By introducing the notation $h_\nu{}^i f_{i\beta}{}^\alpha = (\gamma_\nu)_\beta{}^\alpha$ and $\eta^{ij} h_i{}^\nu f_{j\beta}{}^\alpha = (\gamma^\nu)_\beta{}^\alpha$, the last two equations can be rewritten as

$$(\gamma_\nu)_\beta{}^\alpha \tilde{R}^{*\beta\mu\nu} = 0 \; [\text{or } (\gamma^\nu)_\beta{}^\alpha R_{\mu\nu}{}^{*\beta} = 0], \qquad (26)$$

$$R_\mu{}^{*\lambda} = \tfrac{1}{4}\chi h_\nu{}^\alpha (\gamma_5\gamma_\mu)_{\alpha\beta} \tilde{R}^{*\beta\lambda\nu}, \qquad (27)$$

where $\tilde{R}^{*\beta\mu\nu} = (1/2h)\epsilon^{\mu\nu\lambda\sigma} R_{\lambda\sigma}{}^{*\beta}$ and $h = \det(h_\mu{}^i)$.

The complete algebra of infinitesimal transformations that leave the action I^* invariant will be presented in a subsequent publication. This approach to gravitation and supergravity clearly exhibits the intimate connection between the two theories. Indeed, it becomes apparent that they have exactly the same geometric structure, differing only on the choice of gauge groups for the bundle spaces.

We remark that our formulation of the theories of gravitation and supergravity in the absence of matter fields is done completely in terms of the components of the curvature tensor without introducing a metric tensor, that is, it depends on the geometric properties of the affine bundle spaces. Of course, one can introduce a metric tensor $g_{\mu\nu} = h_\mu{}^i h_\nu{}^j \eta_{ij}$ which will be invariant under parallel transport. Such a tensor might actually be needed when matter fields are incorporated into the theory. It will be interesting to speculate on the possibility of extending this geometrical formulation of supergravity to spaces with more complex structures, including internal degrees of freedom for the matter fields.

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Threshold Electroproduction of Charged Pions from Light Nuclei

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Double-differential cross sections for the electroproduction of pions of both charges have been measured. We compare the data obtained for production near threshold from ^{12}C and ^{16}O with theoretical treatments employing both shell-model and sum-rule nuclear descriptions with full inclusion of the final-state interaction.

Recent results for low-energy elastic scattering of pions off nuclei[1] have confirmed that standard Kisslinger optical potentials,[2] used successfully to correlate the strong-interaction shifts and level widths of pionic atoms,[3] yield cross section predictions that disagree sharply with experiment. Although there is some indication that improved treatments of kinematic effects and short-range correlations may substantially reduce the discrepancies,[4] the need for further, independent probes of the pion wave function in nuclear matter is clear.

The modification of threshold electroproduction cross sections for complex nuclei by the final-state interaction of the emitted pion provides one such test of the Kisslinger potential. Experimental studies indicate that we have a reasonably good understanding of electroproduction, as well as production by real photons, of low-energy pions from the single nucleon.[5] The elementary amplitude for threshold production can be described through order m_π/M_N by the soft-pion theorems arising from the partially conserved axial-vector current (PCAC) hypothesis and from current algebra.[6] Thus the quality of information that can be extracted on the strong final-state interaction, under the distorted wave impulse approximation, depends largely on our ability to properly treat both the elementary amplitude when imbedded in a complex nucleus and the formidable structure uncertainties arising when transitions to many final nuclear states may be important.

