gion, i.e., at $p^2=-m_c^2$. The use of this mass is also preferable from the point of view of application of renormalization group, and our definition of mass m_c refers exactly to the point $p^2=-m_c^2$ [from an explicit calculation we find that in the Landau gauge $m_c(p^2=-m_c^2)/m_c(p^2=+m_c^2)=1-(2\alpha_s \ln 2)/\pi$].

Then, the correction factor to the sum rules depends only weakly on n for $n=1$, 2, 3, and 4 used in our analysis and varies from $1+0.7\alpha_s$ to $1+0.2\alpha_s$. This correction does not explain the breaking of the sum rules for $n\geqslant 5$ discussed above. There are two other sources of corrections, however. These are the terms of higher order in α_s and terms of the order $\mu^2/4m_c^2$. For high n, terms of the order α_s^3 are calculable and are indeed essential. Terms of the order $\mu^2/4m_c^2$ are not calculable at the moment.

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$D^{0*}\to D^0\gamma$ and Other Radiative Decays of Vector Mesons*

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With use of SU(4) as a spectrum-generating group, the radiative decay rates of the charmed vector mesons and of J (ψ) are calculated. With the known decay rates of the "old" mesons $\Gamma(\omega\to\pi\gamma)$, $\Gamma(\varphi\to\eta\gamma)$, $\Gamma(\rho\to\pi\gamma)$, and $\Gamma(K^{0*}\to K^0\gamma)$ as input, one obtains $\Gamma(K^{+*}\to K^+\gamma)=2.6$ keV, $\Gamma(\omega\to\eta\gamma)=220$ eV, $\Gamma(\rho\to\eta\gamma)=4.8$ keV, $\Gamma(\psi\to\chi\gamma)=1.6$ keV, $\Gamma(D^{0*}\to D^0\gamma)=350$ eV, and $\Gamma(D^{+*}\to D^+\gamma)=22$ eV.

In an earlier paper we have discussed the radiative decay of the J (ψ) in an approach in which SU(4) is considered as a spectrum-generating group.[1] This method with use of the spectrum-generating group is a nonperturbative approach to broken SU(4),[2] similar to that in which SU(4) is considered as the dynamical stability group of the velocity operator[3] $P_\mu M^{-1}$.

As a consequence of this assumption the amplitude contains, in addition to the SU(4) Clebsch-Gordan coefficients, a symmetry-breaking factor (suppression factor) Φ, which is a function of the masses involved. The precise form of Φ as a function of the vector- and pseudoscalar-meson masses m_V and m_P appearing in the radiative decays $V\to P\gamma$ depends upon the assumption about the SU(4) property of the "current" operators V_μ^{α}.[4] This assumption should be chosen such that, in the limit when the spectrum-generating group $SU(4)_E$ goes into the SU(4) symmetry group the V_μ^{α} become SU(4) tensor operators. Since there are many possible generalizations away from this limit, we determined in Ref. 1 the precise functional form of the suppression factor Φ phenomenologically from the known radiative decay rates of the "old" vector mesons $\Gamma(\omega\to\pi\gamma)$, $\Gamma(\varphi\to\eta\gamma)$, $\Gamma(\rho\to\pi\gamma)$, and $\Gamma(K^{0*}\to K^0\gamma)$. The three functions which fitted these decay rates are[5]

$$\Phi(m_V, m_P)=(m_V^{\,p}+m_P^{\,p})/(m_V m_P) \tag{1}$$

for $p=\frac{1}{2}$, 1, and $\frac{3}{2}$. The decay rate for the process $V\to P\gamma$ is given by

$$\Gamma(V\to P\gamma)=|g_{VP}|^2[\tfrac{1}{24}\alpha m_V^{\,3}(1-m_P^{\,2}/m_V^{\,2})^3], \tag{2}$$

with

$$g_{VP}=g\langle P|V^{\rm el}|V\rangle\Phi(m_V, m_P), \tag{3}$$

where α is the fine-structure constant, g is an overall constant which could have been absorbed into the reduced matrix elements of $V_\mu{}^\alpha$ and $\langle P|V^{\rm el}|V\rangle$ is the $SU(4)_E$ matrix element of the SU(4) part of the electromagnetic current. In the symmetry limit $\Phi=1$, (2) becomes the usual expression for magnetic dipole transitions.

