Sum Rules for Charmonium and Charmed Mesons in Quantum Chromodynamics

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We consider the contribution of charmed quarks to vacuum polarization and to $\gamma\gamma$ scattering. In quantum chromodynamics the calculation is reliable for small photon momenta provided that the quark mass m_c is large compared with the scale of the hadronic mass μ, $4m_c^2 \gg \mu^2$. By the use of dispersion relations, the calculation is converted into a set of sum rules which impose model-independent upper bounds on the charmonium leptonic and photonic decay rates and enables one to estimate the hadronic widths as well. Weak leptonic decays of the charmed mesons are also estimated.

Because of the simplicity of the quark structure, the richness of the level system, and the large mass, charmonium is uniquely suited for the study of strong interactions. A lot of effort has been made recently in this direction, mostly within the framework of the model-dependent potential approach. In this Letter we will derive sum rules for the charmonium decay rates which rest only on such general assumptions as the validity of quantum chromodynamics (QCD) at short distances and analyticity. The sum rules refer to the integrals over the cross section of charm production in e^+e^- and $\gamma\gamma$ collisions.

In the case of e^+e^- annihilation the sum rules are in agreement with the existing data. Moreover, the sum rules turn out to be very sensitive to the contribution of the low-lying states and some of them are practically saturated by the J/ψ-meson contribution.

Encouraged by this confirmation of the sum rules, we consider some processes not accessible to a direct experimental study such as charm production in photon-photon collisions, and extract in this way predictions for the two-γ-decay widths of charmonium. Coupled with the usual assumptions of the charmonium model, the sum rules produce definite predictions for the total hadronic widths as well.

Let us consider first the charmed-quark contribution to vacuum polarization. For the bare quarks it is given by the graph of Fig. 1(a). As is well known, within QCD the graph can be trusted for large negative[1,2] or complex[3] values of the square of the photon momentum. The crucial observation is that for heavy quarks the bare graph is reliable for the vanishing photon momentum as well. The line of reasoning is the same as in the case of heavy-particle photoproduction[4] and rests on the large virtuality of heavy quarks. As a result, all the loop integrations converge for virtual momenta of the order $p^2 = -m_c^2$ and the T product, $\int dx\, e^{iqx} T\{\bar{c}(x)\gamma_\mu c(x), \bar{c}(0)\gamma_\mu c(0)\}$, is determined by the distances of the order $x^2 \sim 1/4m_c^2$ which are supposed to be small as compared to the usual hadronic scale: $4m_c^2 \gg \mu^2$. Therefore, perturbation expansion in the effective coupling constant $\alpha_s(m_c^2)$ makes sense [from analysis of the J/ψ decays, one concludes that $\alpha_s(m_\psi^2) \equiv \alpha_s \simeq 0.2$].

Using analyticity we convert the calculation of the vacuum polarization at $Q^2 \sim 0$ into a set of dispersion sum rules, and we find in this way

$$\int \frac{R_c(s)\,ds}{s^{n+1}} = \int \frac{R_c^{(0)}(s)\,ds}{s^{n+1}} \equiv 3Q_c^2 \frac{A_n^{(0)}}{(4m_c^2)^n}, \quad (1)$$

where m_c is the charmed-quark mass and R_c is the famous ratio $R_c = \sigma_c(s)(4\pi\alpha^2/3s)^{-1}$ for charm production in e^+e^- collisions; $R_c^{(0)}$ is the same

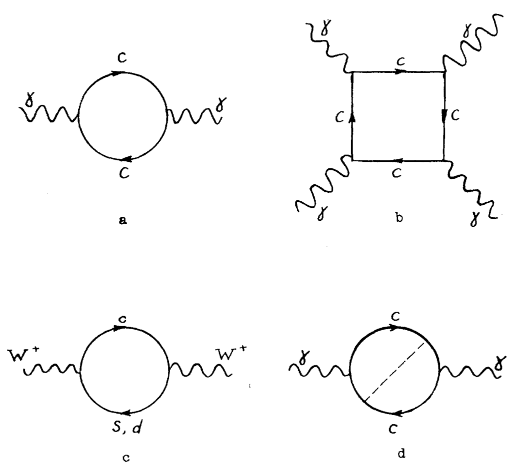

FIG. 1. (a), (b), (c) Feynman graphs for the vacuum amplitudes induced by the c-quark electromagnetic and weak currents. (d) An example of the first-order correction to the vacuum polarization (dashed line denotes a gluon).