We have reasured differential cross sections for pions produced at angles ranging from 32° to 136° from the incident electron beam and with kinetic energies of 21.6 and 30.25 MeV. The beam energy was 280 MeV; and the targets used were ^{12}C(85 mg/cm^2), BeO(139 mg/cm^2), and Be(96 mg/cm^2). The detector system, resting in the focal plane of a double-focusing 180° spectrometer, consists of three plastic scintillators and a plastic Cherenkov counter and has a momentum acceptance of 0.56%. The dimensions of the three scintillators are suitable for pions of kinetic energy above 20 MeV. For detection of the 30.25-MeV pions, thin (2 mm) aluminum moderators were inserted between the scintillation counters. With these detector configurations we were able to achieve good separations between electrons (positrons) and pions in the pulse-height spectra of the different scintillation counters.

A necessary condition for the detection of a pion is the registering of a signal with the proper pulse height in each of the three counters. Electrons are rejected by a veto from the Cherenkov counter, which however is transparent to pions with kinetic energy below 45 MeV. The efficiency of the detecting system was calculated with a Monte Carlo code that took into account the pion energy loss and multiple scattering within the detector materials. This was found to be 0.94 and 0.93 for 21.6-MeV and 30.25-MeV pions, respectively. Pion absorption in the detecting materials, estimated from the absorption cross section,[7] reached values of up to 2%.

The counting rates were corrected for real bremsstrahlung pion production, pion decay, and muon contamination. The contribution of bremsstrahlung pion production, determined from measurements taken with different target thicknesses, was found to range up to 10%. With the distance between the target and the detector in the magnetic spectrometer being 4.60 m, we achieved low background contamination at the expense of appreciable probabilities for pion decay prior to reaching the detecting system. Our computations yielded probabilities of 64% and 57% for 21.6-MeV and 30.25-MeV pions, respectively. The background was determined to be less than 1% from measurements taken with the spectrometer magnetic field adjusted to exclude contributions from pions, but with all other experimental conditions changed as little as possible. In the present energy range it is generally not possible to separate pion signals in the pulse-height spectra from those of the muons produced during pion decay. Only a portion of these muons, however, have both the proper energy and direction to be recorded in the detector. A Monte Carlo code, which included the influence of the magnetic field of the spectrometer by a ray-tracing method, estimated that the muon contribution to the total counting rate was less than 5%.

The double-differential cross section for pion electroproduction, accompanied by the excitation of the target nucleus, is given by[8,9]

$$\frac{d^2\sigma}{d\omega_\pi d\Omega_\pi}=\frac{\alpha^2}{(2\pi)^3}\int d\epsilon_2 d\Omega_2\frac{s_2}{s_1}\frac{9}{k^4}\,\delta(\epsilon_1-\epsilon_2-\omega_\pi-E_x)[(\vec{s}_1\cdot\vec{M})(s_2\cdot\vec{M}^*)+(\vec{s}_2\cdot\vec{M})(\vec{s}_1\cdot\vec{M}^*)-\tfrac{1}{2}k^2\vec{M}\cdot\vec{M}^*], \tag{1}$$

with $(\vec{s}_1,\epsilon_1)$ and $(\vec{s}_2,\epsilon_2)$ the four-momenta of the initial and scattered electrons, $(\vec{q},\omega_\pi)$ the pion four-momentum, $k^2=\omega_k^2-\vec{k}^2$ the square of the four-momentum transferred to the hadronic system, and E_x the nuclear excitation energy. The integrations extend over the energy and angle of the unobserved final electron. The matrix element of the electroproduction operator $\hat{M}$ taken between initial and final states is denoted by $\vec{M}$ in Eq. (1). Employing the distorted wave impulse approximation, we can write $\hat{M}$ as a one-body operator in second quantization,

$$\hat{M}=\sum_{\alpha,\beta}a_\alpha{}^\dagger a_\beta\langle\alpha|e^{i\vec{k}\cdot\vec{x}}\varphi^*(\vec{q},\vec{x})\vec{M}_{sn}|\beta\rangle, \tag{2}$$

with φ^* the outgoing distorted pion wave, with $\vec{M}_{sn}$ the operator appropriate for production off the single nucleon, and with the sums extending over complete sets of single-particle quantum numbers. The threshold single-nucleon operator derived from a low-energy theorem resulting from PCAC and current algebra can be written as an expansion in k^2, with the leading term being the photoproduction operator

$$\vec{M}_\gamma^{(\pi^\pm)} = (1 \mp m_\pi/2M_N)[\vec{\sigma} - \vec{k}(\vec{\sigma}\cdot\vec{k})/\omega_k^2]. \tag{3}$$

