(1962).
[6]A. Narath and D. W. Alderman, Phys. Rev. 143, 328 (1966).
[7]F. Y. Fradin, J. W. Downey, and T. E. Klippert, Mater. Res. Bull. 11, 993 (1976).
[8]The field penetration is essentially complete in the superconducting state of these strongly type-II Chevrel phases; e.g., see R. W. McCallum, D. C. Johnston, S. N. Shelton, and M. B. Maple in *Superconductivity in d- and f-band Metals*, edited by D. H. Douglass (Plenum, New York, 1976), p. 625.
[9]Part of the shift at ^{95}Mo may be due to spin transfer via molecular orbitals. However, this mechanism would not be effective in enhancing H_{c_2}.
[10]J. M. Baker and F. I. B. Williams, Proc. Roy. Soc. London, Ser. A 267, 283 (1962).
[11]Contributions to the hyperfine field from transferred effects due to the other Eu ions will not be significant here since the Eu-Eu distance is very large (~ 6.5 Å). In addition, there may also be a contribution through core polarization from polarized *d* electrons, but this is generally much smaller than that arising from the contact term.
[12]I. Nowik, B. D. Dunlap, and J. H. Wernick, Phys. Rev. B 8, 238 (1973).
[13]F. Gonzalez-Jimenez, P. Imbert, and F. Hartmann-Boutron, Phys. Rev. B 9, 95 (1974); H. H. Wickman and G. K. Wertheim, in *Chemical Applications of Mössbauer Spectroscopy*, edited by V. I. Goldanskii and R. H. Herber (Academic, New York, 1968), p. 548.

Influence of a One-Dimensional Superlattice on a Two-Dimensional Electron Gas*†

Teresa Cole,‡ Amir A. Lakhani,§ and P. J. Stiles‡‖
Physics Department, Brown University, Providence, Rhode Island 02912
(Received 13 December 1976)

We report unusual structure in the dc conductivity below 4.2 K as well as nonperiodic oscillations in the magnetoconductance of a two-dimensional electron gas in Si at low magnetic fields. A model of a superlattice with a gap which varies with carrier density is proposed to interpret the results. The superlattice appears to be of a somewhat general nature although the cause is not yet known.

The recent interest in quasi two-dimensional systems[1] and artificial superlattices[2] stems in a large part from the fact that one may essentially build new physical systems. In the former case one can reversibly change the carrier density or Fermi energy, while in the latter the Brillouin zone and the band structure are modified.

We have studied Si metal-oxide-semiconductor field-effect transistors (MOSFET's) which exhibit behavior different from that usually observed. These observations can be explained within the framework of a model of a superlattice structure at the Si-SiO_x interface. We expect this behavior to occur in general in a certain class of MOSFET systems. These results, therefore, illustrate the approach that many textbooks take in explaining lattice effects and Fermi surfaces.[3]

The consequences of a one-dimensional superlattice can be most easily seen by considering an isotropic two-dimensional electron gas (2D EG) in the presence of a one-dimensional potential with a first Fourier component of

$$V_x = V_1[\exp(2\pi ix/a) + \exp(-2\pi ix/a)].$$

This potential will mix states with k vectors differing by the reciprocal lattice vector $2\pi/a$. For k near π/a, first-order degenerate perturbation theory gives energies $\epsilon = \epsilon_x + \epsilon_y$ with $\epsilon_y = \hbar^2 k_y^2/2m$ and

$$\epsilon_x = (\hbar^2/2m)(k_x - \pi/a)^2 + \epsilon_0\{1 \pm [(4\hbar^2/2m\epsilon_0)(k_x - \pi/a)^2 + \alpha^2]^{1/2}\},$$

where $\epsilon_0 = (\hbar^2/2m)(\pi/a)^2$, and $\alpha = V_1/\epsilon_0$. At $k_x = \pi/a$, we have $\epsilon_x = \epsilon_0(1 \pm \alpha)$, i.e., there is an energy gap of $2V_1$ at the zone boundary.

Some properties of this system will be a strong function of Fermi energy (i.e., n_s). For small values of α and low n_s and also for very large n_s the behavior should be similar to that for the usual 2D EG. As the Fermi energy approaches the top of the first band, the density of states becomes very large and then decreases when the Fermi energy is in the gap. There is a discontinuous increase in the density of states when the Fermi energy enters the second band.

Because of the behavior of the density of states, we expect a w-shaped structure in the conductivity. The conductivity decreases near the band edges because of the increase in scattering caused by the increase in the density of states. In the y direction the conductivity with ϵ_F in the gap should increase with increasing n_s as the scattering is

decreasing and the y component of the velocity is increasing. As the x component of the velocity is decreasing, the effect is diminished in the x direction.

