Simple Model for Characterizing the Electrical Resistivity in A-15 Superconductors*

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A discussion of some of the difficulties with previous analyses of the resistivity of A-15 compounds is given. Precise high-temperature data on α-particle- and electron-damaged Nb_3Ge and Nb_3Sn samples with different defect concentrations are presented here and analyzed in a simple way with use of a phenomenological model based on the idea that the ideal resistivity must approach some limiting value in the regime where the mean free path becomes comparable to the interatomic spacing.

In recent years, the deviations from linearity (dfl) in the resistivity of transition-metal superconductors at high temperatures, and, in particular, those in the high-T_c A-15 superconductors has been a subject of active interest. Some years ago, Cohen, Cody, and Halloran[1] made a "Fermi smearing" model where they argued that if the Fermi level was within $\pm kT$ of sharp structure in the density of states, $N(E)$, then as the temperature was raised the Fermi level could be expected to shift into a region of lower density of states causing a dfl in the resistivity. A specific model was offered, which is now referred to as the RCA model. Recently, this approach has been used by Fradin and co-workers[2] at Argonne. Other work has emphasized that an exponential can be used to fit the data[3]; and most recently, the idea of saturation[4] has been introduced by Fisk and Webb[5] to explain the high-temperature behavior in A-15 materials.

We contend that "Fermi-smearing" models are unreliable because of the following. First, a recent calculation[6] has shown that the approximate solutions used in the "Fermi-smearing" models are inaccurate and, with a more accurate solution of the Boltzmann equation, the density of states at the Fermi level does not appear explicitly in the formula for the resistivity. Second, even with a more accurate solution of the semiclassical Boltzmann equation (SBE) and accurate band-structure information, the nonlinear $\rho(T)$ cannot be explained in Nb. The A-15 metals seem *not* to be fundamentally different in these respects from Nb. The calculated[7] shape of $N(E)$ shows sharp peaks in both cases, but *not* dramatically different in A-15 metals compared with Nb. Band theory provides little supporting evidence for the simplified model used in Refs. 1 and 2.

Because of these difficulties, which are further compounded by the problem of the short mean free path and the breakdown of the SBE, we find that the arguments given by Mooij[4] and by Fisk and Webb[5] at least provided a qualitative explanation of dfl.

Fisk and Webb[5] have recently extended arguments of Mooij to A-15 materials; and they suggest that at a resistivity near 135 $\mu\Omega$ cm the mean free path, l, becomes comparable to the interatomic spacing, and this essentially provides a maximum value for ρ. They call this effect "saturation." Since we find the presently proposed alternative models unacceptable, we are inclined to agree that something of this type is responsible for much or all of the anomalous behavior. We assume that we can describe $\rho(T)$ by a competition between an ideal dependence, ρ_{ideal}, and some limiting value given by $\rho_{\max}$ which is essentially "shunting" ρ_{ideal}. Then we write

$$\rho(T)/\rho_{\max} = A/(1+A), \tag{1}$$

where $A = \rho_{\text{ideal}}/\rho_{\max}$ and

$$\rho_{\text{ideal}}(T) = \rho_{\text{ideal}}(0) + \rho_{e\text{-ph ideal}}(T),$$

with $\rho_{e\text{-ph ideal}}(T)$, as in the usual case, approaching $\rho_1 T$ in the high-temperature limit. There are

