Efimov State in the ^{4}He Trimer

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On the basis of a Faddeev calculation, an Efimov state is predicted to exist in $^4He_3$. This discovery represents the first manifestation of the Efimov effect and may have far-reaching consequences in the statistical mechanics of ^{4}He gas at low temperatures.

In the quantum-mechanical system of three identical bosons interacting via short-range, two-body forces of type $gv(r)$ there occurs a remarkable phenomenon when the coupling strength g approaches, matches, and then exceeds g_0, the critical value required to just bind the two-body system.[1] As the value of g changes, a series of weakly bound $J^p = 0^+$ states appears one after another, their number reaching infinity at "resonance" when $g = g_0$. When g grows beyond g_0, just as quickly as in their appearance, these three-body bound states disappear into the continuum! This startling effect, labeled the Efimov effect after its discoverer,[1] is attributed to the emergence of an effective, attractive, long-range force of radius $|a|$, where a is the two-body scattering length.[2] An equivalent explanation, suggested by Amado and his collaborators,[3] is the infrared divergence of the kernel in the Faddeev-theory treatment of the three-body problem. The transient or Efimov states have large spatial extent and their number is yielded by the formula[1]

$$N = \pi^{-1} \ln|a/r_0|, \tag{1}$$

where r_0 is the effective range of the two-body force.

Although Efimov has speculated on the possible existence of nuclear states possessing "Efimov effect" characteristics, no one has yet pinned that tag to any 0^+ state[2] and it appears unlikely that we will uncover a situation that is favorable for the effect's manifestation in the nuclear domain. Recently, while engaged in the Faddeev solution of the ^{4}He trimer problem,[4] we were struck by the proximity to "resonance" of the ^{4}He interatomic forces[5] we were dealing with, and how well this would bode for any program dedicated to a diligent search for Efimov states in $^4He_3$. Thus, upon completion of our original work, we decided to turn our attention to the task of verifying the presence of Efimov states in the system of three ^{4}He atoms. In this Letter, we report the results of our study—the discovery of one such state in $^4He_3$.

We began by evaluating N from Eq. (1) using, as representative two-body interaction, the potential MDD-2 proposed by Brunch and McGee.[6] Their ^{4}He interatomic potential is generally accepted as one of the more realistic.[4] Its effective range and scattering length were computed with the aid of the expressions for these quantities in Wu and Ohmura[7] and the algorithm of Merrill.[8] We found N to be 1.01, a definite indication that an Efimov state exists. Encouraged, we pressed on to solve the Faddeev equation for its energy.

The definitive work of Amado and co-workers[3] has made it obvious that the essential features of Efimov states are determined by $l = 0$ parts of the two-body interaction. Thus we kept only the $l = 0$ contributions to the kernel and spectator functions in the Faddeev equation, which was thus written as

$$\mu_i \chi_i(\vec{p},\vec{k}) = \iint K(\vec{p},\vec{k},\vec{p}\,',\vec{k}\,',E)\chi_i(\vec{p}\,',\vec{k}\,')\,d^3p'\,d^3k', \tag{2}$$

where the number of three-body bound states is given by the number of eigenvalues μ_i which exceed unity at the scattering threshold, i.e., $E = E_2$, the two-body binding energy. The kernel function $K(\vec{p},\vec{k},\vec{p}\,',\vec{k}\,',E)$ is directly related to the two-body off-shell t matrix[3] while $\chi_i(\vec{p},\vec{k})$ is the spectator function which yields information on the size of the three-body system. In solving Eq. (2), we used the unitary-pole-expansion method of Harms[4,9] to derive a separable form of the potential, and thus we reduced Eq. (2) to the one-variable equation

$$\mu_i \chi_i(p) = \sum_{j,k=1}^{M} \int_0^\infty Z_{ij}(p,p',E)\Delta_{jk}(E - \tfrac{3}{4}p^2)\chi_k(p')p'^2\,dp', \tag{3}$$

where

$$Z_{ij}(p,p',E)=\int_{-1}^{+1}\frac{\psi_i(|\vec{p}'+\frac{1}{2}\vec{p}|)\psi_j(|\vec{p}+\frac{1}{2}\vec{p}'|)}{E-p^2-p'^2-\vec{p}\cdot\vec{p}'}\,d\cos\theta_{pp'} \quad (4)$$

and

$$-\Delta_{jk}^{-1}(E-\tfrac{3}{4}p^2)=\lambda_j\delta_{jk}+\int_0^\infty\frac{\psi_j(q)\psi_k(q)}{E-\frac{3}{4}p^2-q^2}\,q^2dq, \quad (5)$$

with ψ_i and λ_i satisfying the homogeneous Lippmann-Schwinger equation.