At the time Ref. 1 was written, the masses of the charmed mesons were unknown and also the exact expression for the electromagnetic current operator was not experimentally justified. With the discovery of the D meson,[6] both of these deficiencies have been overcome; and from correspondence with the charge in SU(4),

$$Q=I_3+\tfrac{1}{2}Y+\tfrac{2}{3}\chi+\tfrac{1}{2}B,$$

where charm is given by $\tfrac{3}{4}B+\chi$, the electromagnetic current operator $V_\mu{}^{\rm el}$ in SU(4) is—in the phase convention that we shall adopt here—given by[7]

$$V_\mu{}^{\rm el}=V_\mu{}^{\pi^0}+(1/\sqrt{3})V_\mu{}^\eta-\sqrt{\tfrac{2}{3}}V_\mu{}^\chi+V_\mu{}^\sigma. \tag{4}$$

The SU(4) scalar term $V_\mu{}^\sigma$ in (4), whose matrix element between baryon states is proportional to the baryonic charge B and whose matrix element between meson states is zero, is essential in this calculation. Between meson vectors with opposite charge parity (like the pseudoscalar and vector mesons), it is different from zero, $\langle P|V^\sigma|V\rangle\neq 0$, and it is the occurrence of this term which makes it possible to fit the experimental ratio of $\Gamma(\omega\to\pi\gamma)/\Gamma(\rho\to\pi\gamma)$ which otherwise could not be explained by any form of the symmetry-breaking factor $\Phi(m_V, m_P)$. The old SU(3) or naive-quark-model assumption expressed by $\langle$old pseudoscalar$|-\sqrt{\tfrac{2}{3}}V^\chi+V^\sigma|$ old vector meson$\rangle=0$, cannot fit this ratio of the decay rates either.

There are four reduced matrix elements for all $\langle P|V^{\rm el}|V\rangle$. It can be seen that the F-type reduced matrix element is zero as a consequence of the transformation property under charge conjugation. We will further assume, in order to keep the number of parameters as small as possible, that the vector and pseudoscalar mesons belong to a pure 63-plet of $SU(8)\supset SU(4)\otimes SU_{\rm spin}(2)$ and that the particle vectors are given by the basis vectors which are connected to the subgroup chain

$$SU(8)\supset SU(6)\otimes SU_{S_\chi}(2)$$
$$\supset SU_W(4)\otimes SU_{S_Y}(2)\otimes SU_{S_\chi}(2),$$

where SU(6) is the Gürsey-Radicati SU(6), S_χ is the charmed spin, $SU_W(4)$ is the Wigner SU(4) [and distinct from the $SU(4)_E$ or symmetry SU(4) described above], and S_Y is the hypercharged spin. In this approximation, the vector mesons belong to an ideally mixed 16-plet of $SU(4)_E$ and the pseudoscalar mesons belong to a pure 15-plet of $SU(4)_E$; η-η' mixing, deviation from ideal mixing, and isospin mixing are ignored. Deviation from ideal mixing should not be considered separately without considering isospin mixing (ρ^0-ω or π^0-η mixing) because they are of the same magnitude and perhaps of the same origin. However, inclusion of all these mixings by arbitrary mixing angles would introduce too many parameters to result in anything useful.

The narrow width of J (ψ) is usually explained as a consequence of the fact that its decays into old particles are first-forbidden transitions, i.e., they are forbidden if one assumes ideal mixing and the Zweig rule. If one considers the approximation in which the first-forbidden transitions are zero, then one obtains a relation between the D-type reduced matrix elements $D=\langle P(15)|\times|V^{(15)}||(15)V\rangle$ and the reduced matrix element $A=\langle P(15)||V^{(15)}||(1)V\rangle$.[8] All the matrix elements $\langle P|V^{\rm el}|V\rangle$ can then be expressed in terms of the two parameters (reduced matrix elements) $d=(1/\sqrt{3})D$ and $S=\langle P|V^\sigma|V\rangle$, and SU(4) Clebsch-Gordan coefficients[9]:

