Table II. The parameters of high-energy production are a smooth continuation of the trends of the low-energy data[11] with R_ρ now remaining small as Q^2 increases to 2.2 $(\mathrm{GeV}/c)^2$. The phase difference is consistent with zero except, perhaps, in the highest Q^2 bin.

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Relativistic Two-Body Wave Equation and Meson Spectrum

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The meson spectrum is studied with use of an equal-time relativistic two-body equation with electrostatic confinement potentials. A normalizability condition excludes all but a bag solution which gives a spectral pattern in between the SU(6) and the chiral SU(3) $\otimes$ SU(3) limits. Orbital mixing creates a pair of ${}^3S_1+{}^3D_1$ states sandwiching the ${}^1P_1$ level. ρ-ρ' and ψ/J-ψ' pairs fit this picture very well.

Fully relativistic studies of quark-antiquark systems have been done in the past using the Bethe-Salpeter equation, as in the extensive works by Böhm, Krammer, and Joos,[1] which were based on the concept of the effective quark confinement due to large quark masses. The concept may still prove to be a viable one, but in the alternative color confinement scheme the conventional definition of the Bethe-Salpeter amplitude would fail. This is because infinitely high frequencies of the colored intermediate states force such an amplitude to vanish. Therefore, I propose to investigate instead a relativistic two-body wave equation of the form

$$(-i\alpha\cdot\nabla+\beta m_1)\chi(\vec{r})-\chi(\vec{r})(-i\alpha\cdot\nabla+\beta m_2)=[M-V(r)]\chi(\vec{r}). \tag{1}$$

Here the wave function χ is given a 4×4 matrix representation $\chi_{\alpha\beta}$, with indices α and β referring to quark and antiquark spinor components, respectively. I have taken the center-of-mass system and $\vec{r}$ is a relative coordinate. m_1 and m_2 are the quark masses and M is the eigenvalue of the system. The potential $V(r)$, which I assume to be electric in origin as indicated by some confinement models,[2,3] appears in the combination $M-V$, rather than in the combination of $m_{1,2}-V$ for Lorentz-scalar potentials. The equation differs from the Breit equation,[4] based on the one-quantum-exchange approximation, in lacking the current-current interaction term. However, relativistic covariance does not necessarily require the presence of such a term, as can be seen from the following example of a covariant generalization of Eq. (1) for a state of total energy-momentum P_μ:

$$(i\partial\!\!\!/_1 - \tfrac{1}{2}U\!\!\!/ - m_1)\psi(x_1,x_2) + \psi^T(x_1,x_2)(i\overleftarrow{\partial\!\!\!/}_2 - \tfrac{1}{2}U\!\!\!/ + m_2) = 0 \qquad (2)$$

where $U_\mu = n_\mu V(x_1 - x_2)$ with $n_\mu = P_\mu/\sqrt{P^2}$ and $A\!\!\!/ = A_\mu\gamma^\mu$. Equation (2) has a solution of the form $\psi(x_1,x_2) = \delta(P\cdot(x_1 - x_2))e^{-iP\cdot x_1}\chi(x_1 - x_2)n\!\!\!/$, and in the rest frame $\vec{P} = 0$, it reduces to Eq. (1) for χ with $M = \sqrt{P^2}$. In the absence of any viable alternative, I will adopt Eq. (1), which is an obvious, and the simplest, generalization of the Dirac equation, for the purpose of studying the relativistic binding by a confinement potential.

As is well known, the Dirac equation in an electrostatic potential $V(r)$ leads to the Klein paradox[5] if the potential is of confinement type, i.e., if it rises indefinitely with increasing r. Although Eq. (1) appears to suffer from the same disease, we will see that the normalizability condition on the wave function at a singular point $r = R$ such that $V(R) = M$ excludes all but a bag solution,[6] which vanishes for $r > R$. Thus, contrary to the case of the Dirac equation, we have a well-defined eigenvalue problem[7] for Eq. (1). With the existence of the bound states thus assured, I will be discussing in this preliminary note primarily results which are based more on the structure of Eq. (1) than on the detail of the potential, so that I will also include nonconfinement potentials in my discussion.

