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Lossless and Dissipative Current-Carrying States in Quasi-One-Dimensional Superconductors

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We present a global stability analysis of possible states in narrow current-carrying superconductors below T_c within time-dependent Ginzburg-Landau theory. A reversible superconducting-to-normal transition may take place at a current density j_c lower than the maximum supercurrent j_{max}. Localized phase slip occurs spontaneously in a narrow range below j_c.

Current-induced transitions in superconducting filaments continue to be a puzzle. Simple "one-dimensional" situations where the coherence length $\xi(T)$ and the penetration depth $\lambda(T)$ are large compared to the transverse dimensions of the sample can be realized experimentally in the vicinity of the transition temperature T_c. Except very close to T_c, where fluctuation effects dominate, the normal state is approached through successive voltage jumps. The intervening states have been thoroughly investigated by Meyer[1] and Skocpol, Beasley, and Tinkham.[2] These authors relate them to the appearance of localized "phase-slip centers." Hysteresis appears a few millidegrees kelvin below T_c.

No satisfactory theory of these phenomena is available. The simplest time-dependent Ginzburg-Landau (TDGL) theory[3,4] is unable to explain the observed temperature-independent differential resistance presumably introduced by each center,[2] unless inhomogeneities much larger than $\xi(T)$ are significant. Nevertheless, it can provide valuable insight into the situation. We present here a complete picture of possible states in an infinite homogeneous one-dimensional superconductor within that framework. Previous attempts in that direction [5-7] suffered from *ad hoc* assumptions. Recently, Likharev[8] found a special solution describing a superconducting-normal (SN) boundary moving with constant velocity. The latter vanishes at a well-defined current density j_c below the maximum supercurrent i_{max}. We show that the superconducting (alternatively the normal) state is in fact *globally unstable* above (below) j_c. Within a limited range $j_{min} < j < j_c$ we also find a new dissipative state describing *localized phase-slip* oscillations *spontaneously* occurring in a *homogeneous* filament. Boundary effects and results for weak links, which extend those of Likharev and Yakobson,[9] will be reported elsewhere.[10]

Our work is based on numerical and limiting analytic solutions of the one-dimensional TDGL equations,

$$u(\dot{\psi} + i\mu\psi) = \psi'' + (1 - |\psi|^2)\psi, \tag{1}$$

$$j = \operatorname{Im}\psi^*\psi' - \mu'. \tag{2}$$

As in Ref. 9, the complex order parameter ψ is normalized so that its magnitude equals 1 for zero current; distance x, current density j, electrochemical potential μ, and time t are measured in units of $\xi(T)$, $j_0 = (\hbar c/2e)c/4\pi\lambda^2\xi$, $\mu_0 = ej_0\xi/\sigma_N$, and $t_0 = 4\pi\lambda^2\sigma_N/c^2 = \hbar/2\mu_0$, respectively ($\sigma_N$ is the conductivity in the normal state and t_0 is the current relaxation time). Finally, u is the order-parameter relaxation time divided by t_0. Equations (1) and (2) can be rigorously derived in the so-called strong depairing limit for a dirty gapless superconductor; one then has $u = 12$. For weak depairing $u = 5.79$,[3] but microscopic theory predicts important additional terms[11,12] which are not included here. We simply consider u as a parameter. The filament is assumed connected to a dc-current source. To avoid numerical instabilities whenever $|\psi|$ vanishes, it is important to solve for $\mathrm{Re}\psi$ and $\mathrm{Im}\psi$.[9] General solutions were computed using an implicit difference approximation scheme applied to a sufficiently long section with boundary conditions specified as explained below. Once initial transients had died out, stable stationary or periodic solutions were obtained. *Stationary* solutions with spatially varying μ (not necessarily stable once full time dependence is included) were generated via a fourth-order variable-step Runge-Kutta integration started in a region where nonlinear terms are negligible so that $\mu \simeq -jx$; ψ then obeys a complex Airy equation.[7]

