# Rare Muon Decays, Natural Lepton Models, and Doubly Charged Leptons*

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Simple, natural models of leptons are presented in which the rare decays $\mu \to e\gamma$ and $\mu \to 3e$ occur. The models require doubly charged leptons. The experimental lower limit on the mass of doubly charged leptons determines an upper limit on $\Gamma(\mu \to 3e)/\Gamma(\mu \to e\gamma)$. Parity nonconservation in atoms may be small.

The availability of high-intensity meson beams may soon lead to an improvement of the upper limit[1] of $\sim 2\times 10^{-8}$ set on the branching ratio of the famous[2] decay $\mu \to e\gamma$. The first point to realize is that a value for the branching ratio of order $\sim 10^{-9}$ might actually be quite reasonable in the context of modern gauge theories of the weak interaction. Indeed, a theory which leads to a branching ratio of order of $\sim 10^{-9}$ has already been constructed by Cheng and Li.[3] They propose that the electron and muon couple to neutral "heptons[4]" $N$ and $N'$ through right-handed currents. In the standard notation for SU(2)⊗U(1) gauge theories, their proposal is

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L, \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L, N_L, N_L', \begin{pmatrix}N_\varphi\\ e\end{pmatrix}_R, \begin{pmatrix}N_\varphi'\\ \mu\end{pmatrix}_R, \qquad (1)$$

where $N_\varphi \equiv N\cos\varphi + N'\sin\varphi$, $N_\varphi' = -N\sin\varphi + N'\cos\varphi$, with $\varphi$ a mixing angle. The decay $\mu \to e\gamma$ proceeds through the graph in Figs. 1(a), 1(b), and 1(c); the expected branching ratio is readily seen to be of order $(\alpha/\pi)(\sin\varphi\,\Delta m^2/M_W^2)^2$ where the factor $\Delta m^2 \equiv m_N^2 - m_N'^2$ is a consequence of the Glashow-Iliopoulos-Maiani (GIM) mechanism.[5] The rather reasonable value of $\sin\varphi\,\Delta m^2 \sim 1$ GeV$^2$ then gives the branching ratio roughly as quoted above. One feature of this theory, as noted already by Cheng and Li, is that, in general, one would expect that the objects coupling to $e^-$ and $\mu^-$ via left-handed currents would not be purely the massless neutrinos $\nu_e$ and $\nu_\mu$, but some mixture of these with the massive $N$ and $N'$.

In order to avoid disagreement with experiment (large violations of muon-number conservation), such mixing must be exceedingly small. In the present state of the art, the $e$ and $\mu$ acquire mass in this theory by a bare-mass term and by couplings to Higgs triplets and singlets, whose neutral members acquire vacuum expectation values. The couplings which are required to give $e$ and $\mu$ masses inevitably lead to $\bar{\nu}_L N_R$ couplings. These couplings must then be rotated away leading to currents of the typical form $\bar{N}_L \gamma_\mu e_L$.

These considerations suggest that if $\mu^- \to e^-\gamma$ indeed occurs at the rate mentioned above it might be desirable to have a theory which combines the simplicity and order-of-magnitude value of the Cheng-Li theory but which does not mix neutrinos with neutral heptons. We have constructed two such theories, which will be referred to as the doublet and the triplet theory.

Let us discuss the triplet theory first (the doublet theory will be briefly described later): Its SU(2)⊗U(1) multiplet content is

$$\begin{pmatrix}\nu_e\\ e\\ h_\varphi\end{pmatrix}_L, \begin{pmatrix}\nu_\mu\\ \mu\\ k_\varphi\end{pmatrix}_L, e_R,\ \mu_R,\ h_R,\ k_R.$$

Here $h^=$ and $k^=$ are two doubly charged heptons and $\varphi$ is a mixing angle. We now proceed to list some features of this theory:

*Naturalness.*—The theory is natu- .l since the most general mixing (one angle) has been adopted. It is easy to read off the minimal Higgs structure required to generate masses in each theory.[6] Two Higgs triplets with hypercharges $|Y| = 0$, 1 are required. Many scalars, some doubly charged, survive as physical particles.

*$\mu \to e\gamma$ and $\mu \to 3e$.*—The two doubly charged heptons allow the decay $\mu \to e\gamma$ to proceed through the graphs[7] in Figs. 1(d) and 1(e) in addition to the graphs of Figs. 1(a)-1(c). The contributions from physical Higgs exchanges are generally small if the Higgs particles are sufficiently heavy. As remarked above Figs. 1(a)-1(c) lead to a branching ratio $\sim 10^{-9}$ for $\mu \to e\gamma$.

At first sight it seems that the graphs of Figs. 1(d) and 1(e) will be much larger, since only one $W$-boson propagator occurs. However, the GIM cancellation between the graphs with a virtual $h$ or $k$ exchanged insures that the contribution is of the same order up to a logarithmic factor.

