Current Instability and Domain Propagation Due to Bragg Scattering*

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We propose a new mechanism for a current instability in single-valley semiconductors based on the momentum loss of hot carriers by Bragg reflection. We construct a simple model which gives rise to bulk negative differential conductivity of the uniform current state above a critical field E_c associated with a soft dielectric relaxation mode, and to traveling dipole-domain solutions. The possibility for this mechanism to occur in realistic situations is discussed.

We present a new mechanism for a current instability in single-valley semiconductors based on momentum loss of hot carriers by Bragg scattering. With increasing applied field E, the carrier distribution becomes broader and tends to fill the whole Brillouin zone (BZ).[1] Consequently, because of the acceleration by E, an increasing number of carriers reaches the BZ boundary per unit time where they suffer a momentum loss equal to a reciprocal lattice vector K. We show that this mechanism leads to a bulk negative differential conductivity (BNDC) and therefore to an instability of the uniform-current state. In contrast to the Gunn effect, this mechanism is universal and does not depend on peculiarities of the band structure.

There is strong interest in current instabilities as phase-transition-like cooperative phenomena in nonequilibrium systems. A bulk instability in n-type GaAs caused by intervalley scattering of hot carriers into low-mobility valleys was discovered by Gunn,[2,3] who found propagating dipole domains at high fields. This problem was treated theoretically by several authors.[4-9] Some of this work[4-7] rests on purely macroscopic concepts such as field-dependent drift velocity and diffusion constant, and does not depend at all on the mechanism causing the BNDC. Such an approach is unsatisfactory both in principle, because it assumes a field dependence which should be the result of the theory, and in practice, becuase it is doubtful whether the macroscopically determined field dependence is valid in the highly nonuniform field of the nonstationary state. This objection does not apply to a treatment based on phenomenological balance equations for local state variables[8,9] which is also used in the present work.

We consider a nondegenerate, n-type, single-valley semiconductor, and assume that the current states of interest can be described in terms of the carrier density n, the momentum density $n\vec{p}$, and the energy density nw of the carriers for all x, t. Current transport is then governed by the continuity equation

$$\dot{n} + \nabla(nv) = 0,$$

and by balance equations for momentum and energy fields:

$$\dot{\vec{p}} + \vec{v}\cdot\nabla\vec{p} + (\kappa/n)\nabla n = be\vec{E} - \gamma_p\vec{p}, \tag{1b}$$

$$\dot{w} + \vec{v}\cdot\nabla w = e\vec{v}\cdot\vec{E} - \gamma_w(w - w_0). \tag{1c}$$

Here, γ_p and γ_w are relaxation rates describing exchange of momentum and energy with the lattice due to ordinary scattering processes, w_0 being the electronic energy at equilibrium. The lattice is assumed to be kept at constant temperature. We have assumed an electronic stress tensor $\Pi = \kappa n$, and have neglected momentum diffusion in (1b) and electronic heat conduction in (1c) as compared to the relaxation terms. The momentum balance is driven by the acceleration term $e\vec{E}$, and the energy balance by Joule heating $e\vec{v}\cdot\vec{E}$ of the carriers. Bragg momentum loss reduces the acceleration term by a factor b: During dt, a fraction $f(k_b)e\vec{E}\cdot d\vec{S}_k dt/n$ reaches the BZ boundary element $d\vec{S}_k$ at k_b where they lose momentum K (Fig. 1). Therefore, the momentum gain $\dot{\vec{p}}_{\mathrm{field}} = e\vec{E}$ is reduced by a factor

$$b = 1 - n^{-1}\int_{E_n > 0} Kf(k_b)dS_k^{\perp} \leq 1.$$

The value $f(k_b)$ will be mainly determined by the width of the distribution function $f(k)$ and therefore by the carrier energy w yielding $b = b(w)$. For reasons of simplicity and definiteness, we approximate this function by a linear variation $b(w) = (w_m - w)/(w_m - w_0)$ in the region of interest. For the average drift velocity we assume $\vec{v} = \vec{p}/m$ with constant effective mass m.