There are two simple stationary solutions of the TDGL equations (1) The *normal* (N) state: $\psi \equiv 0$, $\mu' = -j$; it is in fact stable (locally, i.e., versus infinitesimal fluctuations) for all $j \neq 0$.[13] (2) The usual current-carrying *superconducting* (S) state with $\mu \equiv 0$,

$$\psi = f_\infty \exp(iqx), \quad q^2 = 1 - f_\infty^2, \quad j = f_\infty^2 q. \qquad (3)$$

For each $j < j_{\mathrm{max}} = 2/(27)^{1/2} = 0.385$ two such solutions exist; they merge at $j = j_{\mathrm{max}}$. That with the larger value of f_∞ is locally stable and becomes unstable at j_{max}; the other solution is unstable.[14] Although j_{max} is usually interpreted as the critical current at which resistance first appears, all one can say is that j_{max} represents the limit of metastability of the zero-voltage S state.

We look for the critical value j_c above which the S→N transition takes place if one waits long enough for all fluctuations with nonvanishing probability (i.e., finite energy) to occur. The S state would then be *globally unstable*. Under equilibrium conditions, the principle of minimum free energy yields a rigorous criterion for global stability. Not knowing of a similar criterion in situations where dissipation and nonlinearity are involved, we use the following reasonable principle: A necessary condition for a locally stable state to be *globally unstable* is the existence of a *finite threshold* solution *localized* about that state which neither decreases nor increases as $t \to \infty$. This leads us to consider *stationary (or periodic)* solutions of Eqs. (1) and (2) asymptotically approaching the stable N or S states described above. This was simulated by imposing $|\psi| = 0$ or $|\psi| = f_\infty$ at sufficiently large distances and verifying that numerical results were insensitive to this procedure.

Stationary threshold solutions localized about the normal state do indeed exist at low currents.

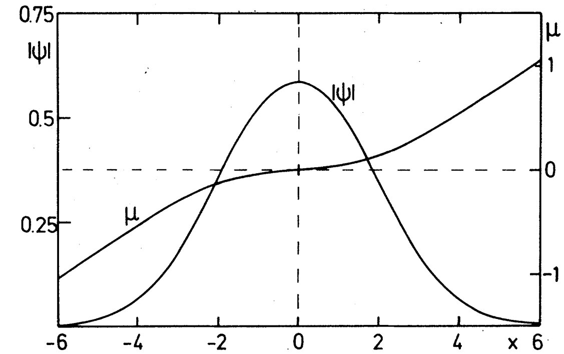

FIG. 1. Spatial dependence of stationary solution associated with the global instability of the normal state at a current density $j = 0.25$. The ratio of the order-parameter and current relaxation times is $u = 5.79$. The order-parameter amplitude $|\psi|$ and the electrochemical potential μ are plotted vs distance x (see text for appropriate units).

An example is plotted in Fig. 1 ($u = 5.79$; $j = 0.25$). In contrast to Winter and Doll,[7] we find that such solutions exist only below the critical current j_c identified previously by Likharev[8] ($j_c = 0.335$ for $u = 5.79$, and $j_c = 0.291$ for $u = 12$). For $j \to 0$ the amplitude of the threshold solution goes to zero (the *Ansatz* $\psi = fe^{i\chi}$ with $f^2 \cong j$ varying slowly compared to χ is a good approximation for $uj \ll 1$; details will be presented in a full paper), whereas for $j \to j_c$ its width diverges, i.e., it degenerates into two widely separated stationary SN boundaries. We verified that these solutions are unstable when the full time dependence is included. Near j_c this is consistent with the observation made by Likharev that a SN boundary moves towards the normal side for $j < j_c$, whereas it moves in the other direction for $j > j_c$.[8] Thus, below j_c the normal state is globally unstable; we expect it to be globally stable above j_c.

