obstacles inertially. The amount of plastic deformation in the latter process is determined by the degree of underdamping. This kind of process is supported by recent computer simulation studies.[20]

*Work supported by the National Science Foundation and by the U. S. Energy Research and Development Administration, both at the University of Illinois, Urbana, Illinois, and at the Argonne National Laboratory.

†Present address: Materials Science Division, Argonne National Laboratory, Argonne, Ill. 60439.

[1]R. E. Jamison and F. A. Sherrill, Acta Metall. 4, 197 (1956).

[2]Z. S. Basinski and D. Dove, in Proceedings of the Fifth International Congress on Crystallography, Cambridge, 1960 (unpublished).

[3]Z. S. Basinski, R. A. Foxall, and R. Pascual, Scr. Metall. 6, 807 (1972).

[4]T. Suzuki and T. Ishii, in *Physics of Strength and Plasticity*, edited by A. S. Argon (The MIT Press, Cambridge, 1969), p. 159.

[5]K. Kamada and I. Yoshizawa, J. Phys. Soc. Jpn. 31, 1056 (1971).

[6]V. I. Startsev, V. V. Pustovalov, and V. S. Fomenko, Trans. Jpn. Inst. Met. Suppl. 9, 843 (1968).

[7]V. V. Pustovalov, A. I. Landau, and T. A. Parkhomenko, in Proceedings of the Fourth International Conference on the Strength of Metals and Alloys, Nancy, France (to be published).

[8]A. Seeger, in *Dislocations and Mechanical Properties of Crystals*, edited by J. C. Fisher, W. G. Johnston, B. Thomason, and T. Vreeland, Jr. (Wiley, New York, 1957), p. 243.

[9]A. V. Granato, Phys. Rev. Lett. 27, 660 (1971), and Phys. Rev. B 4, 2196 (1971).

[10]A. V. Granato, K. Lücke, J. Schlipf, and L. J. Teutonico, J. Appl. Phys. 35, 2732 (1964).

[11]L. J. Teutonico, A. V. Granato, and K. Lücke, J. Appl. Phys. 35, 220 (1964).

[12]A. V. Granato and K. Lücke, to be published.

[13]R. B. Schwarz and A. V. Granato, Phys. Rev. Lett. 34, 1174 (1975).

[14]R. B. Schwarz, to be published.

[15]A. V. Granato, in *International Friction and Ultrasonic Attenuation in Crystalline Solids*, edited by D. Lenz and K. Lücke (Springer, New York, 1975), Vol. II, p. 33.

[16]A. D. Brailsford, in *International Friction and Ultrasonic Attenuation in Crystalline Solids*, edited by D. Lenz and K. Lücke (Springer, New York, 1975), Vol. II, p. 1.

[17]J. A. Garber and A. V. Granato, J. Phys. Chem. Solids 31, 1863 (1970).

[18]R. B. Schwarz, to be published.

[19]Dislocation loops in real crystals are not all of the same length. In the following discussion, L is meant to be an effective length, characteristic of the distribution of loop length in the crystal.

[20]R. B. Schwarz and R. Labusch, to be published.

Composite Magnetic Solitons in Superfluid ^{3}He-A*

Kazumi Maki and Pradeep Kumar
Physics Department, University of Southern California, Los Angeles, California 90007

(Received 7 December 1976)

A new type of domain wall is found where both vectors $\hat{d}$ and $\hat{l}$ rotate from parallel to antiparallel configuration in the same plane but in the opposite sense. The composite soliton has the surface energy smaller by a factor of $\sqrt{5}$ than the pure $\hat{d}$ soliton in a uniform $\hat{l}$ texture. The oscillations of the vector $\hat{d}$ in the composite soliton give rise to satellite resonance frequencies, smaller than the normal magnetic resonance frequencies in the A phase.

In recent papers,[1,2] it has been shown that the $\hat{d}$ texture in superfluid ^{3}He-A may have a planar structure (or domain wall) which has many properties in common with the solitons in other fields of solid state physics.[3] These solitons (we call them magnetic solitons) can be created magnetically and have unshifted resonance frequencies associated with sliding motions of solitons over a uniform $\hat{l}$ background.[2,4] In fact, in all previous analyses,[1,2] it was assumed either explicitly or implicitly that the $\hat{l}$ field constitutes a uniform rigid arena over which the magnetic soliton moves around freely. This assumption may be valid as long as we are concerned with a time scale much smaller than the characteristic relaxation time[5,6] of the vector $\hat{l}$. However, after a lapse of time longer than this, it is very likely that a composite soliton is formed where both $\hat{d}$ and $\hat{l}$ fields are involved, thus providing a natural trapping potential for the $\hat{d}$ soliton (unless, of course, the $\hat{l}$ field is fixed by external constraint). This potential then has significant consequences on the magnetic resonance associated with the soliton.

