these processes are also given in Fig. 3. The single-photon process shown in Fig. 3(a) gives $\chi^{(3)}$, the two-photon process in Fig. 3(b) gives $\chi^{(5)}$ and the three-photon processes in Fig. 3(c) and 3(d) gives $\chi^{(7)}$. Note Haroche and Hartmann's three-photon diagram is corrected here. The fact that the polarization of the probe radiation is perpendicular to that of the pump radiation introduces some complexities. A more detailed description of the theory and experiment will be published elsewhere.

In summary, our observation clearly demonstrates the existence of velocity-tuned multiphoton processes in the laser cavity. Such multiphoton processes have implications in various areas of laser physics: (a) In the theory of the laser the perturbation treatment normally extends only up to the third order.[11] Many "nonresonant" molecules which are not considered in such theories may contribute to the laser power through velocity-tuned multiphoton processes. (b) For many multiphoton experiments, the use of a standing wave will increase the efficiency not only through a more intense field but also through the velocity-tuned multiphoton process. (c) As pointed out by Stenholm[6] a velocity-tuned $(2l+1)$-photon process has a momentum transfer of $2l+1$ photons in a single step. This can be used for efficient deflection of atomic or molecular beams.

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High-Pressure Flux-Conserving Tokamak Equilibria

J. F. Clarke
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

and

D. J. Sigmar
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 26 July 1976)

An analytic theory is developed to calculate poloidal beta β_I and the diamagnetic parameter μ_I for axisymmetric toroidal magnetohydrodynamic equilibria confining high-pressure plasmas $[\beta \sim O(a/R)]$ under the constraint of flux conservation. To satisfy the equilibrium equations, the plasma current increases with pressure as $p^{1/3}$. Previously calculated equilibrium limits on poloidal β are avoided.

Successful auxiliary heating of thermonuclear plasmas requires a characteristic heating time τ_h shorter than the energy containment time τ_E. Using neutral-particle injection, tokamak experiments with $\tau_A < \tau_h \leq \tau_E$, where τ_A is the Alfvén time, have been conducted (ORMAK), and future devices (PLT, ORMAK Upgrade, TFTR) will satisfy this criterion. Experimental evidence so far (ATC, Tuman) indicates $\tau_E < \tau_S$, where τ_S is the magnetic skin penetration time. The resulting condition $\tau_h < \tau_S$ necessitates the study of a series of neighboring magnetohydrodynamic (MHD) equilibria under the constraint of flux conservation.

We will assume, consistent with experiment, that on the time scale of interest both the poloidal flux $2\pi\psi$ and toroidal flux φ are conserved. Since the safety factor q is given by $q(\psi) = (2\pi)^{-1} d\varphi/d\psi$, one concludes that $q(\psi)$ will also be an invariant for these equilibria. Thus, we take $q(\psi)$ to be determined by its value in the low-β initial state and examine the evolution of the plasma equilibrium as the pressure is raised.

The condition $\tau_h < \tau_E$ implies an adiabatic equation of state, augmented in this case by a heat and particle source term due to injection. However, the present problem differs from the well-known "adiabatic compressor" problem[1-3] in that the major radius R remains essentially constant and flux conservation is realized by imposing $d\psi_0/dt = 0$, where ψ_0 is the poloidal flux at the fixed plasma boundary. To keep the analysis simple we drop the coupling between the adiabatic equation of state and the equilibrium equation and assume that the plasma pressure $p(\psi)$ is a free parameter. This assumption is realistic in experimental devices with powerful auxiliary heating. In principle, then, we solve the MHD equilibrium problem with $p(\psi)$ and $q(\psi)$ given.