The result of a detailed computation leads to the branching ratio ($R$)

$$R(\mu\to e\gamma)=\tfrac{75}{32}(\alpha/\pi)[\cos\varphi\sin\varphi\,\Delta m^2/M^2]^2.$$

We neglect terms down by factors $m_e/m_\mu$, $m_\mu/m_{h,k}$, and $m_{h,k}/m_W$.

We next turn to $\mu\to 3e$. Consider the matrix element of the electromagnetic current

$$\langle e|J_\alpha(0)|\mu\rangle = F_1(q^2)\gamma_\alpha+F_2(q^2)\sigma_{\alpha\beta}q^\beta m_\mu+F_3(q^2)q_\alpha,$$

with

$$F_1(q^2)=q^2F_3(q^2)/m_\mu.$$

Clearly, only the transition magnetic moment $F_2(0)$ contributes to the decay $\mu\to e\gamma$. In contrast, the transition charge radius $F_1'(0)$ also contributes to $\mu\to 3e$. Let us estimate the contribution of Fig. 1(d) to the charge radius as follows. Contract the $W$ propagator in the limit of large $M_W$. After a Fierz transformation, the resulting graph is seen to be just the standard vacuum polarization graph in QED. Thus, we obtain a contribution to $F_1'(0)$ of order $eG\ln m_h^2/m_k^2$. This is to be compared with the contribution of the graphs in Fig. (1) to $F_2(0)$ of order $eG(m_h^2-m_k^2)/M_W^2$. In other words, the GIM suppression factor in this case is logarithmic rather than differential. This striking fact is a consequence of the infrared character of the graph.[7] Thus, lepton models which allow the graphs in Figs. 1(d) and 1(e), such as ours, apparently give a larger rate[8] for $\mu\to 3e$ decay than models which do not, such as the Cheng-Li model. Keeping the dominant contribution from Fig. 1(d), we obtain the branching ratio

$$R(\mu\to 3e)=\tfrac{1}{3}(\alpha/\pi)^2[\cos\varphi\sin\varphi\ln(m_h^2/m_k^2)]^2. \quad (2)$$

Our theory determines the two branching ratios $R(\mu\to e\gamma)$ and $R(\mu\to 3e)$ in terms of three parameters, namely the mixing angle $\varphi$ and the masses of the heptons. The question is whether reasonable values of these parameters would allow the theory to be consistent with experiment. If $\varphi$ is not to be unattractively small (say $\lesssim 0.1$) then $\Delta m^2=m_h^2-m_k^2$ would have to be smaller than $m^2=\frac{1}{2}(m_h^2+m_k^2)$ in order to accommodate the experimental upper limit[9] of $\sim 6\times10^{-9}$ on $R(\mu\to 3e)$. For ease of analysis let us take $\Delta m^2/m^2$ to be small compared to unity. Then the ratio of the two decay rates become independent of $\Delta m^2/m^2$ and $\varphi$ so that

$$R(\mu\to 3e)/R(\mu\to e\gamma)=\tfrac{32}{225}(\alpha/\pi)(M_W/m)^4. \quad (3)$$

Electron-positron annihilation experiments certainly give a lower bound of $\sim 4$ GeV on $m$. This then leads to an upper bound

$$R(\mu\to 3e)/R(\mu\to e\gamma)\lesssim 15$$

if we take (for the sake of definiteness) $M_W$ to be $\sim 60$ GeV. Saturating present limits of $R(\mu\to 3e)$ of $\sim 6\times10^{-9}$ and of $R(\mu\to e\gamma)$ of $\sim 2\times10^{-8}$ we obtain $m\sim 11$ GeV. Just to see if reasonable values for $\varphi$ and $\Delta m$ are viable, let us take $M_W\sim 60$ GeV and $m\sim 11$ GeV. In that case $R(\mu\to e\gamma)\sim 1.6\times10^{-7}(\varphi\Delta m)^2$ GeV$^{-2}$ and $R(\mu\to 3e)\sim 10^{-7}(\varphi\Delta m)^2$ GeV$^{-2}$. As an example, if $R(\mu\to 3e)\sim 6\times10^{-9}$ and $\varphi\sim\frac{1}{5}$, then $\Delta m\sim 1.3$ GeV. Detailed and more general analysis without the approximation $\Delta m/m\ll 1$ will not be presented here. The reader is invited to make his own.

*Atomic physics.*—In contrast to the Weinberg-Salam model, the neutral current coupling to the electron and the muon is purely vector. Thus the predicted value of the asymmetry to be observed in atomic parity-nonconsevation experiments is much smaller than in the standard theory (a non-null effect could still arise from interference with an hadronic axial current).

