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Phase Slippage without Vortex Cores: Vortex Textures in Superfluid ^{3}He

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The characteristic dissipation process for conventional superfluid flow is phase slippage: motion of quantized vortices in response to the Magnus force, which allows finite chemical potential differences to occur. Topological considerations and actual construction are used to show that in liquid ^{3}He-A, textures with vorticity but no vortex core can easily be constructed, so that dissipation of superfluid flow can occur by motion of textures alone without true vortex lines, dissipation occurring via the Cross viscosity for motions of $\hat{l}$.

Usually dissipative relaxation of the order parameter of a broken-symmetry condensed system occurs by motion of order-parameter singularities. For instance, magnetic hysteresis involves the motion of domain walls, slip of solids that of dislocations, and self-diffusion that of vacancies or interstitials. These are 2-, 1-, and 0-dimensional "order-parameter singularities." All are characterized by a "core" of atomic dimension where the order parameter departs substantially from its equilibrium value.

One of the clearest examples of this general rule is phase slippage in superconductors (flux flow and creep) and in liquid helium II: The only way in which these superfluides in bulk form can sustain a gradient of chemical potential, and thus flow dissipatively, is by the continual motion of quantized vortex lines transverse to that gradient. The controlling equation is[1]

$$\langle \mu_1 - \mu_2 \rangle = \left\langle \frac{\hbar\, d(\varphi_1 - \varphi_2)}{dt} \right\rangle = h\,\frac{dn_{\text{vortices}}}{dt}, \qquad (1)$$

where $\langle \mu_1 - \mu_2 \rangle$ is the time-averaged chemical potential difference between two points, $\varphi_1 - \varphi_2$ is the phase difference of the mean particle (or pair for superconductors) field, and dn_{vortices}/dt is the rate of passage of quantized vortices across the line joining 1 and 2.

Quantized vortex lines can exist[2] in the anisotropic superfluid ^{3}He-A, but as one of us has shown,[3] the circulation around such a vortex is not a topological invariant, as it is in the simple superfluids; it is, in those, equivalent to the so-called "winding number." In ^{3}He-A, on the other hand, by making a rotation of the order parameter which is continuous everywhere, i.e., by superposing a "texture," one may convert a vortex line of either sign into the opposite one or into a de Gennes disgyration; and two vortex lines of the same sign can in principle annihilate each other. Those topological results demonstrate that quantization of vorticity and vortex line motion are not the keys to dissipative processes that they are in the conventional cases, since they destroy the second equality of Eq. (1). Nonetheless the first equality of Eq. (1) shows that phase slippage by one mechanism or another is necessary in order to feed energy from superfluid flow into dissipation processes (since the energy dissipated is $\Delta\mu\, dN/dt = \Delta\mu\, J_{\text{tot}}$). We describe here some likely mechanisms by which dissipation can occur by phase slippage *without* order-parameter singularities, by the motion of textures alone.

Briefly the geometry of the order parameter in ^{3}He-A is that of the orthogonal triad of vectors $\hat{l}$, $\vec{\Delta}_1$, and $i\vec{\Delta}_2$, where $\hat{l}$ is the orbital angular momentum of the pairs, and $\vec{\Delta}_1$ and $i\vec{\Delta}_2$ are real and imaginary vectors representing the two components of the anisotropic energy gap. Rotations of $\vec{\Delta}_1$ and $i\vec{\Delta}_2$ about $\hat{l}$ are phase changes of the order parameter, while rotations of $\hat{l}$ rotate the anisotropy axis of the system. Rotations of a rigid frame generate the group SO(3) which has the topology of projective space P_3, the three-dimensional sphere S_3 with diametrically opposite points on the surface identified. The rule of Ref. 3 is that one maps paths around the singularities in real space into this gap-parameter space, and if the path cannot be deformed continuously into a point, the singularity is stable. 360° rotations are equivalent to paths between the two identified poles on the sphere and are topologically nontrivial, but a second 360° rotation in any direction returns one to the starting point and the resultant is equivalent topologically to no rotation at all.