We have evaluated the angular integral in Eq. (1) in the "peaking" approximation, in which the dominance of those contributions to the differential cross section in which the electron is scattered in the forward direction is exploited. In this approximation only the first term in the operator expansion, $\vec{M}_\gamma$, contributes. Both this approximation and the use of the threshold operator have been shown to be appropriate in the kinematic region of present interest.[9]

The pion wave function needed in Eq. (2) is taken from the solution of a modified Klein-Gordon equation, with the pion-nucleus potential, consisting of Coulomb and strong-interaction pieces, treated as the fourth component of a four-vector.[10] The Coulomb potential is evaluated for an extended nuclear charge, while the strong-interaction potential is described by the standard nonlocal Kisslinger potential. The explicit form of this potential and the parametrizations that follow from studies of pionic atoms are given in Refs. 2 and 3.

Two approaches to handling the nuclear structure aspects of electroproduction have been pursued. The large amount of energy available for transfer to the nucleus ($\approx$110–120 MeV) suggests that a sum-rule approach, in which a summation over a complete set of final states is performed once a mean excitation energy is defined, would be appropriate. In the present work, the mean excitation energy has been taken from the ratio of the energy-weighted sum rule to the non-energy-weighted sum rule.[11] The cross section can then be expressed in terms of the spin-isospin correlation function and, in the limit of no final-state interaction, is proportional to its Fourier transform $C(\vec{t})$, where $\vec{t} = \vec{k} - \vec{q}$ is the three-momentum transferred to the nucleus. The correlation function has been evaluated using harmonic-oscillator wave functions. The energy-weighted sum rule has been calculated from the usual double commutator of the transition operator and the nuclear Hamiltonian. Using an effective interaction of the Skyrme type to represent the two-body term in the Hamiltonian, we find that this contribution can be incorporated into the kinetic term through an effective mass.[12]

The second approach involves a direct summation of transition strengths to all kinematically allowed shell-model states. Wave functions for the low-lying isospin $T = 1$ levels were taken from Tamm-Dancoff configurations,[13] while the higher level excitations were represented as simple j-j coupling states. We feel that the more realistic treatment of phase-space restrictions in a direct summation provides a valuable check on the range of validity of the sum rule.

In Fig. 1 the experimental double-differential cross sections for production of 21.6-MeV negative pions off ^{12}C and ^{16}O are compared to theoretical predictions. The calculations were performed with five pion partial waves and include the effect of the p-wave interference term[12,14] in addition to the threshold operator of Eq. (3). The error bars shown represent only the statistical experimental fluctuations (the normalization error is approximately 5%). Although the measured angular behavior is reproduced by both the sum-