The principal macroscopic parameters characterizing the tokamak equilibrium are[4]

$$\beta_I = 2\int p\, dV/(I^2 \cdot 2\pi R_c) \tag{1a}$$

$$\mu_I = 2\int dV(8\pi)^{-1}(B_{\varphi_0}{}^2 - B_\varphi{}^2)/(I^2\cdot 2\pi R_c), \tag{1b}$$

$$l_i = 2\int dV(8\pi)^{-1}B_p{}^2/(I^2\cdot 2\pi R_c), \tag{1c}$$

where $I = I(\psi_0)$ is the total current inside the circular flux surface boundary $\psi = \psi_0$ with major radius R_c. $\boldsymbol{B}_\varphi$ and $\boldsymbol{B}_p$ are the toroidal and poloidal magnetic fields and B_{φ_0} is the vacuum toroidal magnetic field. β_I measures the poloidal β, μ_I the plasma diamagnetism, and l_i the internal inductivity (inductance/cm Gaussian) of the plasma column, a geometric factor of order unity determined by the shape of the current profile.

For a complete solution of the equilibrium problem one must solve the Grad-Shafranov equation, where $F \equiv RB_\varphi$ must be expressed through $q(\psi)$, using

$$q(\psi) = (2\pi)^{-1}d\varphi/d\psi = FV'(\psi)\langle R^{-2}\rangle/4\pi^2, \tag{2}$$

where the flux surface average of R^{-2} is defined in Callen and Dory.[5]

In practice, if the global parameters β_I and μ_I of Eq. (1) are known, the problem can be simplified by using the integral form of the virial theorem and the equilibrium equation.

When evaluated on the outer flux surface of an axisymmetric toroidal plasma, these equations

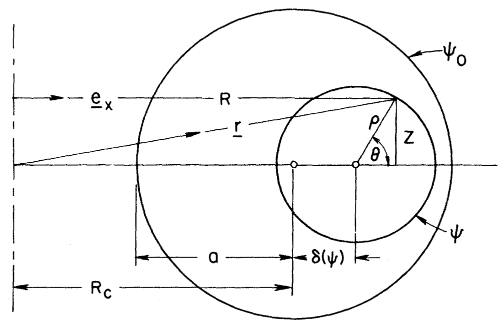

FIG. 1. Circular flux-surface model geometry.

assume the form

$$\int dV[3p + (8\pi)^{-1}(B_p{}^2 + \boldsymbol{B}_\varphi{}^2 - \boldsymbol{B}_{\varphi_0}{}^2)] = \int (8\pi)^{-1}B_p{}^2\vec{n}\circ\vec{r}\,dS_n,$$

$$2\pi\int dS_\varphi[p + (8\pi)^{-1}(B_p{}^2 + B_{\varphi_0}{}^2 - B_\varphi{}^2)] = \int (8\pi)^{-1}B_p{}^2\vec{n}\cdot\vec{e}_x dS_n,$$

where dS_n is a flux surface element, dS_φ a cross-sectional element, and $\vec{r}$ and $\vec{e}_x$ are shown in Fig. 1.

As shown by Shafranov,[4] for a circular flux shell of minor radius a, these relationships reduce to two equations which can be solved for β_I and μ_i to yield

$$\beta_I = S_2 + \tfrac{1}{2}S_1 - \tfrac{1}{2}l_i, \tag{3}$$

$$\mu_I = S_2 - \tfrac{1}{2}S_1 - \tfrac{1}{2}l_i, \tag{4}$$

where, neglecting higher-order terms in a/R_c,

$$2\pi I^2 R_c S_1 = \int_{\psi_0}(8\pi)^{-1}B_p{}^2 a\,dS_n, \tag{5a}$$

$$2\pi I^2 S_2 = \int_{\psi_0}(8\pi)^{-1}B_p{}^2\cos\theta\, dS_n, \tag{5b}$$

and $dS_n = 2\pi R_c(1+\epsilon\cos\theta)a\,d\theta$. In these equations the total plasma current is given by

$$I(\psi_0) = V'(\psi)\langle B_p{}^2\rangle_{\psi_0}/8\pi^2. \tag{5c}$$

Since the average pressure $\bar{p}$ is controlled by auxiliary heating, Eq. (3) is regarded as a relationship connecting the variable $\bar{p}$ with the surface integrals $\langle B_{p_>}{}^2\rangle_{\psi_0}$, S_1, and S_2. Once these are determined, Eq. (4) yields μ_1 as a function of $\bar{p}$ and l_i.

