi^+, \pi^0)^{13}N$ integrated cross sections as a function of the pion energy. The experimental points are from Ref. 1. The dashed line is obtained with use of the PWIA, and the dash-dotted line with use of the DWIA. The solid line is the result of the present isobar-doorway-model calculation. (b) The experimental and calculated $\pi^- + ^{12}C$ total elastic cross sections as a function of the pion energy. The experimental points are from Ref. 2. The solid line is a result of the isobar doorway model and is computed using the same values for $E_\Delta$, $\Gamma_\Delta$, $\bar{\epsilon}_d$, and $\bar{\Gamma}_d$ as in the CE reaction.

The elastic channel and the single CE to the g.s. analog are related by isospin in a simple manner; and, at first, one would expect such a similarity to occur. However, the recent calculations[3-6] do not reproduce a flat excitation function for the CE reaction.

The simplest calculation of the $(\pi^+, \pi^0)$ reaction is performed using the plane-wave impulse approximation[6,7] (PWIA), i.e., using only the first term in the multiple-scattering series

$$T^{\text{imp}} = \langle \vec{k}' | t_1 | \vec{k} \rangle F_1(\vec{q}), \quad (1)$$

where $\vec{k}$ and $\vec{k}'$ are the incident $\pi^+$ and final $\pi^0$ momenta, $\vec{q} = \vec{k} - \vec{k}'$, and $t_1$ is the free-pion–nucleon ($\pi N$) $t$ matrix for CE. The form factor $F_1(\vec{q})$ corresponds to the isovector part of the nuclear density which is equal (neglecting core polarization) to the excess neutron density, and $F_1(0) = N - Z$.

In Fig. 1(a), I show the results of a calculation using Eq. (1). The excess neutron density is simply the density of the $p_{1/2}$ neutron in $^{13}C$. The excitation function is peaked around 160 MeV with a width approximately equal to the width of the $\Delta_{33}$ resonance. The resulting cross sections are much larger than the experimental ones. At maximum the discrepancy is about a factor of 5. The large cross section is attributed to the fact that distortions of the pion waves are neglected. To account for it one constructs a first-order pion-nucleus potential. The CE reaction is then calculated by using the distorted waves impulse approximation[4,6] (DWIA) or the coupled-channels method.[3] Both methods give similar results.

A typical result obtained using Kisslinger's potential[8] and the DWIA is shown in Fig. 1(a). The excitation function has a dip in the resonance region and the cross section around 150 MeV is almost by an order magnitude smaller than the experimental one. Different optical potentials obtained using different off-shell extrapolations give similar results.[3,6] Second-order corrections to the optical potential[6,9] improve somewhat the results but the discrepancy remains still unresolved. The optical-model calculations generate a subsum of the total multiple-scattering expansion. The failure of optical-model calculations in the CE reaction indicates that a different approach is needed in which the multiple-scattering series is summed more effectively. Such a framework exists in the regime of low-energy nuclear physics in the form of the unified theory of reactions[10] applied in particular to the doorway-state case.[11,12] Recently, Kisslinger and Wang have applied this theory to pion-nucleus elastic scattering.[13] I generalize their theory to include the CE, $(\pi^+, \pi^0)$ channel in addition to the elastic one.

In the isobar doorway model of Ref. 13, the $\Delta_{33}$ resonance is introduced explicitly into the description of the $\pi$-nucleus system. The pion forms, together with the nucleon, a $\Delta_{33}$ particle bound in the nucleus. The resulting $\Delta$-particle-nucleon-hole, $(\Delta N^{-1})$ configurations are intermediate states serving as doorways in pion-nucleus scattering. Examples of such $(\Delta N^{-1})$ states for the case of ${}^{13}\mathrm{C}+\pi^+$ are shown in Fig. 2. These doorways are of course not eigenstates of the $\pi$-nucleus system. They are coupled to more complicated states. In the case of a single doorway, it makes its appearance not only in the elastic channel but also in other exit channels. When there are many doorways of the same nature, as it is in the present case, I expect a similar situation to occur.

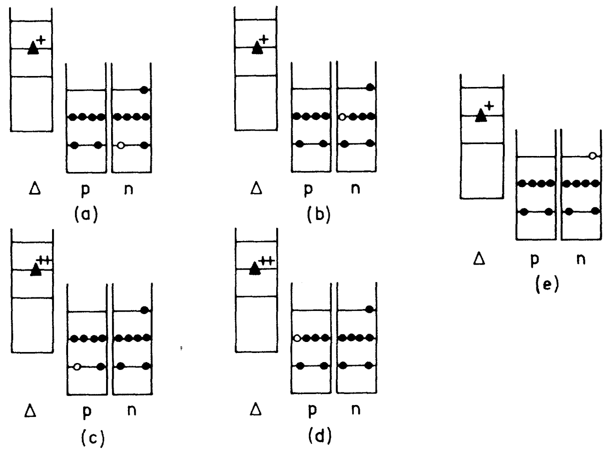


FIG. 2. The isobar doorway states in the $\pi^+ + {}^{13}\mathrm{C}$ system. (The nuclear orbits are the $s_{1/2}$, $p_{3/2}$, and $p_{1/2}$ while the $\Delta$ can be in any orbit possible.)