As for the superconducting state, we notice that for all $j < j_{\mathrm{max}}$, an unstable analytic static solution of Eqs. (1) and (2) which differs only locally from the uniform solution (3) has been found by Langer and Ambegaokar (LA).[14] They identified it as the threshold fluctuation for a single phase slip: As in subsequent treatments[15] it was assumed that the S state was re-established after ψ had gone through zero at one point, thereby reducing the net phase change φ along the filament by 2π.[16]

Starting from a slightly perturbed LA solution (below j_{max}) to initiate a phase slip, we find three types of asymptotic behavior:

(i) Below a current j_{min} somewhat smaller than

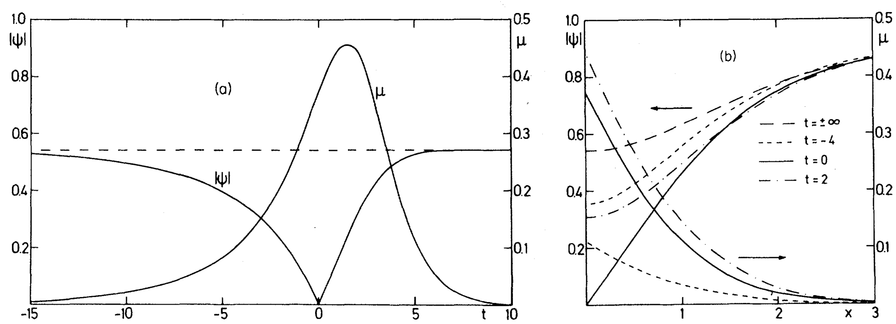

FIG. 2. Localized phase-slip solution in a homogeneous one-dimensional superconductor at a current density $j = 0.326$. For a relaxation-time ratio $u = 5.79$ this corresponds to the lower bound $j_{\rm min}$ of the range where such solutions occur. The order-parameter amplitude $|\psi|$ and the electrochemical potential μ are plotted (a) vs time at the location ($x = 0$) of the phase slip; (b) vs distance x for times preceding, coinciding, and following the instant of phase slip (dashed, full, and dot-dashed curves, respectively). When $t \to \pm\infty$ the corresponding Langer-Ambegaokar solution (Ref. 14) is approached (long-dashed curve). Appropriate units are defined in the text.

j_c ($j_{\rm min} = 0.326$ for $u = 5.79$ and $j_{\rm min} = 0.284$ for $u = 12$), ψ heals back to the pure S state, often, however, after *several phase slips* have occurred.

(ii) Between $j_{\rm min}$ and j_c a new locally stable solution is approached. The filament is superconducting, except for strong localized oscillations in which ψ rapidly goes through zero at one point, while the voltage exhibits a sharp peak. The phase φ increases continuously by 2π between such spontaneous slips. When $j \to j_{\rm min}$ the period goes to infinity; the corresponding solution is localized in space and time. It approaches the static unstable LA solution for $t \to \pm\infty$, and performs *one phase slip* in between. Figure 2(a) shows $|\psi(t)|$ and $\mu(t)$ at the site of the phase slip ($x = 0$). In Fig. 2(b) $|\psi(x)|$ and $\mu(x)$ are plotted at various times; as j is increased above $j_{\rm min}$ the maximum $|\psi(0)|$ decreases.

(iii) Above j_c the order parameter performs oscillations with decreasing amplitude between phase slips. Eventually an expanding normal domain develops. Thus, in this range the LA solution represents the threshold fluctuation for nucleation of the normal state, and the superconducting state is globally unstable.

As a result the LA theory[14] and subsequent improvements[15] describing thermally activated resistance near T_c need revision at high current densities. This does not invalidate experimental verifications of that theory performed to date since they were restricted to $j \ll j_{\rm max}$.