Since the problem centers around the calculation of the surface averages (5a), (5b), (5c) at the circular plasma boundary $\psi = \psi_0 = \text{const}$, we adopt the circular flux-surface approximation

$$\psi = S(\rho^2), \quad \rho^2 = (R - R_\psi)^2 + z^2, \quad R_\psi = R_c + \delta(\psi), \tag{6}$$

describing a set of nested toroidal flux surfaces with circular cross section where R_ψ extends to the center of the circular flux tube $\psi(\rho^2)$, shifted from the geometric center R_c by an amount $\delta(\psi)$, as shown in Fig. 1.

This model equilibrium contains two arbitrary functions $S(\rho^2)$ and $\delta(\psi)$. S is determined from flux conservation and $\delta(\psi)$ or $\delta'(\psi) \equiv \partial\delta/\partial\psi$ from Eq. (3) after the surface integrals are performed. A more complicated flux model with additional free parameters such as ellipticity would require additional moments of the plasma force-balance equations to determine them.[6]

It follows from Eq. (6) that in our simple model,

$$RB_p = |\nabla\psi| = 2\rho\dot{S}/D, \tag{7}$$

where $D = 1 + d\cos\theta$, $d = 2\rho\dot{S}\delta'(\psi)$, and $\dot{S} = d\psi/d\rho^2$, $R = R_\psi(1+\bar{\epsilon}\cos\theta)$, $\bar{\epsilon} = \rho/R_\psi$. The volume inside a flux surface $\psi = $ const is $V(\psi) = 2\pi^2\rho^2 R_\psi$. The integrations of (5c) and the flux surface average of R^{-2} yield

$$I(\psi) = \rho\dot{S}\bar{\epsilon}[1 + \tfrac{1}{2}(\bar{\epsilon}d) + \ldots](1-d^2)^{-1/2}, \tag{8a}$$

$$V'\langle R^{-2}\rangle/4\pi^2 = (\bar{\epsilon}/2\rho\dot{S})(1 - \tfrac{1}{2}\bar{\epsilon}d + \ldots). \tag{8b}$$

The dots indicate higher-order terms in ϵ.

As the flux surfaces shift outward under the increased plasma pressure one expects that $|d|$ becomes of order unity at $\psi = \psi_0$. Concomitantly, the poloidal field (7) has a nonexpandable dependence on $\cos\theta$ in marked contrast to the widely used low-β model[7]

$$B_p = B_{p0}(1 + \epsilon\Lambda\cos\theta).$$

As long as flux is conserved, the singularity is approached asymptotically as the pressure is raised. As shown below, the correct treatment of the θ dependence of B_p eliminates any equilibrium limit on the poloidal β. Using (8a), the remaining surface integrals are

$$S_1 I^2 = (a\bar{\epsilon}\dot{S})^2(1+\bar{\epsilon}d)(1-d^2)^{-3/2}, \tag{9a}$$

$$S_2 I^2 = \frac{R_c}{a}(a\bar{\epsilon}\dot{S})^2\left\{-\frac{d+\bar{\epsilon}}{(1-d^2)^{3/2}} + \frac{\bar{\epsilon}}{d^2}\,\frac{1-(1-d^2)^{1/2}}{(1-d^2)^{1/2}}\right\}. \tag{9b}$$

The $(1-d^2)^{-n}$ terms dominate the high-β equilibrium properties. For example, combining (9a) and (8a),

$$S_1 \simeq (1-d^2)^{-1/2} \gg 1 \tag{10}$$

in the high-β flux-conserving equilibrium versus $S_1 \sim 1$ in Ref. 4.

Defining a pressure variable normalized by the initial low-β toroidal current

$$\bar{\beta}_I = 2\int dV p/2\pi R_c I_i^2,$$

we find for β_I defined in (1a), using (8a),

$$\frac{\beta_I}{\bar{\beta}_I} = \frac{\dot{S}_i^2}{\dot{S}^2}\,\frac{1+\frac{1}{2}(\epsilon d_i)^2}{1+\frac{1}{2}(\epsilon d)^2}\,\frac{1-d^2}{1-d_i^2}, \tag{11}$$

where the subscript i stands for the initial value and $\epsilon = a/R_c$. All quantities on the right-hand side are evaluated at $\psi = \psi_0$.