$$\begin{aligned}
&\langle\pi^0|V^{\rm el}|\omega\rangle=d,\\
&\langle\eta|V^{\rm el}|\varphi\rangle=\sqrt{\tfrac{2}{3}}\,(d+S),\\
&\langle\pi^{0\pm}|V^{\rm el}|\rho^{0\pm}\rangle=S,\\
&\langle K^+|V^{\rm el}|K^{+*}\rangle=S,\\
&\langle K^0|V^{\rm el}|K^{0*}\rangle=d+S,\\
&\langle\eta|V^{\rm el}|\rho^0\rangle=-(1/\sqrt{3})d,\\
&\langle\eta|V^{\rm el}|\omega\rangle=-(1/\sqrt{3})S,\quad \langle\chi|V^{\rm el}|\psi\rangle=\tfrac{1}{2}\sqrt{3}(-d+S),\\
&\langle D^0|V^{\rm el}|D^{0*}\rangle=-d+S,\quad \langle D^+|V^{\rm el}|D^{+*}\rangle=S,\\
&\langle F^+|V^{\rm el}|F^{+*}\rangle=S,\\
&\langle\pi^0|V^{\rm el}|\varphi\rangle=-\langle\pi^0|V^{\rm el}|\psi\rangle\\
&\qquad=-\langle\eta|V^{\rm el}|\psi\rangle\sqrt{3}=0.
\end{aligned} \tag{5}$$

The two free parameters cannot be calculated from any further theoretical assumption and have to be determined from the experimental data of the old mesons $\Gamma(\omega\to\pi\gamma)$, $\Gamma(\varphi\to\eta\gamma)$, $\Gamma(\rho^-\to\pi^-\gamma)$, and $\Gamma(K^{0*}\to K^0\gamma)$.

We use the experimental values of $\Gamma(\omega\to\pi\gamma)$ and

TABLE I. Calculated values and error estimates for the decay rates, in keV.

Decay	Experiment	$p=1/2$	$p=1$	$p=3/2$
$\omega\to\pi\gamma$	870 ± 61[a]	870 ± 61	870 ± 61	870 ± 61
$\varphi\to\eta\gamma$	74 ± 15[b]	51 ± 4	76 ± 6	98 ± 7
$\rho\to\pi\gamma$	35 ± 10[c]	35 ± 10	35 ± 10	35 ± 10
$K^{0*}\to K^0\gamma$	75 ± 35[d]	66 ± 5	87 ± 7	98 ± 8
$K^{+*}\to K^+\gamma$	<80[e]	1.9 ± 0.6	2.6 ± 0.7	3.0 ± 0.8
$\omega\to\eta\gamma$	<50[a]	0.18 ± 0.05	0.22 ± 0.06	0.23 ± 0.07
$\rho\to\eta\gamma$	<152[e]	3.9 ± 0.3	4.8 ± 0.3	5.0 ± 0.4
$\psi\to\chi\gamma$	<2[f]	0.30 ± 0.03	1.6 ± 0.2	7.3 ± 0.8
$D^{0*}\to D^0\gamma$	...	0.10 ± 0.01	0.35 ± 0.04	1.0 ± 0.1
$D^{+*}\to D^+\gamma$	...	0.006 ± 0.002	0.022 ± 0.007	0.07 ± 0.02
$F^{+*}\to F^+\gamma$	...	0.006 ± 0.002	0.022 ± 0.007	0.07 ± 0.02

[a]Ref. 11. [b]Ref. 12. [c]Ref. 13. [d]Ref. 14. [e]Ref. 15. [f]Ref. 16.

$\Gamma(\rho\to\pi\gamma)$ to determine d and S, respectively, and obtain $|S|\simeq\frac{1}{5}|d|$.[10] The experimental values of $\Gamma(\varphi\to\eta\gamma)$ and $\Gamma(K^{0*}\to K^0\gamma)$ then determine the relative phase of d and S, and restrict the possible choices for p. We find that only $p=\frac{1}{2}$, 1, and $\frac{3}{2}$ and $S\simeq\frac{1}{5}d$ give acceptable results.[5] [Note that the old SU(3) or naive-quark-model assumption requires $S=-\frac{1}{3}d$.] The decay rates calculated in this way are shown in Table I (in columns 3, 4, and 5) with $m_\chi=2750$, $m_{D^0}=1867$, $m_{D^0*}=2006$, $m_{D^+}=1871$, $m_{D^+*}=2010$, $m_{F^+}=1925$, and $m_{F^{+*}}=2065$ MeV. (All quantities in Table I are given in units of keV.) The experimental values that have been used are shown in column 2. Our predictions are in the last 7 lines of the table, in columns 3, 4, and 5. Those in column 4, for $p=1$, are considered the best because the old rates are fitted best in that case. In particular, $\Gamma(D^{0*}\to D^0\gamma)=0.35\pm0.04$ keV as compared, for example, to ~80 keV predicted by Hallock, Oneda, and Slaughter.[17] The errors shown were estimated from the experimental errors of the input, $\Gamma(\rho\to\pi\gamma)$ and $\Gamma(\rho\to\pi\gamma)$. Changes in the predictions due to the use of a χ^2 fit to determine d and S, or the use of 751 meV for the ρ mass,[17] lie within these error estimates.