I now briefly outline the derivation of the $T$ matrix for CE, following closely Ref. 13. The states of the pion-nucleus system are divided into three orthogonal subspaces. (1) The $P$ space contains the elastic (i.e., $\pi^+$ and the g.s. of ${}^{13}\mathrm{C}$) and the CE ($\pi^0$ and the g.s. of ${}^{13}\mathrm{N}$) channels; (2) the $d$ space contains the $\Delta N^{-1}$ doorways; and (3) the $q$ space contains all the remaining states. By use of the projection operator technique, the Schrödinger equation is written as set of three equations with states of the $P$, $d$, and $q$ spaces mutually coupled.[11,12] The usual procedure is then to eliminate the $q$ space and to rewrite the three coupled equation as a system of two coupled equation in terms of an effective Hamiltonian $\mathcal{H}$.[11,12] The solution of these equation leads to the following expression for the $T$ matrix:

$$T_{\beta 0} = T_{\beta 0}{}^{\mathrm{NR}} + \langle \Phi_\beta^{(-)} | \mathcal{H}_{Pd}(E - \mathcal{H}_{dd} - \mathcal{H}_{dP} G_P^{(+)} \mathcal{H}_{Pd})^{-1} \mathcal{H}_{dP} | \Phi_0^{(+)} \rangle . \tag{2}$$

(The notation $\mathcal{H}_{XY}$ means $X\mathcal{H}Y$, where $X$ and $Y$ are projection operators.) I denote the elastic and CE channels by 0 and 1, respectively, and $\beta = 0, 1$. The wave functions $\Phi_\beta^{(+)}$ are solutions of $(\mathcal{H}_{PP} - E)|\Phi_\beta^{(+)}\rangle = 0$; similarly, for $\Phi_\beta^{(-)}$. Following Kisslinger and Wang, I postulate that $\mathcal{H}_{PP}$ is generated by the off-resonance part of the $\pi N$ interaction. The matrix $T^{\mathrm{NR}}$ is the nonresonant part calculated using the states $\Phi_\beta^{(\pm)}$, and $G_P^{(+)}$ is the corresponding Green's function. In the doorway-state hypothesis,[11] there is no direct coupling between $P$ and $q$ states, i.e., $H_{Pq} = 0$. By use of this hypothesis, the $T$ matrix becomes[13]

$$T_{10} = T_{10}{}^{\mathrm{NR}} + \sum_d \frac{\langle \Phi_1^{(-)} | H_{Pd} | d \rangle \langle d | H_{dP} | \Phi_0^{(+)} \rangle}{E - \epsilon_d + i\Gamma_{d/2}}, \tag{3}$$

where $\epsilon_d$ is the doorway-state energy plus a shift due to the coupling to both the $P$ and $q$ states.[11,12] The width of the doorway is $\Gamma_d = \Gamma_d{}^{\uparrow} + \Gamma_d{}^{\downarrow}$, where $\Gamma_d{}^{\uparrow}$ is the escape width into the channels in $P$ and $\Gamma_d{}^{\downarrow}$ is the absorption width resulting from the coupling to states in $q$ space.[11,12] Kisslinger and Wang[13] have shown that the numerator in Eq. (3) can be related to the $\pi N$ $t$ matrix. Their expression for the $T$ matrix, adapted to my case, is

$$\langle \vec{k}'| T_{10}|\vec{k}\rangle \simeq \langle \vec{k}'| T_{10}^{\rm NR}|\vec{k}\rangle + \sum_d \frac{\langle \vec{k}'|t_1|\vec{k}\rangle (E - E_\Delta - i\Gamma_{\Delta/2})}{E - \epsilon_d + i\Gamma_{d/2}} F_1^{\,d}(\vec{k}', \vec{k}), \tag{4}$$

where $F_1^{\,d}$ is a nuclear form factor corresponding to doorway $d$ and channel 1, $E_\Delta$ and $\Gamma_\Delta$ are the energy and width of the $\Delta_{33}$ resonance.