From Eqs. (3), (4), (9a), and (9b), large increases in β can be produced if $d \to 1$. One expects a small decrease in l_i as β is increased $(d \to 1)$ since the denominator of Eq. (1c) becomes large while the numerator, which depends only on an integral over $(1-d^2)^{1/2}$, remains finite. Numerical calculations[8] confirm this and we approximate l_i by its low-β value.

The functional form of $S(\rho^2)$ is specified by the invariance of $q(\psi)$ in a flux-conserving system. From Eqs. (2) and (8b), $q(\psi)$ can be written as

$$q(\psi) = (F/2\dot{S}R_c)\{1 - \tfrac{1}{2}\bar{\epsilon}d\}. \tag{12}$$

In general, F is a constant plus an order-β term. Since q is an invariant, $\dot{S}$ must also be an invariant to zero order in β. For similicity, we specialize to the case of q constant on all flux surfaces, which implies that $\dot{S}$ is a constant to zero order in β,

$$S = \psi_0\rho^2/a^2 + O(\beta), \tag{13}$$

and we neglect the change in the functional form of S as pressure is increased.

So far, all surface integrals have been evaluated for the circular flux model, Eq. (6), in conformity with the assumed circular boundary $\psi = \psi_0$. Nevertheless, interior flux surfaces near the magnetic axis $\psi = 0$ exhibit strong elliptical and weaker triangular deformations.[7] Consider a set of shifted elliptic flux surfaces $\psi = S[(R-R_\psi)^2 + \kappa(\psi)Z^2]$, where $\kappa \equiv l_R^2/l_Z^2$ increases from a small number at $\psi = 0$ to unity at $\psi = \psi_0$. In Ref. 2 it was shown that $\delta(\psi)/a \sim O(\epsilon\beta_I)$ and $\dot{\kappa} \equiv \psi_0 d\kappa/d\psi \sim O(\epsilon^2\beta_I^2)$, in low-$\beta$ ordering $\beta_I \leq \epsilon^{-1}$. Solov'ev[7] discusses a class of arbitrary β equilibria; and combining his Eqs. (2.26) and (2.29), we find near the axis $\kappa(0) \leq \frac{1}{2}$ for $\beta_I \leq 2\epsilon^{-1}$, in agreement with numerical work.[8] For pressure and current profiles vanishing at the edge, the numerical work indicates $\dot{\kappa}(\psi_0) < \dot{\kappa}(0)$. The main effect of including the ellipticity $\kappa(\psi)$ is to change the quantity D of

Eq. (7) and one finds for the current

$$I(\psi_0) \propto \oint \frac{d\theta}{2\pi}\left[1+\frac{d}{1-\dot{\kappa}}\cos\theta+\frac{\dot{\kappa}}{1-\dot{\kappa}}\cos^2\theta\right]^{-1}.$$

Thus, for $0 \leq \dot{\kappa} \leq \frac{1}{2}$ the dominant modification will consist of the replacement of d by $d_{\text{eff}} \equiv d/(1-\dot{\kappa})$. d_{eff} rises faster with β than d does, thus producing a somewhat faster current rise, $I \propto (1-d_{\text{eff}}^2)^{-1/2}$, due to interior elliptical deformations. Thus, incorporating the effect of ellipticity tends to enhance the effects found from the circular flux-surface model, Eq. (6). Henceforth, d should be understood to stand for d_{eff}.