In order to reduce Eq. (1) I introduce the customary representation[8] of the Dirac matrices by two sets of Pauli spin matrices $\{\rho_i\}$ and $\{\sigma_i\}$ such that $\alpha_i = \rho_1\sigma_i$ and $\beta = \rho_3$, and expand χ as

$$\chi = \rho_1\chi_1 + \rho_2\chi_2 + \rho_3\chi_3 + \chi_4, \qquad (3)$$

where the χ_i's are 2×2 matrices spanned by $\{\sigma_i\}$. Equation (1) now gives a set of coupled equations for χ_i's:

$$\begin{aligned}
&[-i\sigma\cdot\nabla, \chi_1]_- - (M - V)\chi_4 = 0,\\
&[-i\sigma\cdot\nabla, \chi_4]_- - 2im\chi_2 - (M - V)\chi_1 = 0,\\
&[\sigma\cdot\nabla, \chi_2]_+ - (M - V)\chi_3 = 0,\\
&-[\sigma\cdot\nabla, \chi_3]_+ + 2im\chi_1 - (M - V)\chi_2 = 0.
\end{aligned} \qquad (4)$$

Here the commutators $[\,]_\pm$ refer to the Pauli spin matrices, with ∇ understood to operate directly on χ_i. I have set $m_1 = m_2 = m$ since I consider only states of definite charge-conjugation parity. I reduce Eq. (4) further by assigning a specific spin and orbital configuration to each χ_i for a few low-lying states of physical interest. The spin-orbit structure of χ_i's is dictated by their transformation properties under space reflection and charge conjugation for a given angular momentum J. For a state of parity ϵ_P and charge-conjugation parity ϵ_C, χ_i's transform like $\chi_i(\vec{r}) = \pm\epsilon_P\chi_i(-\vec{r})$ (plus for $i = 3$ and 4 and minus for $i = 1$ and 2) and $\chi_i(\vec{r}) = \pm\epsilon_C C\chi_i^T(-\vec{r})C^{-1}$ (plus for $i = 1$, 2, and 3, and minus for $i = 4$), where $C = \sigma_2$. Thus, χ_1 and χ_2 have the same spin-orbit configuration while χ_3 and χ_4 have different ones. As a customary convention I label a state by the configuration of χ_1 and χ_2, which are represented by the spectroscopic notation ${}^{2S+1}L_J$ or a combination of two of these (orbital mixing).

For a singlet state ${}^1L_{J=L}$, one can write $\chi_i = Y_{LM}(\theta,\varphi)F_i(r)$ (for $i = 1, 2$). From (4), one sees that $\chi_4 = 0$ and $\chi_3 = {}^3L_{L+1} + {}^3L_{L-1}$. Eliminating χ_3, one obtains

$$F_1 = -2im(M - V)^{-1}F_2 \qquad (5)$$

and

$$F_2'' + \left(\frac{2}{r} + \frac{V'}{M - V}\right)F_2' + \left[\frac{1}{4}(M - V)^2 - m^2 - \frac{L(L+1)}{r^2}\right]F_2 = 0. \qquad (6)$$

The term $\frac{1}{4}V^2$ is responsible for the Klein paradox if $V \to \infty$ as $r \to \infty$. For such a potential, there appears a new singular point at $r = R$, where $V(R) = M$. One readily sees from (6) that a regular solution at $r = R$ behaves like $F_2 \sim (r - R)^2$, while a mildly singular second solution behaves like $F_2 \sim \text{const}$. The normalizability condition on the wave function excludes the second solution because the corresponding F_1 amplitude is not normalizable according to Eq. (5). Thus the boundary conditions

$$F_2(0) = \text{const and } F_2(R) = 0, \qquad (7)$$

are necessary and unique conditions to obtain a finite solution. Since $F_2'(R) = 0$ also, F_2 connects smoothly to $F_2 = 0$ outside of R, where $\chi_1 = \chi_3 = 0$ also by (4) and (5). Although χ_3 is discontinuous at $r = R$, no physical principle is violated by this bag solution. The Klein-pardox solution violates the boundary condition (7) and must be excluded.[9] By introduction of an amplitude $f_2 = r^{L+1}(M - V)^{-1/2}F_2$ one obtains the one-dimensional Schrödinger equation

$$f_2'' - U_L(r)f_2 = 0, \qquad (8)$$

with the effective potential U_L given by

$$U_L = m^2 - \frac{(M-V)^2}{4} + \frac{L(L+1)}{r^2} + \frac{V'}{r(M-V)} + \frac{3}{4}\frac{V'^2}{(M-V)^2} + \frac{1}{2}\frac{V''}{(M-V)}. \quad (9)$$