ratio for the bare quarks, given by

$$R_c^{(0)} = 3Q_c^2 v(3 - v^2)/2. \quad (2)$$

Here $Q_c = \frac{2}{3}$ is the charmed-quark charge, v is its velocity, $s(1 - v^2) = 4m_c^2$, and the factor 3 is due to color. Numerically, one gets

$$A_1^{(0)} = \tfrac{4}{5},\quad A_2^{(0)} = \tfrac{12}{35},\quad A_3^{(0)} = \tfrac{64}{315},\ \text{and}\ A_4^{(0)}\tfrac{32}{231}$$

(corrections of the first order in α_s are considered below).

On the other hand, the physical value of R_c is contributed by the production of mesons with hidden charm (J/ψ, ψ',...) and of pairs of charmed particles. The effective threshold for the pair production is about 4 GeV, and we approximate R_c in this region by the step function, $R_c = \frac{4}{3}\theta(s - 16)$. Since integrals (1) are well convergent, the detailed structure of R_c in this region is not of much importance.

Equations (1) define a set of values of m_c, one for each n. The sum rules with $n = 1$, 2, 3, and 4 are self-consistent and produce practically the same m_c. This is also true when we take account of the gluon corrections of first order in α_s, and our best fit is

$$m_c = 1.25\ \text{GeV}.$$

For higher n this value of m_c does not fit the sum rules well and the discrepancy approaches a factor of 2 for $n = 8$. This is an indication of the growing role of the neglected terms for high n, a phenomenon which is expected on rather general grounds. We can check the sum rules in an alternative way; namely, for $n = 3, 4$, the continuum contributions are about 8% and 4%, respectively. Therefore, we can saturate these sum rules by the J/ψ-meson contribution $R_\psi(s) = 9\pi M_\psi \Gamma_{ee}\delta(s - M_\psi^2)/\alpha^2$ and eliminate m_c to find the leptonic width of J/ψ,

$$\Gamma_{ee} \simeq \frac{\alpha^2}{9\pi} 3Q_c^2 \frac{(A_3^{(0)})^4}{(A_4^{(0)})^3} M_\psi \simeq 5\ \text{keV}, \quad (3)$$

which coincides with the experimental number (the correction due to the ψ' meson is negligible). Thus, the sum rules for the e^+e^- annihilation are in excellent agreement with the data.

Let us notice that the value of m_c found above is several hundred MeV lower than that usually accepted in the literature.[2,3,5,6] It is worth emphasizing, therefore, that our determination of m_c refers to small distances, while in the potential approach to charmonium one deals with the c quark at relatively large distances.

The value of m_c found above can be used to determine the charmonium decay widths into some other channels. In particular, the two-γ-decay widths can be extracted from the sum rules for the amplitude of scattering of light by light [see Fig. 1(b)] which are as follows:

$$\int \frac{\sigma_a(s)\,ds}{s^n} = \int \frac{\sigma_a^{(0)}(s)\,ds}{s^n} = 8\pi\alpha^2 3Q_c^4 \frac{B_a^{(n)}}{(4m_c^2)^n}, \quad (4)$$

where σ_a is the charm production cross section for photons with parallel ($a = \parallel$, $\vec{e}_1 \parallel \vec{e}_2$) and perpendicular ($a = \perp$, $\vec{e}_1 \perp \vec{e}_2$) linear polarizations, respectively, and $\sigma_a^{(0)}$ refers to the partonlike cross section[7]:

$$\sigma_\parallel^{(0)} = 3Q_c^4 \frac{\pi\alpha^2}{s}\left[(5 + 2v^2 - 3v^4)\ln\frac{1+v}{1-v} - 2v(5 - 3v^2)\right], \quad (5)$$

$$\sigma_\perp^{(0)} = 3Q_c^4 \frac{\pi\alpha^2}{s}\left[(7 - 2v^2 - v^4)\ln\frac{1+v}{1-v} - 2v(3 - v^2\right]. \quad (6)$$