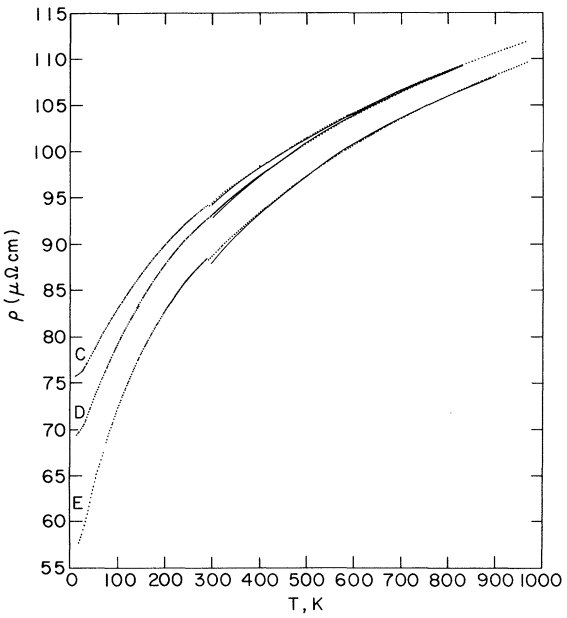

FIG. 1. ρ vs T for sample irradiated with about 3×10^{16} cm^{-2} α particles for three different annealing temperatures. Relevant fitting data for solid curves are given in Table I.

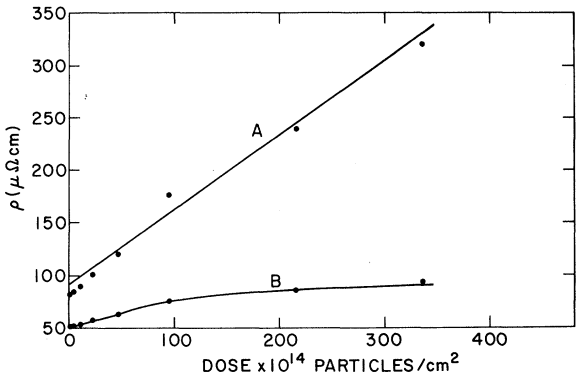

FIG. 2. Curve A, calculated $\rho_{\text{ideal}}(0)$ vs dose. Note that $\rho_{\text{ideal}}(0)$ is calculated from Eq. (2) and it is assumed that $\rho(0)$ is well approximated by the measured value which was taken at 25 K. Curve B, measured $\rho(25) \sim \rho(0)$ vs dose. The first six points were taken without warming the sample above about 77 K. The two highest-dose values of $\rho(25)$ were measured after a 10-h anneal at room temperature. In order to compare all the points in a consistent manner, these last two $\rho(25)$ points should be increased by about 3 $\mu\Omega$ cm to account for the room-temperature anneal.

essentially two arbitrary parameters in using Eq. (1) to fit the data, ρ_1: the coefficient of T and $\rho_{\max}$. It is interesting that Eq. (1) can also be written as two parallel resistors, i.e., $1/\rho(T) = 1/\rho_{\text{ideal}} + 1/\rho_{\max}$. $\rho_{\text{ideal}}(0)$ is given through Eq. (1) as

$$\rho_{\text{ideal}}(0) = \frac{\rho(0)\rho_{\max}}{\rho_{\max} - \rho(0)} \qquad (2)$$

and $\rho(0)$ is the measured residual resistivity. At our present level of understanding, we can think of $\rho_{\max}$ as essentially the limiting resistivity when l has become an interatomic spacing. Of course, in metals where $l \gg a$, e.g., copper, then $\rho_{\text{ideal}} \ll \rho_{\max}$ and $\rho(T)$ approaches ρ_{ideal}.

The experiments reported here[8] were performed by irradiating a ~8000-Å-thick Nb_3Ge film at cryogenic temperatures with 2.6-MeV α particles at the Brookhaven National Laboratory small van de Graaff accelerator. The data shown in Fig. 1 are for a sample damaged with a dose of 3×10^{16} cm^{-2} α particles. Successively higher temperature anneals were performed and the sample was allowed to come to equilibrium at the designated annealing temperature (Table I). Upon cooling, the ρ-vs-T curves were measured along with T_c. Shown in Fig. 1 are the three highest-temperature anneals, the last anneal being sufficient to recover 97% of the original T_c (19.6 K).

In Fig. 1 we show data on α-particle-damaged[9] Nb_3Ge, which illustrates how $\rho(0)$ increases, and also illustrates how the curves appear to flatten with increased damage. This general behavior has already been previously shown.[10,11] An analogous behavior to the dfl found with increasing temperature is also found if $\rho(0)$ is plotted against α-particle dose,[12] as shown in curve B in Fig. 2. In curve A, ρ_{ideal} is calculated from Eq. (2) and

TABLE I. Fitting parameters for data in Fig. 1.