With our model completely specified, Eqs. (3) and (4) serve to determine β_I and μ_I. Neglecting $\dot{d_i}^2 \sim \epsilon^2$ terms and using Eqs. (9) and (11) with $\dot{S} = \dot{S}_i$, Eq. (3) assumes the form

$$\tfrac{1}{2}l_i + \bar{\beta}_I(1-d^2) = \frac{(1-d^2)}{(1+\frac{1}{2}\epsilon d)^2}\left\{\frac{[-\epsilon^{-1}(d+\epsilon)+\frac{1}{2}(1+\epsilon d)]}{(1-d^2)^{3/2}} + \frac{1}{d^2}\,\frac{1-(1-d^2)^{1/2}}{(1-d^2)^{1/2}}\right\}, \quad (14)$$

which yields the desired relation between the parameter d and the pressure variable $\bar{\beta}_I$, and, in the high-β limit, reduces to

$$\bar{\beta}_I \simeq (-d/\epsilon)(1-d^2)^{-3/2}. \quad (15)$$

Solved for d, the inversion of (15) for high β is

$$-d \simeq [1-\tfrac{1}{3}(\epsilon\bar{\beta}_I)^{-2/3}]^{3/2}, \quad (16)$$

showing that $d^2 \to 1$ for $\bar{\beta}_I \to \infty$ as discussed above. The low-β limit follows from (12), $\beta_I \simeq \bar{\beta}_I$.

To calculate β_I as a function of the pressure variable $\bar{\beta}_I$ in the high-β limit, we use Eqs. (15) and (16) to obtain

$$\beta_I/\bar{\beta}_I \simeq 1-d^2 = 1-[1-\tfrac{1}{3}(\epsilon\bar{\beta}_I)^{-2/3}]^3. \quad (17a)$$

In the regime $\epsilon\bar{\beta}_I > 1$, this can be expanded to yield

$$\beta_I = \epsilon^{-2/3}(\bar{\beta}_I)^{1/3}. \quad (17b)$$

The pressure dependence of the plasma current follows from (8a):

$$I(\psi)_0 = \begin{cases} \psi_0/R_c \text{ for low } \beta, & (18a) \\ \psi_0/R_c(1-d^2)^{1/2} \simeq (\psi_0/R_c)(\epsilon\bar{\beta}_I)^{1/3} \text{ for high } \beta. & (18b)\end{cases}$$

From (4), one obtains for the diamagnetic parameter

$$\mu_I + \tfrac{1}{2}l_i = -(1+\tfrac{1}{2}\epsilon d)^{-2}[\epsilon^{-1}(d+\epsilon)+\tfrac{1}{2}(1+\epsilon d)]/(1-d^2)^{1/2} + d^{-2}(1+\tfrac{1}{2}\epsilon d)^{-2}[(1-d^2)^{1/2}-(1-d^2)]. \quad (19a)$$

In the high-β regime $(-d/\epsilon > 1,\ d^2 \to 1)$,

$$\mu_I + \tfrac{1}{2}l_i \to -(d/\epsilon)(1-d^2)^{-1/2} = (1-d^2)\bar{\beta}_I > 0, \quad (19b)$$

where the dependence of d on pressure is given in (16). Comparing this with (17a) shows $\mu_I \to \beta_I$ in asymptotic agreement[9] with Eq. (40) of Ref. 4.

An upper bound for μ_I exists as $\int dV B_\varphi^2/B_{\varphi_0}^2 \to 0$, or, equivalently, $\beta \to 1$. With $\beta = 1$, from Eq. (17b) we find a maximum $\beta_I = q^{2/3}\epsilon^{-4/3}$.

The exact behavior of β_I, μ_I, and the plasma current $I(\psi_0)$ as a function of the pressure variable $\bar{\beta}_I$ is shown in Fig. 2.

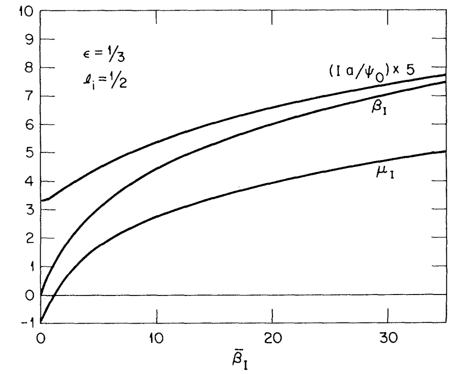

FIG. 2. The behavior of β_I, μ_I, and I is shown as a function of $\bar{\beta}_I$ for $\epsilon = \frac{1}{3}$, $l_i = \frac{1}{2}$.

