Quadrupole Interaction and High-Spin States in the Lead Region

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The energies of high-spin states formed by adding a few valence nucleons to ^{208}Pb can be accurately calculated by using empirical interaction matrix elements. It is shown that the energies of high-spin states with ^{207}Pb, ^{206}Pb, and ^{204}Pb as the core can also be accurately calculated if an additional quadrupole interaction with a strength proportional to the increased polarizability of the core is included. This interaction may contribute to the formation of yrast traps from states with a few valence nucleons with aligned angular momenta.

There is presently a great interest in nuclear structure at very high spins. The experimental study of nuclear properties at very high spins would be greatly helped if there would exist isomeric states along the yrast line.[1] Shell-structure irregularities may give rise to such "yrast traps" in some oblate nuclei, but calculations[2] indicate that shell effects may not be sufficiently strong to overcome the rapid rise in the average rotational energy with increasing spin for $I \gtrsim 30\hbar$.

The shell correction to the liquid-drop energy is usually calculated[2] by summing the fluctuating part of the single-particle energies in a deformed potential well. The interactions between nucleons near the Fermi surface are in this way only partially included. It has been pointed out[3] that the residual interactions may be especially large in states with maximum overlap of nucleonic wave functions by alignment (MONA) of the single-particle angular momenta. This additional effect may help to produce yrast traps.

Several cases of yrast traps with spins in the range $(10-20)\hbar$, which are formed by the MONA mechanism, have been found in spherical nuclei near ^{208}Pb. One simple example[4] is the 3.6-min isomer with $J^{\pi} = 12^-$ in ^{206}Tl, which has the two-hole configuration $\pi h_{11/2}{}^{-1} \nu i_{13/2}{}^{-1}$. The spatial overlap of the single-hole wave functions is largest in the aligned $J = 12$ state, and the ^3S attraction brings this state down well below the $J = 11$, 10, 9, and 8 states of the same configuration.

In the region with $Z > 82$, $N > 126$ above ^{208}Pb, the lowest spherical orbitals are $h_{9/2}$, $f_{7/2}$, and $i_{13/2}$ for protons and $g_{9/2}$, $i_{11/2}$, and $j_{15/2}$ for neutrons, all with large angular momenta. One may expect a number of high-spin isomers in this region by alignment of single-particle angular momenta. The 45-sec isomer[5] of ^{212}Po at 2.93 MeV with the assignment[6] $(\pi h_{9/2}{}^2 \nu g_{9/2} \nu i_{11/2})\ 18^+$ is one experimentally established case. The expected four-proton–four-neutron state $(\pi h_{9/2}{}^3 \pi f_{7/2} \nu g_{9/2}{}^3 \times \nu i_{11/2})\ 30^+$ in ^{216}Rn is another example of a possible yrast trap formed by the interactions among the valence nucleons. It should have an excitation energy of about 6 MeV, and probably decays by α emission to high-spin states in ^{212}Po.

The interactions between the valence nucleons are of two kinds: a direct interaction by the effective two-nucleon force, and an indirect polarization interaction from the virtual excitation of the core by one nucleon and de-excitation of the core by another nucleon. The polarization interaction can be viewed as the effect of a small adjustment of the core by the joint action of two nucleons. This description is well defined only if the core is sufficiently rigid to be weakly perturbed by the valence nucleons. That seems to be the case for the ^{208}Pb core with a few valence nucleons. One is then justified to calculate multiparticle states by standard shell-model matrix diagonalization in the valence-particle space. Such calculations give remarkably good results if one uses empirical single-particle energies *and* empirical two-nucleon interactions derived from experimental spectra of two-particle nuclei. This fact is illustrated in Table I by a comparison of calculated and experimental energies[7] for six well-established high-spin states in ^{205}Pb. The calculation was done in the space formed by three neutron holes in the $p_{1/2}$, $f_{5/2}$, $p_{3/2}$, $i_{13/2}$, $f_{7/2}$, and $h_{9/2}$ orbitals. The single-hole energies and the most relevant two-hole interaction matrix elements were obtained from ^{207}Pb and ^{206}Pb (cf. Table II), while the remaining ones were taken from the Kuo and Herling calculations.[8] The fine agreement in Table I shows that the perturbation of the ^{206}Pb core by the valence nucleons is small enough to make three-body effects negligible.