The local electric field $\vec{E}$ is coupled to the carrier density by Poisson's equation

$$\epsilon_L \nabla\cdot\vec{E} = 4\pi e(n - n_D), \tag{1d}$$

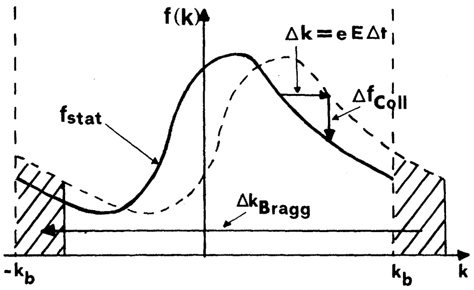

FIG. 1. Change of distribution function due to acceleration by the field E, to ordinary scattering processes (Δf_{coll}), and to Bragg scattering.

where ϵ_L is the lattice dielectric constant and n_D is the donor density.

Setting all derivatives equal to zero in (1a)–(1d) we find a static j-E characteristic of the form

$$\vec{j} = (n_D e^2/\gamma_p m) b_s \vec{E}_s, \tag{2a}$$

$$b(E_s) \equiv b_s = 1/[1 + (E_s/E_c)^2], \tag{2b}$$

(subscript s for static), which has BNDC above a critical field

$$eE_c = [m\gamma_p\gamma_w(w_m - w_0)]^{1/2}. \tag{2c}$$

At $E > E_c$, the momentum gain $be\,\delta E$ due to an incremental field δE is overcompensated by the Bragg loss $eE\,\delta b$ caused by the additional Joule heating. It should be noted that this mechanism is different from that of a negative effective-mass instability which requires that m_{eff}^{-1} averaged over the distribution takes on negative values.[10] The instability found by Schlup[11] for a simple periodic $E(k)$ band in a Boltzmann-equation approach with relaxation approximation, on the other hand, is probably caused by the mechanism presented here.

A mode analysis (frequency ω, wave vector q) of the uniform state yields drifting relaxational modes with frequency

$$\omega_q = -i\Gamma + v_s q - iDq^2 + O(\Gamma^2, q^2\Gamma, q^4). \tag{3}$$

Thus, a drifiting charge distribution decays by two competing processes, dielectric relaxation with rate

$$\Gamma = \frac{(\omega_p^2/\gamma_p)(2b_s - 1)b_s}{1 + (\omega_p^2/\gamma_p\gamma_w)b_s^2}, \tag{4}$$

and carrier diffusion with diffusion constant

$$D = \frac{(\kappa/m\gamma_p)b_s}{1 + (\omega_p^2/\gamma_p\gamma_w)b_s^2}, \tag{5}$$

where $\omega_p = (4\pi e^2 n_D/\epsilon_L m)^{1/2}$ is the plasma frequency. For E increasing toward E_c one finds Γ decreasing toward 0 whereas D stays positive. The uniform state thus becomes unstable at E_c against a soft dielectric relaxation mode.[9] The Debye length

$$l_D = \left(\frac{D}{\Gamma}\right)^{1/2} = \left\{\frac{\kappa/m\omega_p^2}{2b_s - 1}\right\}^{1/2}, \tag{6}$$

diverges for E increasing toward E_c indicating that the system loses its capability to screen long-wavelength charge fluctuations.

We search for one-dimensional traveling-pulse solutions of (1a)–(1d) which depend only on $Z = x - ut$ with u = pulse velocity ("solitary" solutions). We find localized dipole-domain solutions traveling with velocity $u = v_s(E_{as})$ where $E_{as} < E_c$ is the uniform field at large distance from the domain. The form of the domain is given by

$$p = mv_s(E_{as}), \tag{7a}$$

$$\frac{(n - n_D)}{n_D} - \ln\left(\frac{n}{n_D}\right) = \frac{1}{2}\left(\frac{l_d}{l_D}\right)^2 \left(\frac{\Delta E}{\Delta E_m}\right)^2 \left[1 - \left(\frac{\Delta E}{\Delta E_m}\right)\right], \tag{7b}$$

$$w = w_{as} + (w_{as} - w_0)\Delta E/E_{as}, \tag{7c}$$

$$\left(\frac{1}{n}\right)\frac{dn}{dZ} = \left(\frac{l_d}{l_D^2}\right)\left(\frac{\Delta E}{\Delta E_m}\right)\left[1 - \left(\frac{3\Delta E}{2\Delta E_m}\right)\right], \tag{7d}$$

where $\Delta E = E - E_{as}$ is the excess field in the domain with maximum $\Delta E_m = \frac{3}{2}(E_c^2 - E_{as}^2)/E_{as}$, and $l_d = \epsilon_L \Delta E_m / 4\pi e n_D$ is a length equal to the thickness of a depletion layer sufficient to screen the maximum excess field ΔE_m. The n-E characteristic (7b) is displayed in Fig. 2(a), and Fig. 2(b) shows the density field $n(Z)$ obtained by integrating (7d).