Curve	Annealing temperature (K)	$\rho(0)$[a] ($\mu\Omega$ cm)	T_c (midpoint)	ρ_1 ($\mu\Omega$ cm/K)	$\rho_{\max}$ ($\mu\Omega$ cm)
E	1044	57.0	19.2	0.532	133.1
D	962	69.2	16.2	0.505	136.1
C	836	75.5	14.6	0.452	136.8

[a]Extrapolated from low-temperature data.

plotted against dose, where it is seen to be essentially linear. Hence in both cases, whether the disorder is due to phonons or defects, the saturation properties are apparently described by an equation of the form of (1). Similar behavior is obtained for Nb_3Sn. In Fig. 1, the solid lines are two-parameter fits to the data using Eq. (1), where ρ_1 and ρ_{max} are allowed to vary and $\rho(0)$ is estimated from $\rho(25)$. The values of ρ_1 and ρ_{max} are given in Table I.

Perhaps it is even more impressive that a fit to about 1% above 200 K can be obtained by taking some average ρ_1 and ρ_{max} ($\rho_1 \sim 0.56$, $\rho_{max} = 132$) for all three curves in Fig. 1, and then only changing $\rho(0)$, which is determined from the low-temperature data. This illustrates that the gross features of the data can be explained mainly by the variation of $\rho(0)$. We also mention that data for Nb can be fitted to less than 2% up to 2000 K by using Eq. (1) with $\rho_1 = 0.05$ $\mu\Omega$ cm/K and $\rho_{max} \sim 230$ $\mu\Omega$ cm. Furthermore, it should be mentioned that if the analysis on the basis of Eq. (1) is meaningful, analysis of the resistivity data at low temperatures would be modified in large-$\rho(0)$ samples.

In the case of Nb_3Sn, curves similar to Fig. 1 were taken for α-irradiated samples. In this case with a constant ρ_{max} and ρ_1, the curves up to an anneal of 600°C could be fitted to about 5% by just changing ρ_0, which varied from 12 $\mu\Omega$ cm in its original condition to 58 $\mu\Omega$ cm after irradiation and a 300°C anneal. There were problems with the quantitative fits in this data due to copper impurities introduced from the holder during the high-temperature anneals. Therefore, to illustrate the resistivity model, in Fig. 3 we show data for an unannealed Nb_3Sn sample in its original condition and after electron irradiation with 2.5-MeV electrons. The data can be fitted to better than 1% with a constant $\rho_{max} = 133$ $\mu\Omega$ cm and $\rho_1 \sim 0.572$ $\mu\Omega$ cm/K. In this case, ρ changes almost a factor of 2, and the good fit obtained by only changing ρ_0 nicely illustrates the saturation idea.

The idea of saturation has been discussed by Mott and Davis.[13] In the regime where $l > a$, they argue as Allen does,[6] that σ is not a function of N. However when $l \sim a$, they argue that l does not keep decreasing, and instead the resistivity becomes[14]

$$\rho = 12\pi^3\hbar[N(E_F)_{free}/N(E_F)]^2/S_F e^2 a. \quad (3)$$

This saturation value is temperature independent except for the dependence of $N(E)$ on T; the large phonon contribution is absent.