The straightforward extension of this type of calculation to nuclei with more than three valence nucleons is sometimes feasible for high-spin states, where the number of possible configura-

TABLE I. Comparison of calculated and experimental energies for high-spin states in ^{205}Pb. The main configuration and the corresponding wave-function amplitude for each state are given in columns 2 and 3.

J^π	Main configuration	Amplitude	E_{calc} (keV)	E_{expt} (keV)	$E_{expt}-E_{calc}$ (keV)
$(33/2)^+$	$i_{13/2}{}^3$	1.000	5158	5161	+ 3
$(29/2)^+$	$i_{13/2}{}^3$	0.998	5072	5064	− 8
$(29/2)^-$	$f_{5/2}i_{13/2}{}^2$	0.994	3624	3626	+ 2
$(25/2)^-$	$p_{1/2}i_{13/2}{}^2$	0.936	3199	3196	− 3
$(21/2)^-$	$p_{1/2}i_{13/2}{}^2$	0.946	3178	3168	−10
$(19/2)^+$	$p_{1/2}f_{5/2}i_{13/2}$	0.966	2026	2020	− 6

tions is low. In general, however, such calculations are impossible because of the large sizes of the matrices. Also, configuration mixing becomes more important and the errors due to the incomplete knowledge of the interaction correspondingly larger.

In some states in such nuclei it is possible to bring about a great simplification by "freezing" the degrees of freedom of some of the particles, thus describing the states in terms of a smaller number of valence particles outside of a new core, different from ^{208}Pb. Consider, e.g., the lowest 11^- state of ^{208}Po. A complete calculation in terms of two protons and two neutron holes outside ^{208}Pb is unpractical, since there are too many configurations with $J^\pi = 11^-$. Instead, the state may be described as two protons (in the $h_{9/2}$ and $i_{13/2}$ orbitals) outside the ^{206}Pb core. The energy is then given by the sum of two single-proton energies and two-proton interaction energy. Both of these will be different from the corresponding quantities with ^{208}Pb as the core, because the polarization contributions are different. The new single-proton energies can be directly obtained from ^{207}Bi (cf. Table III). For the interaction energy one may take as a first approximation the empirical value derived from ^{210}Po, i.e., with ^{208}Pb as the core. The additional contribution, caused by the polarization of the two neutron holes, will be represented by a quadrupole-quadrupole interaction with an adjustable strength parameter.

The results of a comparison of energies for nine high-spin states with two-, three-, or four-particle configurations coupled to the ^{206}Pb core are given in Table II. The calculated energies in column 4 include experimental single-particle energies with ^{206}Pb as the core, and experimental interaction energies with ^{208}Pb as the core. The differences between experimental and calculated energies are fairly large and sometimes positive, sometimes negative. According to the previous

TABLE II. High-spin states built on the ^{206}Pb core. The calculated energies in column 4 include experimental single-particle energies (Refs. 9, 10) from ^{207}Bi and ^{207}Pb and interaction energies (Refs. 11–13) from ^{210}Po, ^{211}At and ^{210}Bi, together with ground-state binding energies (Ref. 14). $\chi = -0.7$ MeV.

Nucleus	J^π	Particle configuration	E_{calc} (keV)	E_{expt} (keV)	$E_{expt}-E_{calc}$ (keV)	$\sum\chi P_2$ (keV)	Ref.
^{208}Po	6^+	$\pi h_{9/2}{}^2$	1431 ± 10 [b]	1524 ± 1	93 ± 10	82	15
^{208}Po	8^+ [a]	$\pi h_{9/2}{}^2$	1515 ± 10 [b]	1533 ± 5	18 ± 11	31	15
^{208}Po	11^- [a]	$\pi h_{9/2}i_{13/2}$	2803 ± 11 [b]	2708 ± 5	− 95 ± 12	−102	15
^{209}At	$(17/2)^-$	$\pi h_{9/2}{}^3$	1237 ± 17 [b]	1322 ± 1	85 ± 17	90	16
^{209}At	$(21/2)^-$ [a]	$\pi h_{9/2}{}^3$	1333 ± 17 [b]	1428 ± 1	95 ± 17	129	16
^{209}At	$(23/2)^-$	$\pi h_{9/2}{}^2\pi f_{7/2}$	1940 ± 17 [b]	1852 ± 1	− 88 ± 17	− 82	16
^{209}At	$(29/2)^+$ [a]	$\pi h_{9/2}{}^2\pi i_{13/2}$	2554 ± 18 [b]	2429 ± 1	−125 ± 18	−105	16
^{209}Po	$(31/2)^-$ [a]	$\pi h_{9/2}\pi i_{13/2}\nu g_{9/2}$	4670 ± 150 [c]	4266 ± 1	−400 ± 150	−296	17
^{210}At	19^+ [a]	$\pi h_{9/2}{}^2\pi i_{13/2}\nu g_{9/2}$	4380 ± 150 [c]	4028 ± 1	−350 ± 150	−330	18