In order to consider this general situation, we start with the following kinetic energy term of the free energy valid in the Ginzburg-Landau (GL) regime[7]:

$$F_{\rm kin}=\tfrac{1}{2}K\int d^3r\,(\partial_i A_{\mu i}\partial_j A_{\mu j}^*+\partial_i A_{\mu j}\partial_i A_{\mu j}^*+\partial_i A_{\mu j}\partial_j A_{\mu i}^*), \tag{1}$$

where $A_{\mu i}$ (μ is the spin index; and i, orbital) are nine complex order parameters describing the condensate of superfluid ^{3}He, and

$$K=\tfrac{6}{5}(8m^*)^{-1}N(K_BT_c)^{-2}\zeta(3)/4\pi^2,$$

in the GL regime, with m^* the effective mass and N the ^{3}He density. In the A phase, where the condensate is the axial state (Anderson-Brinkman-Morel), the order parameter $A_{\mu i}$ is given as

$$A_{\mu i}=\hat{d}_\mu\cdot(\vec{\delta}_i^{\,1}+i\vec{\delta}_i^{\,2})\Delta_0/\sqrt{2}, \tag{2}$$

where $\hat{d}$ is a unit vector describing the spin coordinates, while $\vec{\delta}^1$, $\vec{\delta}^2$, and $\hat{l}$ ($\equiv\vec{\delta}^1\times\vec{\delta}^2$) are mutually orthogonal unit vectors describing the orbital coordinates. We can then rewrite Eq. (1) in terms of $\hat{d}$ and $\vec{\Delta}=(\vec{\delta}^1+i\vec{\delta}^2)\Delta_0/\sqrt{2}$ as[8,9]

$$F_{\rm kin}=\tfrac{1}{2}K\int d^3r\,\{3|\nabla\cdot\vec{\Delta}|^2+|\nabla\times\vec{\Delta}|^2+2\nabla[(\vec{\Delta}\cdot\nabla)\vec{\Delta}^*-\vec{\Delta}(\nabla\cdot\vec{\Delta}^*)]+2|\vec{\Delta}\cdot\nabla\hat{d}|^2+|\vec{\Delta}|^2\partial_i\hat{d}_\mu\partial_i\hat{d}_\mu\}. \tag{3}$$

In the following we will specialize to the configuration where a static magnetic field $\vec{H}$ is applied in the z direction. Furthermore, we limit ourselves to the case that $\vec{H}$ is large enough so that $\hat{d}$, in the equilibrium configuration, lies in the x-y plane. Then we can take

$$\hat{d}=\sin\psi\,\hat{x}+\cos\psi\,\hat{y}\ \text{and}\ \hat{l}=\sin\chi\,\hat{x}+\cos\chi\,\hat{y}, \tag{4}$$

where ψ and χ are functions of position to be determined later. The vector order parameter consistent with Eq. (4) is then given by

$$\vec{\Delta}=(-\cos\chi\,\hat{x}+\sin\chi\,\hat{y}+i\hat{z})\Delta_0e^{i\Phi}/\sqrt{2}. \tag{5}$$

Substituting (5) and (6) into (4), we have

$$\begin{aligned}F=\tfrac{1}{2}A\int d^3r\,\{&(2\sin^2\chi+1)(\partial\chi/\partial x)^2+(2\cos^2\chi+1)(\partial\chi/\partial y)^2+4\sin\chi\cos\chi(\partial\chi/\partial x)(\partial\chi/\partial y)+(\partial\chi/\partial z)^2\\&+2[|\nabla\psi|^2+(\partial\psi/\partial z)^2+(\cos\chi\,\partial\psi/\partial x-\sin\chi\,\partial\psi/\partial y)^2+4|\nabla\Phi|^2-2(\sin\chi\partial\Phi/\partial x+\cos\chi\partial\Phi/\partial y)^2\\&+2(\partial\chi/\partial z)(\sin\chi\partial\Phi/\partial x+\cos\chi\partial\Phi/\partial y)-6(\sin\chi\partial\chi/\partial x+\cos\chi\partial\chi/\partial y)\partial\Phi/\partial z+\lambda_0\sin^2(\chi-\psi)\},\end{aligned} \tag{6}$$