In addition to the usual low-field magnetoconductance and Shubnikov–de Haas (SdH) oscillations, the behavior in a magnetic field should show the effects of open orbits and of magnetic breakdown. For large values of the energy gap and $\omega_c\tau$, the conductivity in the x direction should tend to zero as $(\omega_c\tau)^{-2}$ and in the y direction it should saturate when the second band is occupied. If the gap were small and magnetic breakdown did occur, the conductivity should tend to zero as $(\omega_c\tau)^{-2}$ in both directions. At densities where the second band is occupied, one would expect in the magnetic breakdown regime to see simultaneously SdH oscillations from the free-electron circle and the second-zone surfboard orbits.

The density of states of a 2D EG in the absence of a superlattice is independent of energy and equal to $n(\epsilon)=m/\pi\hbar^2$. For a superlattice which produces a negligibly small gap, the sum of the densities of states for the two zones should be the zero-gap value, the values being proportional to the Fermi perimeters. Thus

$$n_2(\epsilon)=[(2/\pi)\cos^{-1}(\pi k_F/a)](m/\pi\hbar^2).$$

The density-of-states mass (also the cyclotron mass) is thus

$$m_{2D}=m_{2C}=m(2/\pi)\cos^{-1}(\pi k_F/a).$$

Some of the properties of the samples used in our experiments are summarized in Table I. The linear samples have been described elsewhere.[4] Most of the data reported here were taken on samples a and c, both n-channel (811) samples, with the majority on the former.

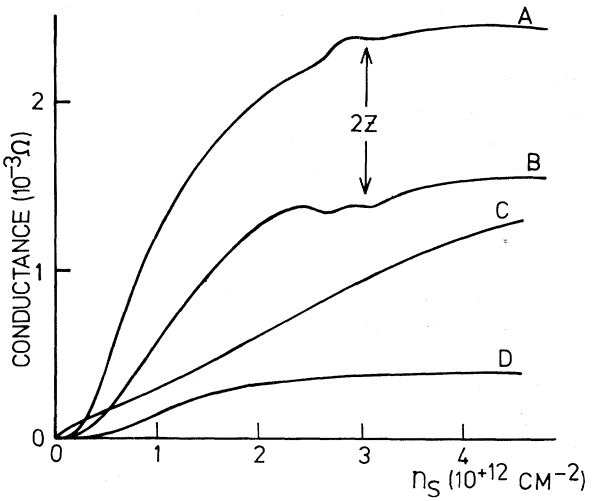

FIG. 1. Conductance vs n_s. Curve A, sample a at 1.7 K; curve B, sample c at 1.7 K; curve C, sample a at 77 K; curve D, sample e at 4.2 K.

The conductance as a function of n_s for samples a, c, and d is plotted in Fig. 1 at two temperatures. Curve A of sample a exhibits the general characteristics of a good quality Si MOSFET at 1.7 K with extra structure around $n_s=3\times10^{12}$ cm^{-2}. The point labeled $2Z$ is the density at which carriers begin to occupy the second band. Curve B is the conductance of sample c ($I\parallel y$) at the same temperature and shows clearly the expected w-shaped structure in the conductivity. The structure in curve A ($I\parallel x$) is less pronounced, and more Ω shaped. As a and c are different samples, one should not ascribe the difference in background conductivity to an anisotropic scattering time although the difference agrees fairly well with the expected anisotropy. Curve C of sample a illustrates that at 77 K the structure is not present (the case for all samples tested). Both the mean free path and kT have changed from 1.7 K. Since the mobility is only 30% less than that of sample c at 4.2 K, the disappearance of the structure must be due to kT. Curve D is for a p-channel (811) sample which showed no structure. Its mean free path is about 15 times less than sample a at the appropriate density.

Fang and Fowler[5] have studied (511) samples and have seen similar structure in circular samples for n_s between 6×10^{12} and 9×10^{12} cm^{-2}. Lakhani and Stiles[6] have studied other surfaces, (311), (211), (322), (111), (110), and (100), and have seen no structure in n or p channels on these surfaces. It would appear that the effect occurs only for the vicinal planes of (100).

The application of a magnetic field perpendicular to the surface of a Si (811) MOSFET results in behavior which is markedly different for different magnetic field strengths and carrier densi-

TABLE I. Sample properties.