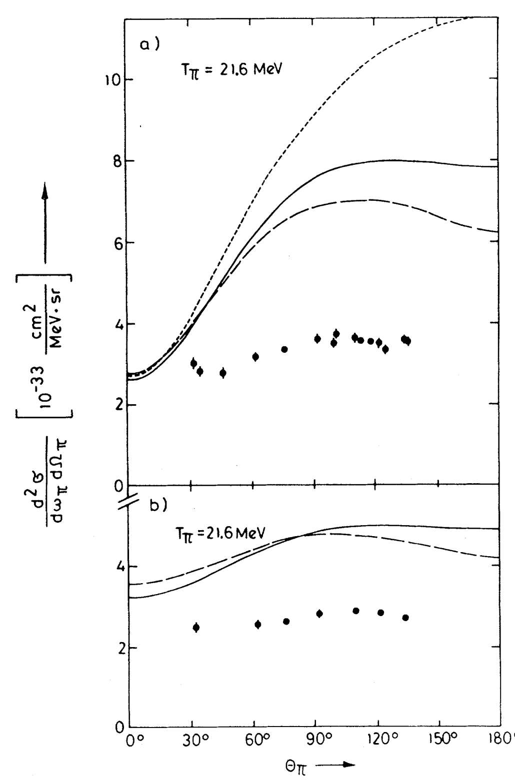

FIG. 1. Double-differential cross sections for production of negative pions by 280-MeV electrons from (a) ^{16}O and (b) ^{12}C as a function of the pion angle. The shell-model results (solid line) are compared with the predictions of the sum-rule results when evaluated with (dashed line) and without (dotted line) the effective two-body Skyrme II (Ref. 14) interaction.

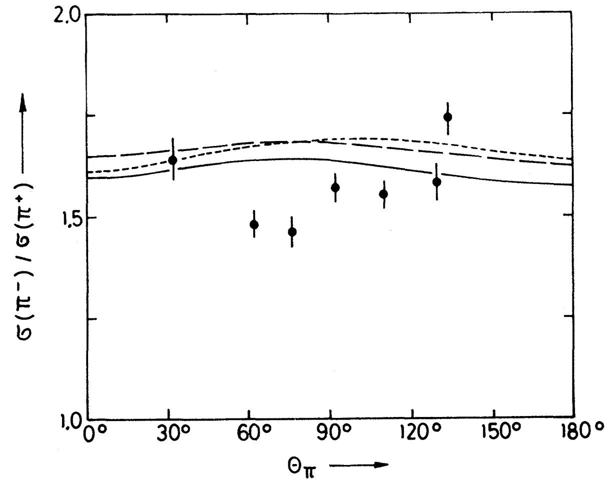

FIG. 2. The ratio of the double-differential cross sections for production of negative and positive 21.6-MeV pions from ^{12}C as a function of the pion angle. The dashed and solid curves are the shell-model results that follow on using a pion optical potential parametrized (Ref. 2) with and without the Lorentz-Lorenz effect, respectively. The dotted line is the Skyrme II sum-rule result with no Lorentz-Lorenz effect.

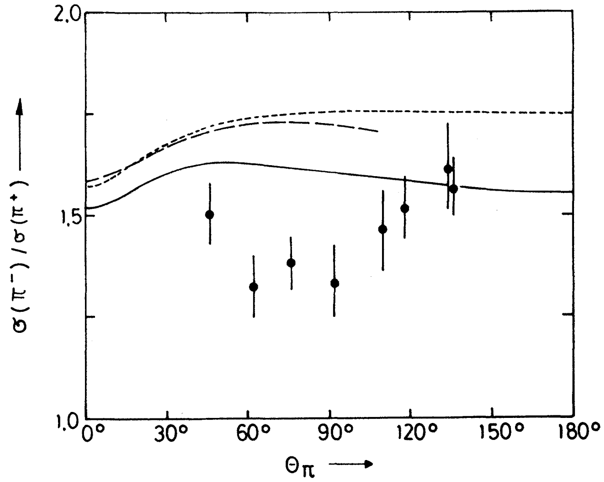

FIG. 3. Same in Fig. 2, but for production of 30.25-MeV pions from ^{16}O. The theoretical curves are as in Fig. 1. The sum-rule calculation with the Skyrme II interaction breaks down for angles above 110°, as discussed in Ref. 15.

rule and the shell-model calculations, each yields a result approximately a factor of 2 too high.[15] The strikingly different small-angle behavior of the cross sections for ^{16}O and ^{12}C is due to the presence of a strong forward-peaked spin-flip transition within the $1p$ shell for the latter nucleus. The two curves shown for the sum-rule treatment correspond to the energy-weighted sum rule evaluated with only the kinetic term in the nuclear Hamiltonian ($M^*=M$) and with the Skyrme II interaction of Ref. 14 ($M^*=0.63M$). Skyrme interactions fitted to ground-state properties of nuclei can give widely varying effective masses. The present choice, and the rather low corresponding effective mass, gives a relatively good description of deep-lying single-particle levels which we expect to contribute importantly in electroproduction by 280-MeV electrons.