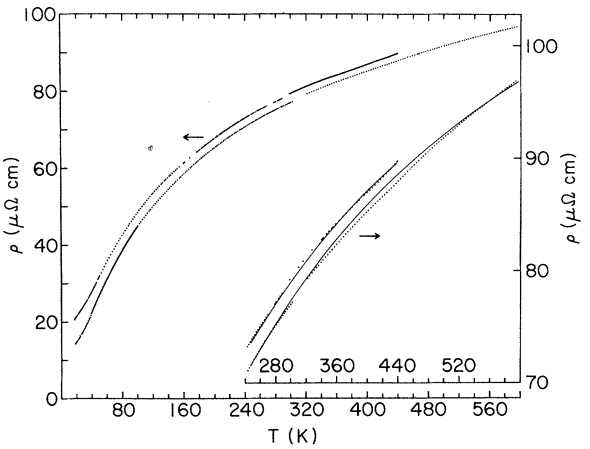

FIG. 3. Nb_3Sn sample in original condition and after irradiation with dose of about 10^{19} 2.5-MeV electrons. Inset shows fit using $\rho_{max} = 133$ $\mu\Omega$ cm and $\rho_1 = 0.57$ $\mu\Omega$ cm/deg. Solid curves are obtained by only changing ρ_0.

It should also be mentioned that, in our work and in the recent work of Testardi, Poate, and Levenstein,[10] data are shown where T_c is very low and $\rho(0)$ is greater than the ρ_{max} that we have used to fit our data. This is not surprising in the small-mean-free-path limit where large amounts of damage start drastically modifying the electronic properties and large changes of N are expected, which would raise ρ_{max} through Eq. (3) or some comparable result. Evidence that damaged samples with lower T_c's have smaller N has already been given,[11] and this is confirmed by critical-field measurements on the sample discussed here.

In concluding it is well to mention that the above scheme for analyzing resistivity data is not unique and, in fact, the approach to a constant ρ_{sat} can also be described nicely by assuming the conductivity goes as $1/T^n$. The saturation of $\rho(0)$ with dose can also be explained with a picture of independent defects becoming more complex as a result of the high damage levels. However, the relative simplicity with which Eq. (1) puts both phonon disorder and a structural disorder on a similar basis leads us to believe that there is some physical basis in this phenomenological approach.

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[14]See, in particular, Eq. (3.17) on p. 81 of Mott and Davis (Ref. 13) for the definition of S_F.

Series Study of a Spin-Glass Model in Continuous Dimensionality*

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A high-temperature series expansion for the Edwards and Anderson spin-glass order-parameter susceptibility is computed for Ising spins on hypercubic lattices with nearest-neighbor interactions. The series is analyzed by Padé approximants with Rudnick-Nelson–type corrections to scaling. The results agree with the first-order ϵ expansion of Harris, Lubensky, and Chen. The critical exponent γ_Q increases monotonically with decreasing dimension, d, for $d<6$, and apparently tends to infinity at $d=4$; however, the critical temperature does not appear to go to zero at $d=4$.

Recently, there has been great theoretical interest in various models for spin-glasses.[1-4] The Hamiltonian is usually of the form

$$\mathcal{H}=\sum_{i,j} J_{ij}\vec{S}_i\cdot\vec{S}_j, \tag{1}$$

where $\{\vec{S}_i\}$ are n-component vectors and the $\{J_{ij}\}$ are randomly distributed over some probability distribution which is assumed to be translationally invariant. Edwards and Anderson[1] (EA) have given a mean-field analysis for a model in which the probability distribution for the $\{J_{ij}\}$ is symmetric, so that $[J_{ij}]_{\rm av}=0$, where $[\ ]_{\rm av}$ denotes a configurational average over the probability distribution of the $\{J_{ij}\}$. The state of spin-glass order is characterized by the conditions

$$M(\vec{q})=\left[\lim_{N\to\infty}\frac{1}{N}\left|\sum_{i=1}^{N}\exp(i\vec{q}\cdot\vec{r}_i)\langle\vec{S}_i\rangle\right|\right]_{\rm av}=0 \tag{2}$$

for all $\vec{q}$, and

$$Q^2=\left[\lim_{N\to\infty}\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}Q_{ij}{}^2\right]_{\rm av}\neq 0, \tag{3}$$

where Q_{ij} is defined by

$$Q_{ij}{}^2=\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\langle S_i{}^{\alpha}S_j{}^{\beta}\rangle^2, \tag{4}$$

where α and β label spin components. The first condition, Eq. (2), indicates that there is no long-range ferromagnetic or antiferromagnetic order,