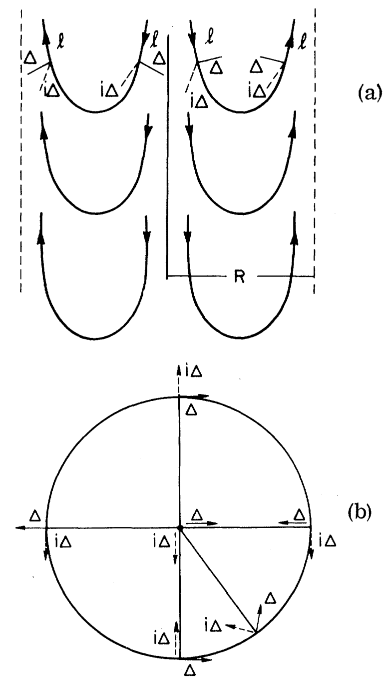

FIG. 1. Sketch of the 4π vortex texture. Lines are stream of $\hat{l}$, and $\hat{\Delta}$ and $i\hat{\Delta}$ rotate as sketched. (a) Side view; (b) view along the length.

Thus, in principle a 720° vortex requires no core.

These ideas led us to search for a texture which can play the role of a double vortex line, of which an example is the following (see Fig. 1). On the axis of a circular cylinder $\hat{l}$ is in the $-\hat{z}$ direction, $\vec{\Delta}_1$ and $i\vec{\Delta}_2$ in say the $\hat{x}$ and $\hat{y}$ directions, respectively. Along radii of this cylinder $\hat{l}$ rotates about the $\hat{\varphi}$ direction through an angle $f(r)$, with $f(R) = \pi$, $f(0) = 0$. At the cylinder R, we will find $\hat{l}$ pointing in the $+\hat{z}$ direction and the phase rotating by 4π as we circumnavigate the cylinder. (This is a simple modification of the Brinkman-Osheroff texture.[4]) A generalization of this to a texture equivalent to a vortex sheet can easily be made.

Another peculiar property of this texture is that it can terminate in a "hedgehog" or pointlike object at which the lines of $\hat{l}$ splay out in all directions. The phase rotates by 360° while $\hat{l}$ rotates by 360° around the circumference of the hedgehog (see Fig. 2).

These vortex textures have two properties which suggest that they may play a role in dynamical processes. First, having no core, the energy will be less than that of two normal 2π vortices by a factor of order $\frac{1}{2}\ln(R/\xi)$ (2 for the double quantum) which will usually make them energetically cheaper. Second, they have no localized

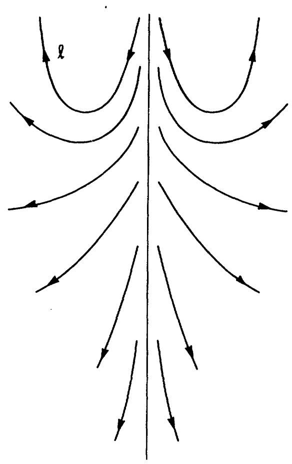

FIG. 2. Termination of 4π vortex texture in a "hedgehog," which is the texture equivalent of a "monopole" attached to the end of a vortex line.

pattern of high superfluid velocity and thus the characteristic and puzzling problem of vortex nucleation will not be as serious.

On the other hand, the phase slippage theorem [the first equality of Eq. (1)] is still valid between any two points in the fluid where the orientation of $\hat{l}$ does not change. This is because the usual argument from the number-phase commutation relation or from gauge invariance[1] that $d\varphi/dt$ equals $\partial E/\partial N$ is universal to all superfluids, but, if $\hat{l}$ is moving too rapidly, it is not safe to equate $\langle \partial E/\partial N\rangle$ with μ: hence the proviso on motions of $\hat{l}$. φ is, of course, not a velocity potential but is uniquely definable when $\hat{l}$ is fixed, as it will be effectively, in most physical situations, at boundaries by the boundary condition on $\hat{l}$, and in many other situations in the bulk of the system because of orienting flows and fields or the Cross "normal pinning" effect.[5] Equation (1), not its character as a velocity potential, is the key to the importance of the phase φ in dissipative processes. For instance, flow between two orifices at which $\hat{l}$ can be expected to be pinned (see, for example, Wheatley[6]) will see a chemical potential difference obeying the phase slippage equation, and dissipation can occur by motions of vortex textures across the path between the orifices. The importance of vortex textures in this context arises from the fact that $\hat{l}$ is fixed in direction in the exterior region, and with $\hat{l}$ fixed in the exterior region *changes* in vorticity are still quantized. The problem of boundary conditions will be discussed in more detail in a future publication.