$$\psi_i(p)=\lambda_i\int_0^\infty\frac{V(p,k)\psi_i(k)}{E_2-k^2}\,k^2dk, \quad (6)$$

with

$$V(p,k)=(2/\pi)\int_0^\infty j_0(pr)V(r)j_0(kr)r^2dr. \quad (7)$$

The binding energies of the bound states were extracted by searching for eigenvalues of unity in Eq. (3) with $M=3$; we used Gauss and Filon quadratures, cubic-spline interpolation, and matrix inversion in the analysis.[4]

Our results are shown in Table I. There are definitely two 0^+ bound states in $^4He_3$, the second of which should be the Efimov state. This excited state has an energy below threshold of only 0.0004 K, indeed a loosely bound state when compared to the ground state whose energy is 0.086 K.[4] A plot of their respective spectator functions (see Fig. 1) reveals graphically that the excited state is concentrated at small values of the momentum. It is overwhelmingly larger in size than the ground state which is itself huge.[10] To obtain conclusive proof that we have an Efimov state, we deepened the MDD-2 potential by a factor of 1.01 (identifying this force as MDD-2D on Table I) so that N fell below unity to 0.80, then solved the Faddeev equation again. The excited 0^+ state disappeared, as predicted for Efimov states:

The Efimov effect is only weakly dependent on the details of the two-body forces.[3] Because most realistic ^{4}He interatomic potentials possess the same characteristics, viz., "near resonance" and large scattering length, as MDD-2—e.g., the Beck potential[5] has $|a|\simeq 600$ Å and $N>1$—it is very likely that they will yield similar results. Three-body forces, such as the Slater-Kirkwood force,[10] are of relatively short range and have no influence on the effect. From the results of this work, it appears that an Efimov state may exist in $^4He_3$. Because this theoretical discovery may have far-reaching consequences in studies of the low-temperature behavior of ^{4}He gas,[11] we commend its search to interested experimentalists.

TABLE I. Eigenvalues and binding energies of the excited 0^+ states in $^4He_3$.

Potential	a (Å)	r_0 (Å)	Eigenvalues μ_1	μ_2	μ_3	E_2 (K)	$E_{\rm Efimov}$ (K)
MDD-2	175	7.3	4.08	1.81	0.80	0.0005	0.0009
MDD-2D	117	9.5	2.45	0.96	0.58	0.0010	Unbound

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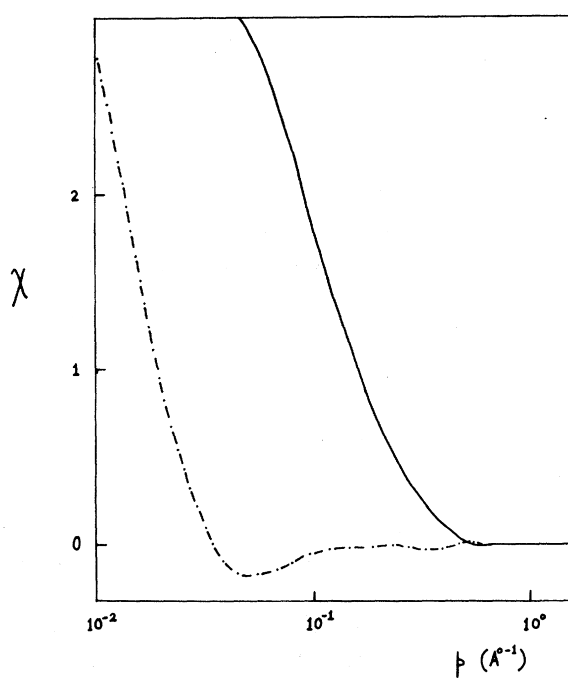

FIG. 1. Plot of the momentum-space spectator functions of the two 0^+ states in $^4He_3$ for MDD-2. The solid line represents the ground state.