This expression can be simplified even more when the closure approximation is used.[13] The energies $\epsilon_d$ and widths $\Gamma_d$ are replaced by average values; and the sum over $d$ in the form factor is performed. The final result is

$$\langle \vec{k}'| T_{10}|\vec{k}\rangle = \langle \vec{k}'| T_{10}^{\rm NR}|\vec{k}\rangle + \frac{E - E_\Delta + i\Gamma_{\Delta/2}}{E - \bar{\epsilon}_d + i\bar{\Gamma}_{d/2}} \langle \vec{k}'|t_1|\vec{k}\rangle F_1(\vec{q}). \tag{5}$$

When applying this equation I will assume that $\mathcal{H}_{PP}$ is diagonal in the two channels, i.e., direct-channel coupling can be neglected in the resonance region.

The doorways for the CE channel are of the type given in Fig. 2(e). (The $\Delta_{33}$ can be in any single-particle state possible.) The doorways for the elastic channel are all the $\Delta N^{-1}$ configurations shown in Fig. 2. The interpretation of the CE reaction is simple. The incoming $\pi^+$ interacts with the $p_{1/2}$ neutron, forming a $\Delta^+ N^{-1}$ configuration. Subsequently, the $\Delta^+$ decays into a $p_{1/2}$ proton while a $\pi^0$ escapes.

Since the doorway states for both channels are of the same type their average energies and widths are approximately the same. I assume that these do not change appreciably in the energy range $E = 80-200$ MeV. I can now use the elastic $\pi + {}^{12}C$ data to estimate $\bar{\epsilon}_d$ and $\bar{\Gamma}_d$. The data of Ref. 2 yield $\bar{\epsilon}_d \simeq 150$ MeV and $\bar{\Gamma}_d \approx 300$ MeV. I use $E_\Delta \approx 170$ MeV and $\Gamma_\Delta \approx 120$ MeV, which are close to the $\Delta_{33}$ resonance values. The values of $t_1$ are calculated using the $\pi N$ phase shifts[14]; and $F_1(\vec{q})$ is determined using the wave function of the $p_{1/2}$ neutron orbit. The solid line in Fig. 1(a) is the result of our calculation. The theoretical curve agrees well with the experimental excitation function. The solid curve in Fig. 1(b) is the theoretical total elastic cross section calculated using Eq. (5) with $t_1$ replaced by $t_0$, the elastic $\pi N$ $t$ matrix, and $F_1(\vec{q})$ replaced by the elastic form factor $F_0(\vec{q})$ evaluated using harmonic-oscillator wave functions for $^{12}$C. The values for $E_\Delta$, $\Gamma_\Delta$, $\bar{\epsilon}_d$, and $\bar{\Gamma}_d$ were the same as in the CE calculation. Again, the agreement with experiment[2] is satisfactory.

Several remarks should be made: (a) Although the resonant term in Eq. (5) is equal to $T^{\rm imp}$ times a simple factor, this resemblance between Eq. (1) and Eq. (5) is only formal. As stressed in Ref. 13, the $T$ matrix in Eq. (5) corresponds to an entire sum of the multiple-scattering series, and is not merely a trivial modification of the first term in this series. The advantage of the doorway-state approach lies in the fact that complicated nuclear structure effects can be expressed in terms of several parameters, such as the shift or absorption width $\Gamma_d^{\downarrow}$. For example, if we approximate $\Gamma_d^{\uparrow} \approx \Gamma_\Delta$ and $\bar{\epsilon}_d \approx E_\Delta$, then for $E = \bar{\epsilon}_d$ the resonant term in Eq. (5) is equal to $T^{\rm imp}$ times the absorption factor, $1 - \Gamma_d^{\downarrow}/\Gamma_d$. (b) In the present description there is a simple relation between the CE to analog channel and the elastic one. This is consistent with the fact that the excitation functions in these two channels have similar shapes. (c) In the present approach, the doorways are simple $\Delta N^{-1}$ states, and $\mathcal{H}_{dd}$ is assumed to be diagonal. An extension to the case in which the various $\Delta N^{-1}$ states are correlated is of interest. (d) The present approach can be generalized to include inelastic scattering and "inelastic" CE to nonanalog states.

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[1]Y. Shamai *et al.*, Phys. Rev. Lett. 36, 82 (1976).

[2]F. Binon *et al.*, Nucl. Phys. B17, 168 (1970).

[3]N. Auerbach and J. Warszawski, Phys. Lett. 45B, 171 (1973); see also the work of G. A. Miller and J. E. Spencer, Ann. Phys. (N.Y.) 100, 562 (1976) in which an extensive study of the single- and double-pion charge-exchange reactions is presented.

[4]D. Tow and J. M. Eisenberg, Nucl. Phys. A237, 441 (1975).