where $A=\frac{1}{2}K\Delta_0^{\,2}$ and $\lambda_0=\chi_n\Omega_A/A$. Here we have dropped pure divergence terms, which vanish identically in the present case. Furthermore, we have included the dipole energy term[7] [the last term in Eq. (6)], which is given as $F=-\frac{1}{2}\chi_n\Omega_A^{\,2}(\hat{l}\cdot\hat{d})^2$, with χ_n the spin susceptibility and Ω_A is the longitudinal resonance frequency. If we neglect the χ variable from Eq. (6), we obtain the equation describing a pure magnetic soliton (or $\hat{d}$ soliton) in the static limit considered previously.[1,2] We shall look for a planar solution of Eq. (6) of the form $\chi=\chi(s)$, $\psi=\psi(s)$, and $\Phi=\Phi(s)$, with $s=k_1x+k_2y+k_3z=\hat{k}\cdot\vec{x}$ and $|\hat{k}|^2=1$. Then $\Phi(s)$ is easily minimized to give $\Phi_s=k_3a\chi_s/(2-a^2)$ with $a=(k_1\sin\chi+k_2\cos\chi)$, where suffixes s mean the derivative in s. By substitution of this the free energy is reduced to

$$F/\sigma(\hat{k})=\tfrac{1}{2}A\int ds\{[1+2a^2-2k_3^{\,2}a^2(2-a^2)^{-1}]\chi_s^{\,2}+4(1-\tfrac{1}{2}a^2)\psi_s^{\,2}+\lambda_0\sin^2(\chi-\psi)\}, \tag{7}$$

where $\sigma(\hat{k})$ is the area of the surface with normal vector $\hat{k}$. We shall see later that $\chi_s^{\,2}>\psi_s^{\,2}$. Therefore the planar solution with minimum energy must correspond to the case $k_\perp^{\,2}=k_1^{\,2}+k_2^{\,2}=0$, and $k_3=\pm 1$. Introducing new variables u and v by $u=\chi+4\psi$ and $v=\chi-\psi$, we obtain

$$\begin{aligned}F/\sigma(\hat{k})=\tfrac{1}{2}A\int ds\{&\tfrac{1}{5}(1-\tfrac{1}{5}2a^2k_3^{\,2})u_s^{\,2}+\tfrac{4}{5}[1+a^2(\tfrac{3}{2}-\tfrac{8}{5}k_3^{\,2}(2-a^2)^{-1})]v_s^{\,2}\\&+\tfrac{4}{5}a^2[1-\tfrac{4}{5}k_3^{\,2}(2-a^2)^{-1}]v_su_s+\lambda_0\sin^2 v\}.\end{aligned} \tag{8}$$

For $k_\perp=0$, and $k_3=\pm 1$, we have pure twist solutions

$$u=\text{const and }\tan(u/2)=\exp[(5\lambda_0)^{1/2}z/2], \tag{9}$$

with the surface energy

$$f(\hat{l},\hat{d})=F/\sigma_3=4(\lambda_0/5)^{1/2}A, \tag{10}$$

which is compared with the $\hat{d}$ soliton energy[2]

$$F_{\parallel}(\hat{d})/\sigma = 2(2\lambda_0)^{1/2}A, \quad F_{\perp}(\hat{d})/\sigma = 4(\lambda_0)^{1/2}A, \tag{11}$$

where $\parallel$ and $\perp$ denote the orientation of the soliton plane to the vector $\hat{l}$. Since the composite soliton is thinner by a factor of $(5)^{-1/2}$, the energy is reduced by the same factor as compared with a pure $\hat{d}$ twist soliton. The pure twist soliton is shown in Fig. 1. We estimate the energy increase to a nonzero $k_\perp$ perturbatively and find

$$F/\sigma(\hat{k}) = f(\hat{l},\hat{d})[1+\tfrac{7}{40}(1-\tfrac{25}{39}\cos\tfrac{1}{5}\pi)k_\perp^2 + O(k_\perp^4)] \cong f(\hat{l},\hat{d})(1+0.1804k_\perp^2), \tag{12}$$

with $f(\hat{l},\hat{d})$ given by Eq. (10).

Therefore, once $\hat{d}$ solitons are created magnetically, it is very likely that they relax into the composite soliton, unless the vector $\hat{l}$ is constrained to be uniform by some external perturbation. When the composite soliton is formed, the $\hat{d}$ component cannot move freely, since the $\hat{l}$ texture provides the trapping center for the vector $\hat{d}$. This gives rise to nonvanishing frequency shifts in the magnetic resonance.