Sample	Plane (normal)	Channel type	Current direction	Conductivity mobility peak (cm^2/V sec)
a,b	(811)	n	$\parallel(01\bar{1})$	13000
c,d	(811)	n	$\perp(01\bar{1})$	7000
e	(811)	p	$\perp(01\bar{1})$	1700
f	(811)	p	$\parallel(01\bar{1})$	1700
g	(511)	n	Circular	3300

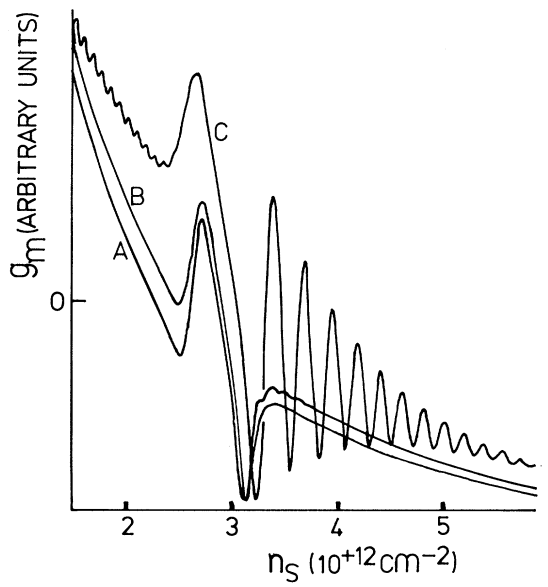

FIG. 2. Transconductance of sample a as a function of n_s at $T = 1.5$ K. Curve A, $H = 0$ T; curve B, $H = 0.3$ T; curve C, $H = 0.8$ T.

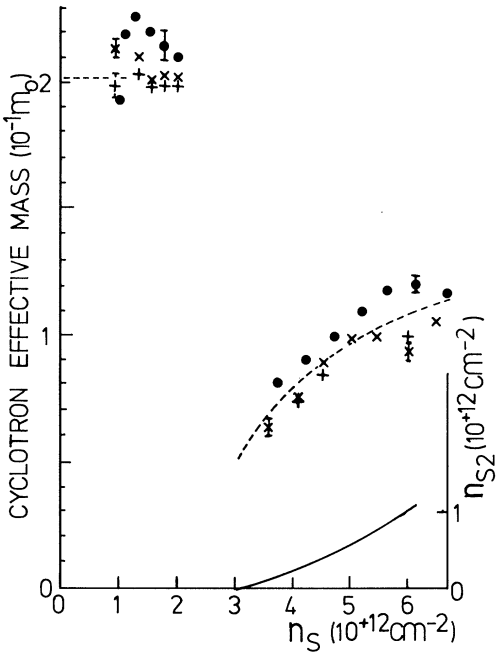

FIG. 3. Measured effective masses as a function of carrier density for samples a (+, run 1; ×, run 2) and c (●). Error bars shown are typical for each case. The solid curve is the measured number of carriers in the second zone. The dashed curves are the expected values for the model discussed in the text (excluding many-body effects).

ties. For $0 < H < 0.3$ T and for all densities, $\omega_c \tau < 1$ and one observes a classical type of magnetoconductance. The derived μ for the lowest fields equals the conductivity μ. When $0.3 < H < 0.7$ T, SdH oscillations occur for carriers in the second band as seen in Fig. 2 for several magnetic fields. From the magnetic field dependence a mobility can be extracted which is 2.5×10^4 cm^2/V sec for n_s slightly greater than 3×10^{12} cm^{-2}. μ decreases with increasing n_s. The oscillations as a function of n_s of the conductivity extrapolate to the point $2Z$ for all fields. The measured mobilities for the carriers in the second band give an average scattering time differing by not more than a factor of 2 from those in the first band at the same density. For $H < 0.7$ T, SdH oscillations occur for carriers in the first band as seen in curve C of Fig. 2. Mobilities derived from the field dependence are consistent within the uncertainties (± 10%) with the conductivity mobilities. As the magnetic field is increased beyond 0.9 T, the low density oscillations extend to higher and higher densities and eventually coexist with the oscillations from the second band. The period in n_s of these first-band oscillations does not change when they extend beyond the point $2Z$. This rules out the possibility that the effect is due to two different quantized levels of the perpendicular electric field.[7] The behavior is consistent, however, with magnetic breakdown.

If we assume the degeneracy of the second band to be the same as that of the first band, we can determine the number of carriers going into the second band from the period in gate voltage of the oscillations (solid curve in Fig. 3). By comparing this number with the number calculated for different values of α, we find that the gap increases from 4± 1 meV at 3×10^{12} cm^{-2} to 8± 3 meV at 6×10^{12} cm^{-2}. The magnetic breakdown regime occurs over a wide magnetic field range. For metals, easy breakdown occurs when $E_g \ll \beta(\hbar\omega_c \epsilon_F)^{1/2}$, where β contains angular factors and is usually 1 or less.[8] At 6×10^{12} cm^{-2}, first-band oscillations are observed for $H > 2.5$ T, indicating that $\epsilon_g \ll 12$ meV. The value of ϵ_g at 3×10^{12} cm^{-2} agrees well with that estimated from the spread in n_s of the structure in the conductivity ($0.2\epsilon_F \sim 4$ meV).