We are confident of the accuracy of our computations since they are consistent with similar ones for long weak links (length $a \gtrsim 20\xi$). The latter were simulated by imposing $|\psi| = 1$ and the Josephson relation $\Delta\mu = -\dot{\varphi}$ at the boundaries. As j increases above j_c we find a changeover from behavior (ii), which Likharev and Yakobson[9] failed to notice, to a solution of the form[9]

$$\psi(x,t) \simeq g(x) + g^*(a-x)e^{i\varphi(t)}, \qquad (4)$$

where $g(x)$ is a stationary solution of Eqs. (1) and (2) *localized near one boundary* and satisfying $g(0) = 1$; the link is then normal except near its ends. When uj or $(uj)^{1/3}x$ are sufficiently large, nonlinearities may be neglected, and $g(x)$ is proportional to the Airy function $\mathrm{Ai}[e^{-i\pi/6}(uj)^{1/3}(x - i/uj)]$. We verified that (4) remains a good approximation except just above j_c. The transition at j_c becomes smeared out in shorter links. In the limit where $a \lesssim \xi$ and $u(a/2\xi)^2 \ll 1$, the computed normalized voltage $\dot{\varphi}(t)$ agrees well with the analytic approximation presented in Ref. 9. When $u(a/\xi)^2 \gtrsim 36$ we find hysteresis and strong deviations[10] from an unjustified extension[17] of that approximation.

In conclusion, we wish to emphasize the following points: Although thermal fluctuations are not included explicitly in our treatment, their qualitative effects may be ascertained. In particular, if sufficient time is allowed, the SN transition should occur reversibly at j_c. The times involved may, however, be astronomically large in practice: Hysteretic behavior may occur. Clearly, it should be influenced by end effects.[10]

In real quasi-one-dimensional superconductors phase slips will presumably occur at weak spots, so that several phase-slip centers may coexist. This is the standard explanation for the experimentally observed voltage steps.[1,2] Further calculations including inhomogeneities are needed.

Within TDGL theory, we see no possibility for substantial "time-dependent superconductivity" in a long *homogeneous* filament above j_c, let alone j_{max}, contrary to previous suggestions.[5-7] Rieger, Scalapino, and Mercereau[5] obtained large amplitude oscillations for $j > j_{max}$ by *forcing* 2π phase slips whenever an *ad hoc* criterion motivated by analogy with LA theory was satisfied. Whether thermal fluctuation effects can be simulated in this way, for that range, seems doubtful. The "solutions" of the TDGL equations given in Ref. 6 are, in fact, neither solutions (because discontinuities in ψ' have been artificially introduced), nor do they approximate any actual solutions.[18]

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Fine Structure of the Luminescence from Excitons and Multiexciton Complexes Bound to Acceptors in Si†

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My high-resolution studies of the bound-exciton luminescence in Ga-doped Si reveal a triplet structure. Also, I report for the first time bound multiexciton complexes associated with Ga, which are similar to those previously reported for Si doped with P, Li, and and B. In the case of Ga, however, the $m=2$ bound-multiexciton complex luminescence is found to be split by an amount equal to the splitting between two of the bound-exciton lines, strongly suggesting that the $m=2$ complex decays into the bound exciton.

The nature of the bound-multiexciton complexes (BMEC), whose luminescence has been observed in lightly doped Si at low temperatures,[1,2] is an unresolved problem in semiconductor physics. We have undertaken high-resolution photoluminescence studies of Si doped with a variety of impurities in order to obtain information as to the multiplicities of these states. In this Letter I report a number of new results obtained in Ga-doped Si, including the resolution of the Ga bound-exciton (BE) luminescence into a triplet structure, the observation of BMEC associated with Ga, and the doublet structure of the luminescence associated with one of the BMEC. These results provide important new information for any theoretical models of the BMEC. Completely analogous results for Al-doped Si, as well as those for some other impurities, will be published elsewhere.

The results reported here were obtained from a sample containing 1×10^{15} cm^{-3} Ga, along with a small amount of P, immersed in either liquid He between 1.6 and 4.2 K or liquid H_2 at 15 K. The luminescence was excited by ≤ 1.5 W of Ar-ion laser light in an unfocused beam ($d \sim 4$ mm). The luminescence was analyzed by a Perkin-Elmer