In this situation at least, the textures will experience the characteristic Magnus force in the presence of a background superfluid flow field v_s,

$$F_{\rm M}=\rho\vec{v}_s\times\vec{\Omega}, \tag{2}$$

where Ω is the circulation, in this case equal to h/m in magnitude. But much more general situations can be understood if we simply retain the idea of a locally definable phase, except possibly where $\hat{l}$ is moving too fast, and rely on the second fundamental equation

$$-\frac{dN_{\rm pairs}}{dt}=\frac{1}{\hbar}\frac{\partial\langle E\rangle}{\partial\varphi}. \tag{3}$$

Equation (3) implies the more general remark[7] that a current source may always be inserted as a phase-dependent term in the Hamiltonian:

$$H_{\rm source}=-dN/dt_{\rm ext}\,\varphi.$$

Hence, a current source exerts an appropriate force on any texture whose motion can allow phase slippage.

The final physical fact is that textures move relatively slowly and dissipatively in ^{3}He because of the Cross "normal pinning" effect,[5] so that vortex textures can only affect very low-frequency phenomena. We envisage three flow regimes.

(1) For high-frequency phenomena such as fourth sound and vibrating wires, or for large chemical potential differences, ^{3}He will behave like a conventional but anisotropic superfluid, since orbital motion will be pinned and phase slippage can only occur by motion of conventional vortices.

(2) In the absence of a magnetic field and for moderately low frequencies, $\hat{l}$ will be free to orient at will and vortex textures will play a great role in causing dissipation. The critical velocity for nucleating textures should be extremely low, of order h/mR where R is an apparatus size, or $\sim 10^{-2}$ cm/sec. This is even lower than what is observed in heat flow experiments by Wheatley's group, but in the right range. It seems possible that in the absence of a magnetic field almost any flow will fill the sample with enough vorticity to damp out fluctuations and the relatively quiet behavior at low fields could be a turbulent regime.

(3) In a magnetic field $\hat{l}$ is oriented by the dipolar energy for structures larger than the length R_S of order $\sim 100\ \mu$m[4] determined by the ratio of dipolar to current energy. The vortex structures must be of this order and thus contain velocities of order $h/mR_S\sim 10^{-1}$ cm/sec. This is again the right order of magnitude, but a bit fast relative to some critical velocities measured in resonance

and other experiments. In both of these cases, it is easily possible that a regular motion of vortex structures can be set up under appropriate flow conditions, which we speculate may be related to certain observations of regular and irregular orbital fluctuations.[6]

It is interesting to speculate on the outcome of a measurement of quantized vorticity in ^{3}He-A by the Vinen[8] vibrating-wire experiment or otherwise. At a Vinen wire the boundary condition will require $\hat{l}$ to be radial and the phase may rotate by any integer number of units 2π; a texture in, for instance, a cylinder can simply add or subtract 4π to this, so that any integer amount of vorticity is possible. However, the results might be chaotic in the absence of a field because of vortex textures throughout the liquid. With a field the wire can again have integer vorticity but in the surrounding liquid the 4π double vorticity is the most stable vortex line, consisting of a "core" which is a Fig. 1 texture of size $\sim R_s$, and a conventional outer region. Thus, one may tend to add or subtract *double* units.

In summary, the most important point to be made is that dissipation in this superfluid is qualitatively different from that in other superfluids and in most broken-symmetry systems, in that it can occur by the motion of textures (rather like "topological solitons") and not only by singularities of the order parameter. Thus, the property of superfluidity takes a very novel form in this case.

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Condensation of Optically Excited Carriers in CdS: Determination of an Electron-Hole-Liquid Phase Diagram

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We demonstrate formation of an electron-hole liquid in CdS by phase separation of optically excited carriers. Measurement of the electron-hole-liquid density as a function of pump intensity and sample temperature determines the liquid portion of the liquid-gas coexistence curve giving a low-temperature liquid density of 2×10^{18} cm^{-3} and $T_c = 55$°K.

We present measurements which demonstrate the phase separation that occurs when an electron-hole liquid (EHL) forms by condensation from a less dense gas of excitation in CdS. These measurements allow construction of the liquid portion of the gas-liquid coexistence curve. Experimental identification of the EHL phase, bound by 13 meV, in highly excited CdS has been recently reported[1-3] and these measurements are in good agreement with the calculations of Beni and Rice.[4] The liquid chemical potential was found to be independent of pump intensity at 2°K,[1] suggesting that phase separation was occurring but no other evidence for this phenomenon was provided. The new results reported here provide additional evidence for phase separation and establish that the critical temperature, above which the liquid does not condense, is $T_c \simeq 55$°K. In addition the low-