We conclude with a remark concerning the deviation from ideal mixing: One can easily see that a small deviation from ideal mixing has an effect of only a few percent upon the radiative decay rates calculated above. However, this small deviation has a sufficiently large effect to explain the value of the first-forbidden transitions. As we have already mentioned above, it does not make sense to include the deviation from ideal mixing without also taking the isospin mixing into account. Thus we should not expect to obtain a prediction if we arbitrarily adapt one formula for the nonideal $|\psi^{\text{nonideal}}\rangle$ from the literature, e.g.,[18]

$$|\psi^{\text{nonideal}}\rangle=-0.9999|\psi\rangle-0.0101|\varphi\rangle-0.0130|\omega\rangle,$$
$$|\eta^{\text{nonideal}}\rangle=0.999|\eta\rangle+0.0115|\chi\rangle.$$

With this admixture of φ and ω in ψ^{nonideal} and of χ in η^{nonideal} one obtains with the values in column 2 of the table $\langle\eta^{\text{nonideal}}|V^{\text{el}}|\psi^{\text{nonideal}}\rangle=-0.0031$ and therewith $\Gamma(\psi^{\text{nonideal}}\to\eta^{\text{nonideal}}\gamma)=0.13$ keV. This is already greater than the experimental value[19] of $\Gamma^{\text{exp}}(\psi\to\eta\gamma)=0.069$ keV. Ignoring the χ admixture in η, one even obtains $\Gamma(\psi^{\text{nonideal}}\to\eta\gamma)=0.94$ keV. This, however, also demonstrates that the small ρ^0-ω or η-π^0 mixing cannot be ignored.

Recently, two sets of values for $\Gamma(\rho^0\to\eta\gamma)$ and $\Gamma(\omega\to\eta\gamma)$ have been obtained[20] in an experiment using photoproduction of ω and ρ^0 from Cu. The destructive-interference values from this experiment seem to be above the present raw upper limit of an experiment in progress at Argonne National Laboratory (Carleton–McGill–Ohio State–Toronto collaboration). The ratio $\Gamma(\omega\to\eta\gamma)/\Gamma(\rho^0\to\eta\gamma)$ of the constructive-interference values agrees both with our prediction of $\sim\frac{1}{25}$ [which comes from the experimental value of $\Gamma(\rho^-\to\pi^-\gamma)/\Gamma(\omega\to\eta\gamma)$] and also with the old SU(3) prediction of $\sim\frac{1}{9}$. However, the absolute values are an order of magnitude higher than our predictions. Whereas the prediction for the ratio is independent of the suppression factor Φ, the absolute values are very sensitive to the form of the suppression factor. The principal purpose of our investigations is to determine the suppression factor and therewith the underlying assumption about the $SU(4)_E$ property of the "currents." If these new experimental values should be confirmed, then the suppression factors used in this paper would have to be modified.

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[6]G. Goldhaber *et al.*, in Proceedings of the Stanford Linear Accelerator Center Summer Institute, 1976 (unpublished).

[7]Because of a misunderstanding of the conventions, none of the three cases considered for X in Ref. 1 is identical to (4). Therefore, our prediction here for $\psi \to \chi\gamma$ differs from the predictions given there. The fit to the old vector mesons, however, is independent of the value for X.

[8]The Zweig rule in this formulation turns out to be the relation $D/\sqrt{6} + A = 0$ between the otherwise arbitrary reduced matrix elements D and A of the vector current 15-plet. This results—with ideal mixing— in $\langle\pi^0|V^{el}|\varphi\rangle = -\langle\pi^0|V^{el}|\psi\rangle = -\langle\eta|V^{el}|\psi\rangle/\sqrt{3} = 0$ and deviations from this relation are usually considered as deviations from the ideal-mixing assumption for φ and ψ.

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