[a] Configurations supported by g-factor measurements.

[b] Only empirical interactions used; errors refer to errors in nucleon separation energies.

[c] Large error because the energy of the $(\pi i_{13/2}\nu g_{9/2})$ 11^+ state in ^{210}Bi is not known experimentally.

TABLE III. Relevant single-nucleon excitation energies used in the calculations.

Core	$\pi h_{9/2}$	$\pi f_{7/2}$	$\pi i_{13/2}$	$\nu g_{9/2}$	Ref.
^{208}Pb	0	897	1609	···	19
^{207}Pb	63(4^+) 0(5^+)	···	1624(6^-) 1667(7^-)	···	20
^{206}Pb	0	992	1605	2728	19
^{204}Pb	0	···	1590	···	9

discussion, these differences should represent the extra polarization interaction between the particles due to excitations of the two neutron holes in ^{206}Pb. Since 2^+ excitations probably are most important, one can try to describe the interaction by a simple quadrupole-quadrupole coupling

$$\sum_{i<j} \chi P_2(\cos\theta_{ij}).$$

By choosing the coupling constant

$$\chi(^{206}\mathrm{Pb}) = -0.7 \text{ MeV},$$

the energy differences are very well reproduced, as seen by comparing columns 6 and 7 in Table II.

A similar description can be applied to states built on cores other than ^{206}Pb. In Table IV we have collected cases where the cores are ^{207}Pb and ^{204}Pb. In the case of a ^{207}Pb core, the single-particle energies can be obtained from suitable averages of relevant multiplets in ^{208}Bi. Good fits are obtained with the coupling constants

$$\chi(^{207}\mathrm{Pb}) = -0.25 \text{ MeV},$$

$$\chi(^{204}\mathrm{Pb}) = -1.1 \text{ MeV}.$$

The coupling constants should be proportional to the softness of the cores. As a measure of the quadrupole polarizability one may take

$$\textstyle\sum_i B(E2,\ 0\rightarrow i)/(E_i - E_0).$$

These quantities can be obtained from experimental Coulomb excitation rates to the first $\frac{5}{2}^-$ and $\frac{3}{2}^-$ states in ^{207}Pb and the first 2^+ states in ^{206}Pb and ^{204}Pb. The ratios of $\sum B_i(E2)/E$ for ^{207}Pb, ^{206}Pb, and ^{204}Pb, 0.051:0.117:0.178e^2 b^2 MeV^{-1} and 0.44:1:1.52 relative units, agree very well with the ratios of the empirical coupling constants 0.25:0.7:1.1 MeV and 0.36:1:1.57 relative units.

Let us return to the question of formation of yrast traps by the MONA mechanism. The additional interaction between a pair of high-spin particles due to the polarization of low-spin particles in the core is attractive for $j_1 + j_2 = J$ and $\chi < 0$. This will enhance the tendency to make yrast traps from states with a number of valence nucleons with aligned angular momenta. The effect increases with the polarizability of the core. Many isomers with spins $< 20\hbar$ have been observed in the Pb region, and it is quite possible that there exist isomers with much higher spins. The search for such states can be supported by energy calculations of the kind discussed here.

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TABLE IV. High-spin states built on the ^{207}Pb and ^{204}Pb cores. The calculated energies include experimental single-particle energies (Refs. 9, 20) from ^{208}Bi and ^{205}Bi and interaction energies (Ref. 11) from ^{210}Po. $\chi(^{207}\mathrm{Pb}) = -0.25$ MeV; $\chi(^{204}\mathrm{Pb}) = -1.1$ MeV.