In order to study the resonance, let us consider small oscillations of the vector $\hat{d}$ around the equilibrium configuration. We assume that $\hat{d}$ is now given by

$$\hat{d} = [\sin(\psi+f)\hat{x} + \cos(\psi+f)\hat{y}](1-g^2)^{1/2} + g\hat{z}, \tag{13}$$

where f and g are assumed to be small. Within the quadratic approximation, the free energy associated with the fluctuations is given

$$\delta F/\sigma_3 = \tfrac{1}{2}A\int dz\big(4(\partial f/\partial z)^2 + \lambda_0\{1-2\,\mathrm{sech}^2[\tfrac{1}{2}(5\lambda_0)^{1/2}z]\}f^2 + 4(\partial g/\partial z)^2 + \lambda_0\{1-\tfrac{6}{5}\,\mathrm{sech}^2[\tfrac{1}{2}(5\lambda_0)^{1/2}z]\}g^2\big). \tag{14}$$

Both f and g modes have bound states with eigenvalues λ_1 and λ_2 with

$$\lambda_1/\lambda_0 = \tfrac{1}{2}(\sqrt{65}-7), \quad f \propto \{\mathrm{sech}[\tfrac{1}{2}(5\lambda_0)^{1/2}z]\}^{(\sqrt{13/5}-1)/2},$$

and

$$\lambda_2/\lambda_0 = \tfrac{4}{5}, \quad g \propto \{\mathrm{sech}[\tfrac{1}{2}(5\lambda_0)^{1/2}z]\}^{1/5}. \tag{15}$$

Here we have considered for simplicity the case with $\lambda = \mu = 0$, $\nu = 1$. In Lagrangian formulation of the spin dynamics,[10]

$$L = T - V, \tag{16}$$

and the above free energy δF provides the potential V, while T is given with the Euler angles describing the spin relation[10]

$$T = \tfrac{1}{2}\chi_N \int d^3r\,[\mathring{\alpha}^2 + \mathring{\beta}^2 + \mathring{\gamma}^2 + 2\mathring{\alpha}\mathring{\gamma}\cos\beta - 2\omega_0(\mathring{\alpha} + \mathring{\gamma}\cos\beta)]. \tag{17}$$

Here, a dot on top of α, β, and γ denotes the time derivative and $\omega_0 = \gamma_0 H$ is the Larmor frequency associated with the static magnetic field.

Assuming that we start with the configuration $\hat{d} = \hat{x}$ with $\alpha = \beta = \gamma = 0$, and that we are studying a small fluctuation around $\alpha = \psi(z)$, $\beta = \frac{1}{2}\pi$, and $\gamma = 0$, we can make the following identifications:

$$f = \alpha - \psi(z), \quad \gamma = g.$$

Then the Lagrangian (16) describes the magnetic resonance associated with the composite soliton. It is easy to find that the composite soliton gives rise to satellite resonance frequencies

$$\omega_l^2 = \Omega_A^2(\lambda_1/\lambda_0)$$

and

$$\omega_t^2 = \omega_0^2 + \Omega_A^2(\lambda_2/\lambda_0)$$

for the longitudinal and the transverse resonance, respectively, and which are separated from the main resonance peaks. We note that in the GL regime, the satellite frequency shifts normalized by Ω_A are independent of temperature and we have

$$(\lambda_1/\lambda_0)^{1/2} = 0.722, \quad (\lambda_2/\lambda_0)^{1/2} = 0.8944 \tag{18}$$

for the longitudinal and the transverse satellites, respectively.