The boundary conditions corresponding to (7) are

$$f_2 \sim r^{L+1} \text{ (for } r\sim 0),$$
$$f_2 \sim (r-R)^{3/2} \text{ (for } r\sim R). \quad (10)$$

We see that $U_L(r)$ has an infinite barrier $\frac{3}{4}(r-R)^{-2}$ at $r=R$ if $V(r)$ is of the confinement type. One condition on $V(r)$ is

$$V(0)=0, \quad (11)$$

which gives from (5) a relation $MF_1(0) = -2imF_2(0)$, which can be shown to be equivalent to the axial-vector divergence relation.[10] By requiring (11), three types of potentials can be considered. The first is of confinement type in which case the boundary conditions (7) or (10) must be applied; we see from (9) that $M > 2m$ in this case (called case I hereafter). The second comprises potential wells such that $V(\infty)=0$, including the attractive Coulomb potential, for which a repulsive core is necessary to satisfy (11). There are no singular points except for $r=0$ and $r=\infty$, where the ordinary boundary conditions apply with $M < 2m$ (case II). The third is a hybrid of the first two (case III). In this Letter I will simply assume that there exists a potential $V(r)$ among the three types which fits the low-lying singlet levels. Then, the singlet levels serve as the reference levels to be compared with triplet states.

Among triplets, the ${}^3P_1$ and ${}^3P_0$ states are the simplest to study. For ${}^3P_1$ one has $\chi_{1,2} = \vec{\sigma}\cdot\vec{\epsilon}\times\vec{r}r^{-1}F_{1,2}$, $\chi_3 = 0$, and $\chi_4 = {}^3S_1 + {}^3D_1$. By introduction of $f_2 = r(M-V)^{1/2}F_2$, one has

$$f_2'' - [U_1(r) - \Delta(r)]f_2 = 0, \quad (12)$$

with

$$\Delta(r) = V'/[r(M-V)]. \quad (13)$$

Similarly for ${}^3P_0$, one has $\chi_{1,2} = \vec{\sigma}\cdot\vec{r}r^{-1}F_{1,2}(r)$, $\chi_4 = 0$, and $\chi_3 = F_3(r)$. For $f_2 = r(M-V)^{-1/2}F_2$, one obtains

$$f_2'' - [U_1(r) - 2\Delta(r)]f_2 = 0. \quad (14)$$

$\Delta(r)$, which is obviously related to the spin-orbit interaction, is positive-definite for case I, but has an alternating sign for cases II and III because of requirement (11). Comparing Eqs. (12) and (14), I conclude that

$$M({}^1P_1) \lesseqgtr M({}^3P_1) \gtrless M({}^3P_0), \quad (15)$$

with the lower inequality being possible only for cases II and III. To first-order perturbation in $\Delta(r)$ one may write

$$2[M({}^3P_1)]^2 = [M({}^1P_1)]^2 + [M({}^3P_0)]^2. \quad (16)$$

Equations (12) and (14) indicate that the spin-orbit interaction $\Delta(r)\vec{L}\cdot\vec{S}$ splits the three otherwise degenerate states. Since my Eq. (1) has no explicit spin-spin interaction, it presents a very plausible picture. However, as we shall see later, the ${}^3P_2$ state is mixed with ${}^3F_2$ and does not belong to this multiplet. Although $B(1235)$ (${}^1P_1$), $A_1(1100)$ (${}^3P_1$), and $\delta(970)$ (${}^3P_0$) satisfy the equal-spacing law (16) very well and indicate $\langle\langle\Delta(r)\rangle\rangle > 0$, the existence of A_1 is very much in doubt,[11] and we may not exclude the possibility that $\langle\Delta(r)\rangle$ is very small or even negative.