From an explicit calculation one readily finds

$$B_\parallel^{(1)} = \tfrac{5}{9},\ B_\parallel^{(2)} = \tfrac{8}{45},\ B_\parallel^{(3)} = \tfrac{124}{1575},\ B_\parallel^{(4)} = \tfrac{68}{1575},$$

$$B_\perp^{(1)} = \tfrac{7}{9},\ B_\perp^{(2)} = \tfrac{14}{45},\ B_\perp^{(3)} = \tfrac{268}{1575},\ B_\perp^{(4)} = \tfrac{524}{4725}.$$

As for the left-hand sides of Eqs. (4), they are contributed by the charmonium levels and by the pair production of charmed particles. The latter we rather arbitrarily approximate by $\sigma_a = \theta(s - 16)\sigma_a^{(0)}$. Then for $a = \parallel$ ($a = \perp$) the continuum gives 60, 30, 20, and 10% (55, 20, 12, and 5%) of the expected total for $n = 1$, 2, 3, and 4, respectively (because of the crossing properties of the amplitude only even moments are of physical meaning, while the odd moments are understood as an analytical continuation).

Apart from the continuum, we keep explicitly the contributions of pseudoscalar mesons η_c and η_c', $\sigma_\perp(0^-) = 16\pi^2\Gamma_{\gamma\gamma}\delta(s - M_{0^-}^2)/M_{0^-}$; of the scalar meson χ_0, $\sigma_\parallel(0^+) = 16\pi^2\Gamma_{\gamma\gamma}\delta(s - M_{0^+}^2)/M_{0^+}$, $\sigma_\parallel(0^-) = \sigma_\perp(0^+) = 0$; and of the tensor one χ_2, $\sigma_\parallel(2^+) \simeq \sigma_\perp(2^+) \simeq 40\pi^2\Gamma_{\gamma\gamma}\delta(s - M_{2^+}^2)/M_{2^+}$, with only the dominant amplitude being retained for the χ_2. From sum rules (4) we find the following upper bounds on the photonic widths of these resonances: $\Gamma(\chi_0 \to 2\gamma) < 9$ keV, $\Gamma(\chi_2 \to 2\gamma) < 4.5$ keV, and $\Gamma(\eta_c \to 2\gamma) < 4.5$ keV if $M_{\eta_c} = 2.85$ GeV, and $\Gamma(\eta_c \to 2\gamma) < 7.5$ keV if $M_{\eta_c} = 3.0$ GeV [here we identify the scalar particle with the $\chi_0(3.41)$ state and the tensor meson with the $\chi_2(3.55)$ state].

Moreover, Eqs. (4) are consistent with the relation[8] $\Gamma(\chi_2\to 2\gamma)\simeq\frac{4}{15}\Gamma(\chi_0\to 2\gamma)$ which is true in the nonrelativistic limit. Accepting this relation, we find $\Gamma(\chi_0\to 2\gamma)=4.6-5.4$ keV. The predictions for the pseudoscalar mesons depend on their masses, and we get $\Gamma(\eta_c'\to 2\gamma)\sim\Gamma(\eta_c\to 2\gamma)\simeq 3.5$ keV if $M_{\eta_c}=2.85$ GeV. On the other hand, if $M_{\eta_c}=3.0$ GeV, then $\Gamma(\eta_c\to 2\gamma)\simeq 6.5$ keV and is substantially larger than that of η_c'.

The photonic decay widths are presumably related to the decay widths into two gluons,[2,8] and the corresponding conversion factor is given by[8] $9\alpha_s^2/8\alpha^2\simeq 845$ if $\alpha_s=0.2$. According to common wisdom the total hadronic widths are given by the widths of two-gluon decays. For the latter we find $\Gamma(\chi_0\to 2$ gluons$)=3.9-4.6$ MeV, $\Gamma(\chi_2\to 2$ gluons$)=1.0-1.2$ MeV, $\Gamma(\eta_c\to 2$ gluons$)\simeq 3$ MeV (for $M_{\eta_c}=2.85$ GeV), $\Gamma(\eta_c\to 2$ glouns$)\simeq 5.4$ MeV (for $M_{\eta_c}=3.0$ GeV). These results are in agreement with the nonrelativistic model predictions,[5,8] and for the $\chi_0(3.41)$ and $\chi_2(3.55)$ states are consistent with the existing data. However, one must keep in mind the uncertainties in the calculation of the hadronic width which are due to the neglect of the interaction between gluons and of the possible admixture of light quarks and/or gluons in the charmonium states, e.g., in a form of molecular charmonium.[9]