Core	Nucleus	J^π	Particle configuration	E_{ex}	$E_{expt} - E_{calc}$ (keV)	$\sum \chi P_2$ (keV)	Ref.
^{207}Pb	^{209}Po	$(13/2)^-$	$\pi h_{9/2}^2$	1418	29 ± 8	29	17
		$(17/2)^-$	$\pi h_{9/2}^2$	1473	13 ± 8	11	17
		$(23/2)^+$	$\pi h_{9/2}\pi i_{13/2}$	2770	− 35 ± 8	− 36	17
^{204}Pb	^{206}Po	6^+	$\pi h_{9/2}^2$	1572	125 ± 26	129	21
		8^+	$\pi h_{9/2}^2$	1581 ± 5 [a]	50 ± 26	48	21
		11^-	$\pi h_{9/2}\pi i_{13/2}$	2652 ± 5 [a]	− 152 ± 26	− 160	21

[a]The 8^+ state not directly observed; γ spectra (Ref. 21) and half-lite (Ref. 21) suggest that as in ^{208}Po (Ref. 17) the separation between the 8^+ and 6^+ levels should be less than 10 keV.

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Fine Structure of the n^2D Series in Rubidium near the Ionization Limit*

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Two-photon, Doppler-free spectroscopy with counter-propagating beams from a cw dye laser and a new type of thermionic detector have been used to study the $^2D_{3/2}$ and $^2D_{5/2}$ levels of Rb for Rydberg states up to $n = 85$, only 16 cm^{-1} from the ionization limit. It is shown that the fine-structure spacing varies as n_{eff}^{-3} for $n > \sim 25$, and as a linear combination of n_{eff}^{-5} and n_{eff}^{-3} for $n < \sim 25$.

Recently, the study of highly excited states of alkali atoms, particularly of the ^2D states which exhibit inversion of fine structure, has attracted considerable attention. Experiments with quantum beats,[1] level crossing,[2] and two-photon absorption[3,4] have been employed. In general, these investigations have been limited to values of the principal quantum number $n < 20$, because of the small oscillator strengths of transitions to high n values, and the long lifetimes of these states.

Here we report the results of experiments on rubidium, with excitation to Rydberg states up to $n = 85$, only 16 cm^{-1} from the ionization limit. It is well known that the lowest ^2D level of rubidium is inverted[5] and its hyperfine structure exhibits anomalies.[6] The fine-structure splitting $n^2D_{5/2} - n^2D_{3/2}$ has been measured by classical spectroscopy[5] up to $n = 13$, and recently, by Doppler-free two-photon absorption[4] to $n = 32$. The present experiments have extended the precise measurement of the ^2D fine structure up to $n = 65$, when the doublets merge. These data now establish that the fine-structure spacing varies as the effective quantum number[7] n_{eff}^{-3}, for $n > 25$, and as a function of n_{eff}^{-5} and n_{eff}^{-3} in the region of crossover of the ^2D states ($n < 25$) from inverted to normal. The present results thus constitute the first complete study of the variation of fine-structure spacing with principal quantum number, for an alkali atom.

The present observations of highly excited Rydberg states in rubidium were made possible by the use of a sensitive thermionic detector in conjunction with the technique of Doppler-free two-photon spectroscopy.[8,9] Figure 1 shows the experimental arrangement. An Ar^+ laser was used to pump a dye laser (Spectra Physics Model 580)

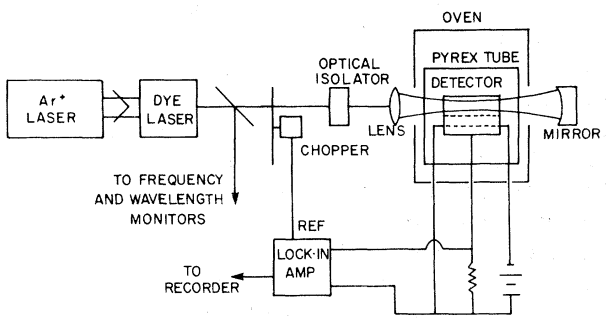

FIG. 1. Schematic diagram of the experimental arrangement.