For E_{as} close to E_c, one obtains a diffusion-

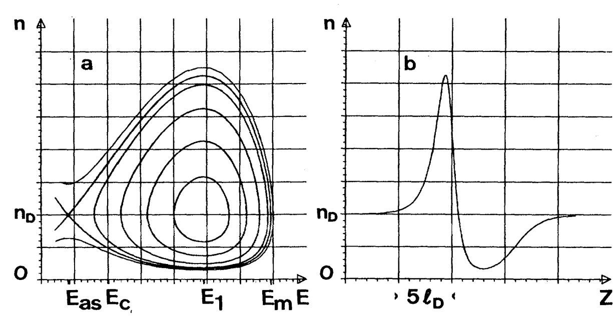

FIG. 2. (a) n-E characteristics of traveling-wave solutions of (1a)-(1d). The solitary solution separates periodic solutions (closed curves) and unbounded solutions (open curves). (b) Carrier density of the dipole domain.

limited small-amplitude domain (SAD) with a width determined by the Debye length $l_D(E_{as})$ which diverges at E_c. The excess field can be given in analytic form:

$$\Delta E(Z) = \Delta E_m \operatorname{sech}^2(Z/2l_D) + O(\Gamma^{3/2}). \tag{8}$$

As E_{as} departs from E_c, the amplitude increases and the domain width decreases. In the large-amplitude domain (LAD), on the other hand, the leading edge is completely depleted, and the domain width is determined by the depletion length l_d which increases with decreasing E_{as}. The minimum width occurs for $l_d \sim l_D$.

For the excess voltage $\Delta V = \int \Delta E(Z) dZ$ we obtain

$$\Delta V = 4l_D \Delta E_m \sim (E_c - E_{as})^{1/2} \text{ for SAD,} \tag{9a}$$

$$= \tfrac{1}{2} l_d \Delta E_m \sim E_{as}^{-2} \text{ for LAD.} \tag{9b}$$

The dynamic current-voltage characteristic can be constructed from the excess voltage[6] and depends strongly on the ratio of the domain width to the sample length.

In order to test the stability of the domain solutions, one studies the time dependence of small deviations $\delta\varphi(Z,t) = \delta\varphi(Z)\exp(-\lambda t)$ $(\varphi = n, p, w, E)$. For infinite external impedance, $Z^{\text{ext}} = \infty$, the boundary condition is constant total current $j + \epsilon_L \dot{E}/4\pi$. Because of translational invariance, there exists always an eigenvalue $\lambda = 0$ with eigenfunction $\partial\varphi_{\text{domain}}/\partial Z$. In order for the domain solution to be stable, all other eigenvalues have to be positive.

For the SAD we find the eigenvalue problem

$$l_D^2 \delta E''(Z) - \{1 - 3\operatorname{sech}^2(Z/2l_D) - \lambda/\Gamma\} \delta E = 0, \tag{10}$$

with discrete spectrum $\lambda_0 = -5\Gamma/4$, $\lambda_1 = 0$, $\lambda_2 = 3\Gamma/4$, and a continuous spectrum $\lambda_q = \Gamma + Dq^2 \geqslant \Gamma$. Thus, at $Z^{\text{ext}} = \infty$, the SAD is unstable.[7] We have shown that it can also not be stabilized by an external circuit. Preliminary results of a full stability analysis for arbitrary amplitudes show that the solution is stable above some critical amplitude, such that a first-order transition occurs at E_c to a LAD.

With $w_m \sim$ conduction-band width ~ 1 eV, $m \sim 0.1$ eV, $\gamma_p \sim \gamma_w \sim 10^{13}$ sec^{-1}, the critical field $E_c \sim 10^5$ V/cm of a pure semiconductor is rather high. Better candidates for observing this instability are possibly the semiconductor superstructures studied by Esaki and Chang[12] and Dingle, Wiegmann, and Henry,[13] where the artificial spatial periodicity causes a splitting of the conduction band into a number of sub-bands separated by energy gaps. The current instabilities observed in these structures[12] have been interpreted in terms of a different mechanism based on tunneling through the gap Δ into the next higher sub-band. This mechanism requires phase coherence of the electron wave function over a distance Δ/eE equal to the spatial extension of the gap, whereas our approach is valid for (electron mean free path) $\ll \Delta/eE \ll l_D$. Thus, by studying the dependence on the mean free path, one may decide which of the two mechanisms causes the instability.

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