For the $J^P = 1^-$ state, one finds the spin-orbit configurations $\chi_{1,2}({}^3S_1 + {}^3D_1)$, $\chi_3({}^3P_1)$, and $\chi_4({}^1P_1)$. Writing $\chi_i = \vec{\sigma}\cdot\nabla[\vec{\epsilon}\cdot\vec{r}F_i(r)] + \vec{\sigma}\cdot\vec{r}[\vec{\epsilon}\cdot\vec{r}G_i(r)]$ (with $i = 1, 2$), where $\vec{\epsilon}$ is a polarization vector, one can reduce Eq. (4) to coupled equations in F_2 and G_1. With the further introduction of $f_2 = r^2(M-V)^{1/2}F_2$ and $g_1 = \frac{1}{2}ir^2(M-V)^{-3/2}G_1$, one obtains

$$f_2'' - U_1(r)f_2 = 2m(M-V)g_1,$$
$$g_1'' - [U_1(r) - \Delta(r)]g_1 = 2m\Delta(r)(M-V)^{-1}f_2. \quad (17)$$

It is remarkable that these equations involve the centrifugal barrier for $L=1$. Thus, in the limit of vanishing coupling between f_2 and g_1 which obtains for $m=0$ in case I, one has two states, $f_2(g_1=0)$ degenerate with ${}^1P_1$ and $g_1(f_2=0)$ degenerate with ${}^3P_1$, both states still being a mixture of ${}^3S_1$ and ${}^3D_1$. (The chiral-symmetry limit $m\to 0$ must be carefully analyzed[12] in conjunction with the boundary condition at $r=R$.) Under the assumption that $\langle\Delta\rangle$ is small, the introduction of the coupling will split the two almost degenerate levels, as can be seen from an approximate diagonalization of Eq. (17),

$$h_\pm'' - [U_1(r) \pm S(r)]h_\pm = 0, \quad (18)$$

where $S(r) = 2m[\Delta(r)]^{1/2}$ and $h_\pm = f_2 \pm bg_1$ with $b = [(M-V)^3 r/V']^{1/2}$. In deriving (18), I have neglected higher-order terms in $\Delta(r)$ and also treated b as if it is a constant. It may be noted that the latter treatment is correct in case V is dominated by a large attractive Coulomb potential $-g^2/r$. Then neglecting M against V, one has $S(r) = 2m/r$ and $b = g$. A splitting much larger than the spin-orbit splitting will occur if $(\langle S(r)\rangle) \gg (\langle\Delta(r)\rangle)$. For example, taking $V=\mu^2 r$ and assuming $\langle r\rangle \sim \frac{1}{2}R$ (with $V(R)=M$), we find that the above inequality is equivalent to $(2m\langle r\rangle)\gg 1$, an inequality which can certainly be satisfied with reasonable values of m and $\langle r\rangle$. The same condition holds for a large attractive Coulomb potential. Thus we propose that the pairs ρ-ρ' and ψ/J-ψ' be identified with such a pair of ${}^3S_1+{}^3D_1$ states, which are separated from the ${}^1P_1$ state because of a large splitting potential $\pm S(r)$. One may thus write a mean mass formula

$$M_B{}^2=\tfrac{1}{2}(M_\rho{}^2+M_{\rho'}{}^2), \tag{19}$$

which is satisfied extremely well. This in turn would support the assumption that the spin-orbit interaction $\langle\Delta(r)\rangle$ is small. A similar formula applied to ψ/J-ψ' will give $M({}^1P_1$ charmonium) $=3410$ MeV. If this value is correct, then inequality (15) would contradict the identification of $\chi(3410)$ and $\chi(3510)$ as the ${}^3P_0$ and ${}^3P_1$ states,[13] respectively. If the identification holds, then one must conclude that the mean mass formula fails in this case. Another characteristic feature of our model is that both ${}^3S_1+{}^3D_1$ states have the same radial quantum number (no radial node), contrary to the conventional picture of ρ' and ψ' being a radially excited state of ρ and ψ, respectively. As a consequence, there will be no suppression of the leptonic decay amplitude of ρ' or ψ' due to higher radial quantum number. I cannot, however, make any numerical prediction at this time.

In completely the same way I expect a pair of $J^P=2^+$ states $({}^3P_2+{}^3F_2)$ which will sandwich the ${}^1D_2$ state. If one identifies $A_2(1310)$ and $A_4(1900)$ with such a pair, one will have $M_{A_3}{}^2=\frac{1}{2}(M_{A_2}{}^2+M_{A_4}{}^2)$, a relation very well satisfied. The conventional L-S coupling scheme, in which A_2, A_1, and δ form split ${}^3P$ states, is obviously grossly violated.

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