Summary.—(1) When the average pressure $\int dV p/V$ is increased by auxiliary heating on a flux-conserving time scale, the equilibrium equations of an axisymmetric toroidal plasma permit a continuous transition from low to high values of β. (2) In a flux-conserving equilibrium, the plasma current increases with pressure as $(\epsilon\bar{\beta}_I)^{1/3}$ [Eq. (18b)]. (3) Consequently, the poloidal β grows slower than linearly with pressure, and the frequently used scaling relation $\beta_I = q^2\beta/\epsilon^2$ does not apply for flux-conserving tokamaks. It is replaced by (17b) asymptotically. (4) The flux-conserving equilibria considered in this paper do not permit formation of a second magnetic axis, since B_p does not vanish on the outer flux surface [see Eqs. (7) and (16)]. Thus, there is no equilibrium limit such as implied by the condition $\beta_I \leq \epsilon^{-1}$ obtained for flux-nonconserving equilibria.[10] (5) In the high-β limit, the diamagnetic parameter μ_I approaches β_I, implying confinement by the toroidal diamagnetic well. At the ultimate limit $\beta = 1$, $\beta_I = q^{2/3}\epsilon^{-4/3}$.

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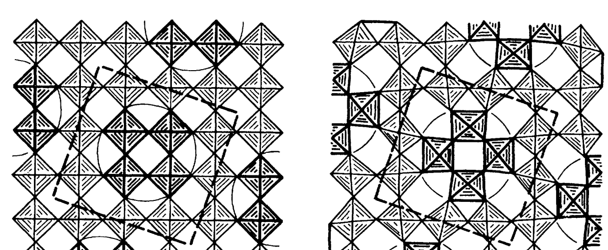
Model of the Ferroelectric Phase Transition in the Tetragonal Tungsten-Bronze–Structure Ferroelectrics

K. L. Ngai and T. L. Reinecke*
Naval Research Laboratory, Washington, D. C. 20375
(Received 19 April 1976)

A microscopic model is presented for the ferroelectric phase transition in the tetragonal tungsten-bronze–structure ferroelectrics (e.g., $Sr_{1-x}Ba_xNb_2O_6$) which, for the first time, provides a description of its essential features, including the fact that it is a displacive transition but has no "soft"-phonon mode. The model employs an interaction between the ferroelectric phonon displacement and local structural changes, which are important in these materials; and it describes well important features of the Raman spectra, dielectric constant, and refractive index.

There is at present no basic understanding of the physical mechanism of the ferroelectric phase transition for the large class of technologically important ferroelectrics in the tetragonal tungsten-bronze $T1$ structure[1] [e.g., $Sr_{1-x}Ba_xNb_2O_6$ (SBN), and $Ba_{4+x}Na_{2-2x}Nb_{10}O_{30}$]. A model, which is based on the interaction of the ferroelectric phonon with specific local structural changes that are important in these disordered[2] materials, is presented and shown to account for this transition.

The tetragonal tungsten-bronze (TTB) $T1$ structure consists of a network of distorted NbO_6 octahedra [shown in Fig. 1(b)] connected together in such a way that there are pentagonal, square, and triangular "tunnels" which can be occupied by the Ba and Sr ions[2] of SBN. Ba and Sr ions are randomly distributed in the pentagonal tunnels, and Sr is randomly distributed in the square tunnels (but neither kind of tunnel is completely occupied). As the temperature is lowered through the ferroelectric T_c, the metallic atoms (including Nb) displace along the c axis into the oxygen layers.[2] Although this transition is displacive, it does not



(a) (b)

FIG. 1. Structure of tetragonal tungsten-bronze materials projected on the ab plane. Pentagonal, square (and triangular) cells contain Ba, Sr, Na, etc. ions. (a) is $T2$ phase and (b) is $T1$ phase. Square shows "unit cell" which is rotated in going from the $T1$ to the $T2$ phase.