When deriving Eqs. (1) and (4) we neglect the contribution of light quarks which arises in higher orders in the quark-gluon interaction. Some doubt may arise whether this is justified since for low external momenta the virtuality of light quarks is small and they interact strongly. It is obvious, however, that although the interaction between light quarks is strong, their interaction with deeply virtual c quarks is weak. The transition $c\bar{c}\to$gluons takes place deep inside the light-quark cloud, where there *is* asymptotic freedom. From the small hadronic widths of J/ψ and ψ' we know that the annihilation of c quarks into gluons is small. The situation in the case of low external momenta is by no means worse than that for charmonium decays where only the exchanged c quark is deeply virtual. Thus, it is quite consistent to neglect the light quarks in calculating electromagnetic decays of charmonium.

The same technique can be applied to the analysis of the weak decays of the charmed mesons. Let us consider to this end the vacuum polarization by the weak currents $j_\mu=\bar{q}\gamma_\mu c$ and $j_\mu{}^5=\bar{q}\gamma_\mu\gamma_5 c$, where q stands for a light quark [see Fig. 1(c)]. Neglecting the mass of the light quark and terms of higher order in α_s we find, for $s\ll m_c^2$,

$$\frac{f_P^2}{M_P^2(M_P^2-s)}+\frac{1}{\pi}\int\frac{\rho_c(s')\,ds'}{s'(s'-s)}=\frac{3m_c^2}{8\pi^2}\int_{m_c^2}^{\infty}\frac{(s'-m_c^2)^2\,ds'}{(s')^4(s'-s)},\qquad(7)$$

$$\frac{\lambda_V^2}{M_V^2-s}+\frac{1}{\pi}\int\frac{\tilde{\rho}_c(s')\,ds'}{s'(s'-s)}=\frac{1}{8\pi^2}\int_{m_c^2}^{\infty}\frac{(s'-m_c^2)(2s'+m_c^2)\,ds'}{(s')^3(s'-s)}.\qquad(8)$$

Here ρ_c and $\tilde{\rho}_c$ represent the continuum contribution, while the poles corresponding to the pseudoscalar (P) and vector (V) mesons are accounted for explicitly; f_P and λ_V are the decay constants of these mesons defined as $\langle 0|\partial_\mu j_\mu{}^5|P\rangle=f_PM_P^2$ and $\langle 0|j_\mu|V\rangle=\lambda_V M_V^2V_\mu$, respectively.

Expanding Eqs. (7) and (8) at small s we get a set of sum rules which impose upper bounds on the coupling constants. In particular, for $M(F)=2.0$ GeV and $M(F^*)=2.15$ GeV we find $f_F\lesssim 180$ MeV $[\Gamma(F\to\mu\nu)\simeq G_F^2\cos^2\theta_c f_F^2m_\mu^2M_F/8\pi<5.3\times10^9\ \mathrm{sec}^{-1}]$, and $\lambda_{F^*}{}^2<2.2\times10^{-2}$. It is worth mentioning that exact SU(4) symmetry leads to a value which is approximately 3 times higher than that determined above. Thus, we expect that the diffractive production of F^* mesons in the neutrino experiment is lower than is implied by the formal application of the SU(4) symmetry. In the case of the D mesons our upper bounds are $f_D<150$ MeV $[\Gamma(D\to\mu\nu)=G_F^2\sin^2\theta_c f_D^2m_\mu^2M_D/8\pi<1.9\times10^8\ \mathrm{sec}^{-1}]$ and $\lambda_{D^*}{}^2<1.4\times10^{-2}$.

The self-consistency of the calculation of the widths presented above can be tested in a number of ways. In particular, it is possible to extract the same vertices as those considered above in an independent way by studying sum rules for other amplitudes. The new relations agree with those obtained above and are described elsewhere.