[5]W. R. Gibbs *et al.*, Phys. Rev. Lett. 36, 85 (1976).

[6]J. Warszawski and N. Auerbach, Nucl. Phys. A276, 402 (1977).
[7]Y. Sakamoto, Nucl. Phys. B10, 299 (1969).
[8]L. S. Kisslinger, Phys. Rev. 98, 761 (1955).
[9]E. Oset, Phys. Lett. 65B, 46 (1976).
[10]H. Feshbach, Ann. Phys. (N.Y.) 19, 287 (1962).
[11]H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. (N.Y.) 41, 230 (1967).
[12]N. Auerbach, J. Hüfner, A. K. Kerman, and C. M. Shakin, Rev. Mod. Phys. 44, 48 (1972).
[13]L. S. Kisslinger and W. L. Wang, Phys. Rev. Lett. 30, 1071 (1973), and Ann. Phys. (N.Y.) 99, 374 (1976).
[14]L. D. Roper, B. M. Wright, and B. T. Feld, Phys. Rev. 138, B190 (1965).

## Quadrupole Moment of the First $3^-$ State in $^{208}$Pb

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The $B(E3;\ 0^+ \rightarrow 3^-)$ and quadrupole moment, $Q_{3^-}$, of the first excited state of $^{208}$Pb have been measured by the reorientation effect in Coulomb excitation, giving $B(E3;\ 0^+ \rightarrow 3^-) = 0.665 \pm 0.035\ e^2 b^3$ and $Q_{3^-} = -0.42 \pm 0.32\ e\,b$. This value for $Q_{3^-}$ is much smaller in magnitude than those obtained by Barnett *et al.*, and is consistent with most theoretical predictions.

Few problems of nuclear structure physics have aroused as much attention in recent years as the dramatic discrepancy between the experimental and theoretical values for the static quadrupole moment $Q_{3^-}$ of the $3^-$ first excited state at 2.61 MeV in $^{208}$Pb. Using the reorientation effect in Coulomb excitation, Barnett and Phillips[1] reported a value of $Q_{3^-} = -1.3 \pm 0.6\ e\,b$ and, in a later measurement,[2] $Q_{3^-} = -0.9 \pm 0.4\ e\,b$ or $-1.1 \pm 0.4\ e\,b$, depending on the value assumed for $Q_{2^+}$ in $^{206}$Pb. With the exception of the work of Krainov,[3] theoretical calculations[4-9] give $Q_{3^-}$ between $-0.09$ and $-0.20\ e\,b$. Guidetti, Rowe, and Chow[7] have shown that it is very difficult to reproduce simultaneously the experimental $B(E3;\ 0^+ \rightarrow 3^-)$ and $Q_{3^-}$ with particle-hole models and models involving the coupling of a quadrupole phonon to particle-hole excitations. In addition, Hamamoto[6] and Bohr and Mottelson[8] have pointed out the inconsistency between the experimental value of $Q_{3^-}$ and the splitting of the $(h_{9/2} \otimes 3^-)$ septuplet in $^{209}$Bi.

The reorientation experiments of Barnett *et al.* involved the measurement of the excitation probability of the $3^-$ level for a number of scattering angles and/or projectiles. It has been acknowledged[10] that at least some of these measurements were performed at bombarding energies sufficiently high for Coulomb-nuclear interference to have a significant effect on the excitation probability, and hence on the value inferred for $Q_{3^-}$. In the first experiment of Barnett and Phillips,[1] $^4$He energies of 17.5 and 18.0 MeV, and an $^{16}$O energy of 69.1 MeV were used. In their later experiment,[2] the various heavy-ion projectiles had an energy of 4.15 MeV/$A$. In principle it is possible[10] to correct for the Coulomb-nuclear interference effect by assuming an appropriate nuclear potential. However, while this may be feasible for $^4$He data, there are serious difficulties in obtaining reliable potentials for heavy ions. Clearly it is preferable to perform the experiments at lower bombarding energies, where nuclear effects will be less important. This Letter reports such a measurement.

The major experimental difficulty lies in the rapid decline of the already small $E3$ excitation probability as the bombarding energies are reduced below those used in Ref. 1; in the present work, the excitation probabilities are a factor of 7 lower for $^4$He and a factor of 4 lower for $^{16}$O. We used an annular high-collection-field surface-barrier detector[11] at a mean laboratory scattering angle of 171.6°. An annular counter gives a large solid angle (42 msr), which is required in view of the small cross sections, and detection at 180° has the advantage of reducing the dependence of the excitation probabilities on scattering angle. The targets were 99.1% enriched $^{208}PbCl_2$ on thin carbon backings, the partial thickness of