About a year ago, the Saclay-Orsay group[11] re-

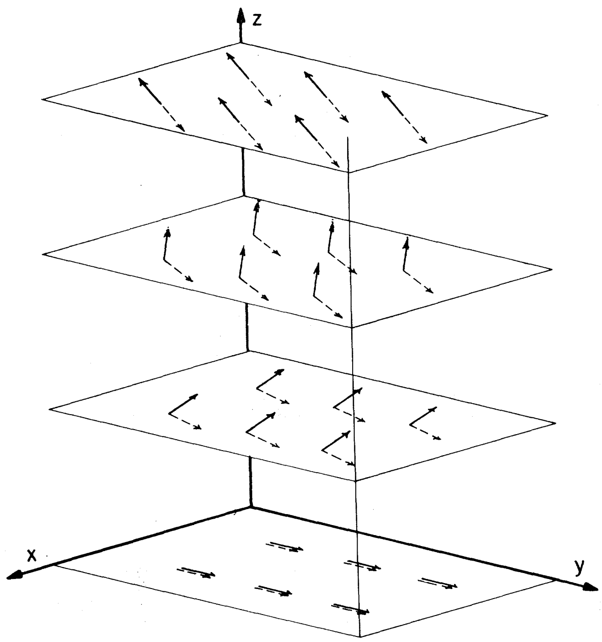

FIG. 1. Pure twist solution is shown schematically. The solid arrows indicate the direction of the vector $\hat{l}$ while the arrows with the dashed line indicate the direction of the vector $\hat{d}$.

ported observation of a satellite peak in their longitudinal resonance experiment with the frequency shift $\Omega_{\text{sat}}/\Omega_A \simeq 1/\sqrt{2}$. More recently, an extensive study of satellites in both transverse and longitudinal magnetic resonance experiments has been carried out by Gould and Lee.[12] In our notation, their experimental results may be summarized as

$$(\lambda_1/\lambda_0)^{1/2} \cong 0.74 - 0.35(1 - T/T_c),$$

$$(\lambda_2/\lambda_0)^{1/2} \cong 0.835.$$

Although their satellite frequency for the transverse resonance is somewhat different from ours, the longitudinal resonance satellite appears in good agreement with the predicted value, if one ignores the small temperature-dependent terms. Furthermore, the temperature dependence can be at least qualitatively understood, if we include the temperature dependence of the Fermi-liquid correction as discussed by Cross[9] in our free energy.

Moreover, the fact that satellite appears only in relatively open systems (which guarantees the free rotation of the vector $\hat{l}$), its relative permanence, as well as the procedure required to produce the satellite, appear to indicate the possibility that at least the longitudinal satellite is associated with the composite soliton discussed here. In this respect, the experiment by Gould and Lee[12] appears to be the first experiment where the existence of solitons in superfluid ^{3}He is established. As to the transverse resonance satellite, we note that the geometrical condition is quite different from the longitudinal experiment. They turn the static magnetic field rather than the radio-frequency field and therefore the accompanied texture could be quite different. Furthermore, absence of any temperature dependence in the observed shift normalized by Ω_A is rather strange, if we assume the transverse satellite arises from the composite soliton discussed here. We believe that if their experiment is repeated but with the static field fixed the composite soliton will produce a satellite predicted in Eq. (18).

We would like to thank Chris Gould and Dave Lee for sending us a copy of their preprint prior to publication.

---

*Supported by the National Science Foundation under Grant No. DMR76-17702.

[1]K. Maki and H. Ebisawa, J. Low Temp. Phys. 23, 351 (1976).

[2]K. Maki and P. Kumar, Phys. Rev. B 14, 118, 3920 (1976).

[3]See for example, J. A. Krumhansl and J. R. Schrieffer, Phys. Rev. B 11, 3535 (1975); M. J. Rice, A. R. Bishop, J. A. Krumhansl, and S. E. Trullinger, Phys. Rev. Lett. 36, 432 (1976).

[4]We are indebted to David Mermin, who pointed out to us that the transverse resonance frequency is unshifted as well.

[5]M. C. Cross and P. W. Anderson, in *Proceedings of Fourteenth International Conference on Low Temperature Physics, Otaniemi, Finland, 1975*, edited by M. Krusius and M. Vuorio (North-Holland, Amsterdam, 1975), Vol. 1, p. 29.

[6]D. N. Paulson, M. Krusius, and J. C. Wheatley, Phys. Rev. Lett. 36, 1322 (1976).

[7]A. J. Leggett, Rev. Mod. Phys. 47, 331 (1975).

[8]V. Ambegaokar, P. G. de Gennes, and D. Rainer, Phys. Rev. A 9, 2676 (1974).

[9]M. C. Cross, J. Low Temp. Phys. 21, 525 (1975).

[10]K. Maki, Phys. Rev. B 11, 4264 (1975).

[11]O. Avenel, M. E. Bernier, E. J. Varoquaux, and C. Vibet, in *Proceedings of Fourteenth International Conference on Low Temperature Physics, Otaniemi, Finland, 1975*, edited by M. Krusius and M. Vuorio. (North-Holland, Amsterdam, 1975), Vol. 5, p. 429.

[12]C. M. Gould and D. M. Lee, Phys. Rev. Lett. 37, 1223 (1976).