The effective mass of the carriers can be derived from the temperature dependence of the SdH oscillations. The results are shown in Fig. 3 for carriers in both bands. The mass derived for carriers in the first band is consistent with that expected from the band mass.[9] The mass of the carriers in the second band, however, is

much smaller than any of the band masses of Si and highly nonparabolic. The variation of mass with n_s is in reasonable agreement with the values calculated from the dispersion relation of our model taking into account the variation of α with n_s (see the dashed curve in Fig. 3).

From the density at the point $2Z$, we calculate from our model a correlation distance of 104 Å [about 3.5 times the (811) surface lattice constant of 31.4 Å] with a half-width of 6% as the mean free path is 1500 Å. If we relate V_1 to a variation in the surface height, $h\cos(2\pi x/a)$, in a manner similar to that used in the treatment of surface roughness scattering,[10] we find $h \sim 0.1$ Å for a normal electric field of 6×10^6 V/cm. This is much less than an interatomic distance or the expected variation in height (surface roughness) of 3–5 Å.[10] The magnitude of the potential and its dependence on normal electric field may be explainable in terms of a correlation of the surface roughness or of the graded region[11] of SiO_x at the Si-SiO_x interface. There may be a correlated variation in density, composition, etc. What causes the correlation is unknown, although periodic steps have been observed on reconstructed surfaces of GaAs and other materials.[12]

To date substrate bias experiments have not been quantitatively consistent. However the trend (with both forward and reverse substrate bias) is that the closer the carriers are confined to the surface, the broader and more pronounced is the structure in the dc conductivity. This trend is consistent with an increase in α with increasing n_s. There are other apparent weak dependences of one parameter on another. More precise measurements are necessary before more comments can be made.

With such a superlattice structure one can study the fermiology of a 2D EG with an additional zone boundary as a function of density. In addition, experiments involving interband spectroscopy, Bloch oscillators, caliper measurements of the Fermi surface, and others appear possible.

We wish to thank Y. Takeishi for samples, T. Ando and J. J. Quinn for helpful discussions, and the Physik Department of the Technische Universität München for facilities and hospitality.

---

*Work supported in part by the National Science Foundation, by the Office of Naval Research, and by the Materials Research Laboratory at Brown University funded through the National Science Foundation.

†Part of the work was performed at the Physik Department, Technische Universität München, 8046 Garching bei München, West Germany.

‡Temporary address: Department de Physique, Université Sherbrooke, Sherbrooke, Quebec J1K 2R1, Canada.

§Present address: Department de Physique, Université Sherbrooke, Sherbrooke, Quebec J1K 2R1, Canada.

‖Senior Humboldt Fellow.

[1]G. Dorda, Festkörperprobleme 13, 215 (1973); F. Stern, Crit. Rev. Solid State Sci. 4, 499 (1975); G. Landwehr, Festkörperprobleme 15, 49 (1975); Proceedings of the International Conference on Electronic Properties of Quasi-Two-Dimensional Systems, edited by J. J. Quinn and P. J. Stiles, Surf. Sci. 58, No. 1, 1 (1976).

[2]R. Dingle, Festkörperprobleme 15, 21 (1975); L. Esaki and L. L. Chang, Crit. Rev. Solid State Sci. 6, 195 (1976).

[3]See, e.g., C. Kittel, *Quantum Theory of Solids* (Wiley, New York, 1963), p. 254.

[4]T. Sato, Y. Takeishi, and H. Hara, Phys. Rev. B 4, 1950 (1971).

[5]F. F. Fang and A. B. Fowler, private communications.

[6]Amir A. Lakhani and P. J. Stiles, to be published.

[7]For such a discussion see K. von Klitzing, G. Landwehr, and G. Dorda, Solid State Commun. 14, 387 (1974); Amir A. Lakhani and P. J. Stiles, to be published.

[8]M. H. Cohen and L. Falicov, Phys. Rev. Lett. 7, 231 (1961); E. I. Blount, Phys. Rev. 126, 1636 (1962); A. B. Pippard, Proc. Roy. Soc. London, Ser. A 270, 1 (1962).

[9]The band effective mass is in fact slightly anisotropic; $m_y = m_t$ and $m_x = 1.12m_t$ ($m_t = 0.1905m_0$). We do not mention this in the text but take it into account in all calculations.

[10]Y. Matsamoto and Y. Uemura, Jpn. J. Appl. Phys. Suppl. 2, 237 (1974).

[11]F. Stern, to be published.

[12]D. L. Rode, J. Cryst. Growth 27, 313 (1974).