In conclusion, let us consider the effect of the first-order corrections in α_s on the sum rules for e^+e^- annihilation [see Fig. 1(d)]. To evaluate these terms one has to replace the ratio $R_c^{(0)}$ in Eqs. (1) and (2) by $R_c^{(1)}$, where[10]

$$R_c^{(1)}=R_c^{(0)}\left[1+\frac{2\pi\alpha_s}{3v}-\frac{3+v}{3}\left(\frac{\pi}{2}-\frac{3}{4\pi}\right)\alpha_s\right].\qquad(9)$$

Moreover, the question arises as to which mass, in the field-theoretical language, enters the sum rules. Equation (9) is true, in fact, if m_c is understood as a position of the pole. In the theories with quark confinement it seems more appropriate to introduce mass in the deep Euclidian region, i.e., at $p^2 = -m_c^2$. The use of this mass is also preferable from the point of view of application of renormalization group, and our definition of mass m_c refers exactly to the point $p^2 = -m_c^2$ [from an explicit calculation we find that in the Landau gauge $m_c(p^2 = -m_c^2)/m_c(p^2 = +m_c^2) = 1 - (2\alpha_s \ln 2)/\pi$].

Then, the correction factor to the sum rules depends only weakly on n for $n = 1, 2, 3,$ and 4 used in our analysis and varies from $1 + 0.7\alpha_s$ to $1 + 0.2\alpha_s$. This correction does not explain the breaking of the sum rules for $n \geq 5$ discussed above. There are two other sources of corrections, however. These are the terms of higher order in α_s and terms of the order $\mu^2/4m_c^2$. For high n, terms of the order α_s^3 are calculable and are indeed essential. Terms of the order $\mu^2/4m_c^2$ are not calculable at the moment.

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$D^{0*} \to D^0\gamma$ and Other Radiative Decays of Vector Mesons*

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With use of SU(4) as a spectrum-generating group, the radiative decay rates of the charmed vector mesons and of J (ψ) are calculated. With the known decay rates of the "old" mesons $\Gamma(\omega \to \pi\gamma)$, $\Gamma(\varphi \to \eta\gamma)$, $\Gamma(\rho \to \pi\gamma)$, and $\Gamma(K^{0*} \to K^0\gamma)$ as input, one obtains $\Gamma(K^{+*} \to K^+\gamma) = 2.6$ keV, $\Gamma(\omega \to \eta\gamma) = 220$ eV, $\Gamma(\rho \to \eta\gamma) = 4.8$ keV, $\Gamma(\psi \to \chi\gamma) = 1.6$ keV, $\Gamma(D^{0*} \to D^0\gamma) = 350$ eV, and $\Gamma(D^{+*} \to D^+\gamma) = 22$ eV.

In an earlier paper we have discussed the radiative decay of the J (ψ) in an approach in which SU(4) is considered as a spectrum-generating group.[1] This method with use of the spectrum-generating group is a nonperturbative approach to broken SU(4),[2] similar to that in which SU(4) is considered as the dynamical stability group of the velocity operator[3] $P_\mu M^{-1}$.

As a consequence of this assumption the amplitude contains, in addition to the SU(4) Clebsch-Gordan coefficients, a symmetry-breaking factor (suppression factor) Φ, which is a function of the masses involved. The precise form of Φ as a function of the vector- and pseudoscalar-meson masses m_V and m_P appearing in the radiative decays $V \to P\gamma$ depends upon the assumption about the SU(4) property of the "current" operators V_μ^{α}.[4] This assumption should be chosen such that, in the limit when the spectrum-generating group $SU(4)_E$ goes into the SU(4) symmetry group the V_μ^α become SU(4) tensor operators. Since there are many possible generalizations away from this limit, we determined in Ref. 1 the precise functional form of the suppression factor Φ phenomenologically from the known radiative decay rates of the "old" vector mesons $\Gamma(\omega \to \pi\gamma)$, $\Gamma(\varphi \to \eta\gamma)$, $\Gamma(\rho \to \pi\gamma)$, and $\Gamma(K^{0*} \to K^0\gamma)$. The three functions which fitted these decay rates are[5]

$$\Phi(m_V, m_P) = (m_V^p + m_P^p)/(m_V m_P) \tag{1}$$

for $p = \frac{1}{2}$, 1, and $\frac{3}{2}$. The decay rate for the process $V \to P\gamma$ is given by

$$\Gamma(V \to P\gamma) = |g_{VP}|^2[\tfrac{1}{24}\alpha m_V^3(1 - m_P^2/m_V^2)^3], \tag{2}$$

with

$$g_{VP} = g\langle P|V^{\rm el}|V\rangle\Phi(m_V, m_P), \tag{3}$$