*A boost to R.*—The most spectacular prediction of the theorires presented here (aside from the nonconservation of muon number!) is the existence of doubly charged heptons. Needless to say, these heptons would show up in a spactacular way in $e^+e^-$ annihilation experiments. Once they were produced, their decays would enable us to check many of the details of the weak currents we propose. The doubly charged heptons could also be produced (weakly) in $e^-p$ and $\mu^-p$ deep inelastic scattering, leading to spectacular final states with $\mu^-\mu^-$, $e^-e^-$, or $\mu^-e^-$ plus hadrons. There are also rare processes, to be sure, such as $K_L\to e\mu$ and $K^\pm\to\pi^\pm e\mu$. The expected rate for these decays will be of the same order as

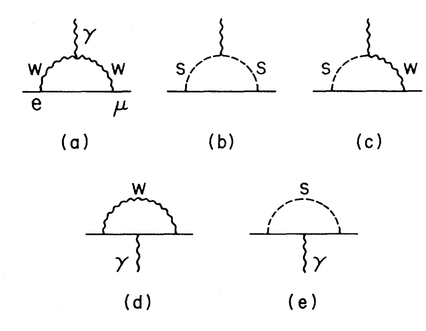


FIG. 1. Graphs contributing to $\mu\to e\gamma$. The unphysical Higgs field is denoted by $s$. In the Cheng-Li theory, the graphs in (d) and (e) are absent.

that given in Ref. 3. One could also look for $\mu^-\mu^-\mu^+\mu^+$, $\mu^-e^-\mu^+\mu^+$, etc., in hadronic collision from decays of a pair of heptons. The expected weak contribution to muon $(g-2)$ is of order $(1/\pi^2)Gm_\mu^2<10^{-8}$.

*Universality*.—Universality requires that left-handed quarks should also fall into weak SU(2) triplets, leading to new quark flavors with charges $+\frac{5}{3}$ or $-\frac{4}{3}$. Those who would rather maintain universality with left-handed quarks in doublets may prefer the following variation of our theory:

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L,\ \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L,\ \begin{pmatrix}e\\ h_\varphi\end{pmatrix}_R,\ \begin{pmatrix}\mu\\ k_\varphi\end{pmatrix}_R,\ h_L,\ k_L.$$

This doublet theory shares many of the features of the triplet theory listed above with one significant exception: The axial part of the neutral current coupling to electrons and muons is twice as large as in the Weinberg-Salam model and thus in this theory one expects approximately twice as much parity nonconservation in atoms.

Let us put our theories in perspective and summarize. If the decay $\mu^-\to e^-\gamma$ is confirmed, of course, some hitherto unknown interaction is involved. We can, however, build theories in which this new interaction is of a familiar type—simply a new weak current. This would put nonconservation of muon number *on a similar footing* to, say, nonconservation of strangeness, charm, strong $I_3$, and so forth. Our theories are relatively tightly constrained. The lower bound on doubly charged hepton mass from $e^+e^-$ experiments places an upper bound on $\Gamma(\mu\to 3e)/\Gamma(\mu\to e\gamma)$. With reasonable values of the parameters, we obtain values for these decay rates close to the present experimental upper limit. One theory also predicts that atomic parity nonconservation is much smaller[10] than predicted by the Weinberg-Salam model. Thus, measurements of rare muon decays, of atomic parity nonconservation, and of $e^+e^-$ annihilation could do much to clarify the spectrum of leptons. If, on the other hand, the limit on $\mu\to 3e$ could be substantially lowered, the theories we propose here would require mass ratios for the leptons very close to unity or very small mixing angles, and might appear less attractive.

The forward-backward asymmetry in $\mu^+\to e^+\gamma$ with a polarized $\mu$ would tell us the helicity of the outgoing $e^+$ and could also distinguish between different models.[11]

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[1]S. Parker, H. L. Anderson, and C. Rey, Phys. Rev. 133B, 768 (1964). Our work is stimulated by preliminary reports that experiments performed at Schweizerisches Institut für Nuklearforschung may have observed this decay.

[2]G. Feinberg, Phys. Rev. 110, 482 (1958).

[3]T. P. Cheng and L.-F. Li, Phys. Rev. Lett. 38, 381 (1977).

[4]We eschew the oxymoronic " heavy lepton."

[5]S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1972).

[6]We are not convinced that the Higgs mechanism with elementary scalar fields necessarily represents the dynamical breakdown of the weak SU(2)⊗U(1) gauge symmetry correctly. Nevertheless, we will discuss the consequence flowing from this hypothesis, which at least enables us to construct a consistent theory in which definite computations can be done, and even may be correct.

[7]The identical Feynman graphs arise in other contexts. See, e.g., M. Gaillard and B. Lee, Phys. Rev. D 10, 897 (1974); N. Vasanti, Phys. Rev. D 13, 1889 (1976).

[8]Our result for $R(\mu\to 3e)$ is based on the contribution from this apparently dominant contribution only. If the hepton mass gets sufficiently large, a more refined calculation is necessary.

[9]S. M. Korechenko *et al.*, Yad. Fiz. 13, 1265 (1971) [Sov. J. Nucl. Phys. 13, 728 (1971)]. This bound is a factor ~ 20 improvement over all previous experiments.

[10]P. E. G. Baird *et al.*, Nature (London) 264, 528 (1976); E. N. Fortson *et al.*, private communication.

[11]See, e.g., F. Wilczek and A. Zee, Nucl. Phys. B106, 461 (1976).