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Polarization Effects in the Differential Quenching Cross Section of Na* by Diatomic Molecules

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The dependence of scattering intensities for the quenching process $Na^*(3^2P) + N_2(v=0) \rightarrow Na(3^2S) + N_2(v')$ on the polarization of the exciting laser light has been studied. A distinct anisotropy is observed at small scattering angles, which is most pronounced where the scattering cross section is largest. It is found that the quenching cross section σ_1 for a $3p\pi$ state is smaller that σ_0 for the $3p\sigma$ state and thus electronic angular momentum is transferred to collisional and/or rotational angular momentum.

In a recent Letter[1] we have reported first experiments on the quenching of laser-excited $Na(3^2P_{3/2})$ at thermal energies in the differential scattering process:

$$Na(3^2P_{3/2}) + N_2(v=0, j_{therm}) + E_{therm} \rightarrow Na(3^2S_{1/2}) + N_2(v', j') + E_{kin}'. \quad (1)$$

A crossed-beam apparatus is used. The primary sodium beam is supersonic with a full width at half-maximum (FWHM) of 20%, and the thermal molecular beam effuses from a capillary array at 80 or 300°K. The initial c.m.-system energy, E_{therm}, is defined by the relative average energies and kinematics of the two beams. The amount of energy transferred into vibrational and/or rotational energy is found from the measured final kinetic energy E_{kin}' and from the electronic excitation energy E_{el} (= 2.1 eV), and is $\Delta E_{vib\ rot} = E_{el} + E_{therm} - E_{kin}'$. A typical energy-transfer spectrum is shown in Fig. 1, displaying the c.m.-system kinetic energy after collision.

It has been stated that the experimental findings are in qualitative agreement with theoretical models proposed by Bjerre and Nikitin[2] and by Bauer, Fisher, and Gilmore.[3] The theoretical calculations are tentative for several reasons. Among other shortcomings, no attention has been paid to the influence of rotational-energy transfer, and the general assumption is that only vibrational energy levels are excited (the v' scale in Fig. 1 corresponds to this assumption). The present

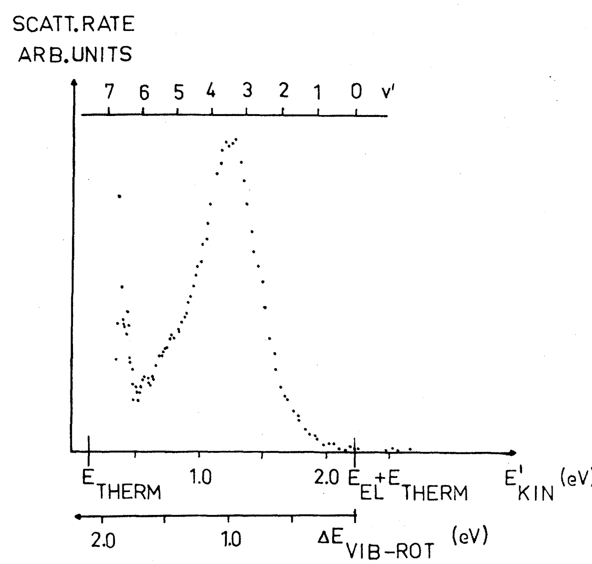

FIG. 1. Energy-transfer spectrum for the quenching process $N_2 + Na^*(3P) \rightarrow N_2 + Na(3S)$. Displayed is the difference "light on" − "light off" as a function of the energy after collision in the c.m. system. Initial Na velocity is 1370 m/sec, 20% FWHM; N_2 temperature is $\approx 80°$ K, $\theta_{lab} = 10°$ corresponding to $\theta_{c.m.s.} = 5°$ at $\Delta E_{vibrot} = 1$ eV. The rise of the scattering rate below 0.5 eV indicates elastic scattering from the excited sodium which cannot be distinguished unambiguously from a remaining energy transfer to $\nu' = 7$.