In Figs. 2 and 3, we show the ratio of the π^- to π^+ differential cross sections for production of 21.6-MeV and 30.25-MeV pions from ^{12}C and ^{16}O, respectively. We feel that investigating such ratios may prove to be the most realistic approach for extracting information on the strong final-state interaction. While little sensitivity to the final-state interaction is lost by forming the ratio due to the differing signs for the strong-Coulomb interference, we expect that many of the nuclear structure uncertainties plaguing total-cross-section predictions may tend to cancel. Figure 2 illustrates the sensitivity of the ratio to optical potentials parametrized with and without the Lorentz-Lorenz term. Work on the effects of Pauli and short-range correlations has advocated the use of strong Lorentz-Lorenz terms in low-energy pion optical potentials.[4] Much more complete examinations of the sensitivity of electroproduction cross-section predictions to variations in the optical potential will be given elsewhere.[16] We note that the depression in the experimental ratios at intermediate angles is not reproduced by the theoretical curves shown in Figs. 2 and 3. It perhaps should not be overlooked that some approximations have been made that could affect π^- and π^+ cross sections differently. Recent work on threshold pion production has emphasized that terms of higher order in the pion momentum can make important contributions to the threshold-production operator if the local pion momentum, as opposed to the asymptotic momentum, is used.[17]

We are presently preparing to expand our measurements to provide cross sections for a much more extensive range of pion kinetic energies below 50 MeV. We feel that the kinematic region closer to threshold may be particularly interesting due to the increased strength of the strong-Coulomb final-state interference. With the present calculations indicating that the nuclear structure difficulties are most easily manageable at low pion-emission angles, effort also will be expended to determine π^-/π^+ ratios closer to the forward direction. We intend to provide cross sections both for additional doubly even nuclei and for targets with $N \neq Z$. In the latter nuclei the contributions of isovector and isotensor components of the spin-isospin correlation functions can be investigated.

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Charge Exchange of Stopped π^- in Deuterium: Experiment and Theory*

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The branching ratio for the reaction $\pi^- d \to \pi^0 nn$ has been measured for the first time by stopping the π^- in a liquid-deuterium target and detecting the two decay γ's in two large NaI detectors. A branching ratio (B.R.) $= \omega(\pi^- d \to \pi^0 nn)/\omega(\pi^- d \to \text{all})$ of $(1.45 \pm 0.19) \times 10^{-4}$ has been measured; the predicted B.R. lies in the range 1.39×10^{-4} to 1.59×10^{-4}.

The charge-exchange reaction (π^-, π^0) for stopped negative pions is known[1-6] to occur for only two targets, ^{1}H and ^{3}He, although it is energetically allowed for most medium-heavy-mass nuclei. Several searches for the reaction $\pi^- d \to \pi^0 nn$ have been conducted using either counters[4,5] or internal conversion in a bubble chamber[6] to detect the π^0 decay γ's, and upper limits of 2×10^{-3} and 1.2×10^{-3}, respectively, for the branching ratio (B. R.) has been established. The strong suppression of this reaction was used to prove[4] that the π^- and π^0 were both pseudoscalar mesons ($J^P = 0^-$). Since the initial capture occurs only from s orbits, J^P initial $= 1^-$ and therefore conservation of angular momentum and parity requires a final state $= {}^3P_1(2n)$, p wave (π^0) if the π^0 has $J^P = 0^-$, and this would suppress the reaction. If, however, the π^0 has $J^P = 0^+$, then a ${}^3P_1(2n)$, s-wave (π^0)