dent than the production rate for D mesons which has a peak near 4.03 GeV. They can be misidentified as heavy leptons.

The weak current obtained from (3) and (5) will cause the decay of the blue quarks to baryons. For instance, the decays $u^3 \to \Xi^0\pi^+$, $u^3 \to \Xi^0 e^+ \nu_e$, and—less frequently—$u^3 \to \Lambda\pi^+$ are predicted.

Beside the Cabibbo rotation of the quark fields in the weak current, further mixing effects are possible. In particular, a small mixing of $\nu_L{}^e$ with $u_L{}^1$ and of $\nu_L{}^\mu$ with $s_L{}^{2C}$ appears likely (in the convention used here, ν^e, e^- and ν^μ, μ^- carry lepton number -1):

$$\nu_L{}^e \to \nu_L{}^e + \epsilon_1 u_L{}^1, \quad u_L{}^1 \to u_L{}^1 - \epsilon_1 \nu_L{}^e,$$
$$\nu_L{}^\mu \to \nu_L{}^\mu + \epsilon_2 s_L{}^{2C}, \quad s_L{}^{2C} \to s_L{}^{2C} - \epsilon_2 \nu_L{}^\mu, \qquad (8)$$
$$\epsilon_{1,2} \ll 1.$$

If such a mixing occurs, the charged weak current obtained from (3) and (5) will also be responsible for the decays of red and white quarks: $s^1 \to \nu^\mu e^- \bar{\nu}^e$, $s^1 \to \nu^\mu \mu^+ \bar{\nu}^\mu$, $s^2 \to \mu^+ +$ hadrons, $d^1 \to \nu^e +$ hadrons, $d^1 \to e^- +$ hadrons, etc. These decays are similar to the ones expected for heavy leptons but with different transition rates and hadron multiplicities. The same mechanism will then also cause the decays of those diquark states which are simultaneously baryons and leptons leading, for example, to the unusual $\Xi\mu$ final states. The values of the mixing parameters ϵ are restricted. An upper limit may be obtained by considering the process $K^+ \to \pi^+ \nu_e \bar{\nu}_\mu$ which is of order ϵ^4. With the experimental number[14] $R(K^+ \to \pi^+ 2\nu) \lesssim 0.6 \times 10^{-6}$ one finds $\epsilon \lesssim 0.05$.[15]

The existence of quarks and diquarks would lead to remarkable effects. Perhaps it is worthwhile to look for them, if for no other reason than to be sure that they are not there.

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Asymptotic Estimates in Scalar Electrodynamics

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Lipatov's estimates for large orders of perturbation theory are extended to scalar electrodynamics.

Lipatov has recently obtained estimates for large orders of the renormalized perturbation series in scalar field theories using semiclassical methods.[1-3] He has beautifully extended the pioneering work done on the quantum-mechanical harmonic oscillator by Bender and Wu[4] and by Loeffel *et al.*[5] We present similar results pertaining to scalar electrodynamics.

One-particle irreducible Euclidean Green's functions behave as

$$\Gamma_n=\sum_k \Gamma_n^{(k)}(-e^2/4\pi)^k, \quad \Gamma_n^{(k)}=k!A^k k^{b_n} c_n[1+O(1/k)], \tag{1}$$

where k is the order of perturbation, and the constant A is estimated as

$$A=0.0808. \tag{2}$$

The real numbers b_n and c_n depend on the process described by Γ_n. This implies that the series may be resummed by use of a Borel transformation:

$$\Gamma_n(e^2/4\pi)=\int_0^\infty du\, e^{-u}\tilde{\Gamma}_n(e^2u/4\pi), \quad \tilde{\Gamma}_n(u)\sim\sum_k(-uA)^k k^{b_n}c_n, \tag{3}$$

where $\tilde{\Gamma}_n(u)$ is analytic inside a circle of radius A^{-1} with a power-type singularity on the negative real axis, controlled by the exponent b_n. The extension of these estimates to the case of fermionic charged fields will be the subject of a future publication.

The generating functional $G(j)$ of connected Green's functions is represented in Euclidean space-time as a path integral:

$$\exp G(j)=\int D(\varphi)\exp[-A_0(\varphi)-A_1(\varphi)-\int dx\, j\varphi], \tag{4}$$

where A_0 and A_1 denote the free and interacting parts of the action. For the time being we ignore the effect of renormalization which can be accounted for by using the methods of Refs. 1 and 2.

In the simplest case, $A_1(\varphi)$ is proportional to a coupling constant g. We rewrite it as $gA_1(\varphi)$ and expand its exponential in powers of g, thus obtaining the kth order of perturbation theory as

$$\exp G(j)=(-g)^k C_k(j), \quad C_k(j)=\int D(\varphi)\exp[-A_0(\varphi)-\int dx\, j\varphi](k!)^{-1}[A_1(\varphi)]^k. \tag{5}$$

Under the assumption that $A_1(\varphi)$ is a positive functional, $C_k(j)$ is dominated for large k by the contribution of those φ's for which $A_1(\varphi)$ is maximal given $A_0(\varphi)$. In the typical case of renormalizable scalar massless field theory with

$$A_0(\varphi)=\int d^dx\tfrac{1}{2}(\partial\varphi)^2, \quad A_1(\varphi)=\int d^dx(\varphi^2)^N, \tag{6}$$

where $N=d/(d-2)$, one has the Sobolev bound[6]

$$[A_1(\varphi)]^{1/N}\leq[8/d(d-2)]S_d^{-2/d}A_0(\varphi), \tag{7}$$

with $S_d=2\pi^{(d+1)/2}[\Gamma(\frac{1}{2}(d+1))]^{-1}$, the area of the unit sphere in $d+1$ dimensions. In other words, the Sobolev bound controls the growth of the interaction part of the action in terms of the free one. In the special case of a massless field, the bound is reached for the set of conformal transformed functions

$$\varphi_{a,\lambda}(x)=\left[\frac{2\lambda}{\lambda^2+(x-a)^2}\right]^{(d-2)/2}. \tag{8}$$

If we add a mass term to A_0, a strict inequality sign would hold in (7), and one could only ascertain the existence of a sequence (without limit) of functions approaching the bound as closely as one wants.

From (7) we deduce that the appropriate Borel transform

$$\sum_k(-b^N)^k C_k(j)\frac{k!}{(Nk)!}=\sum_{p=0}^{N-1}\int D(\varphi)\exp[-A_0(\varphi)-e^{2i\pi p/N}bA_1^{1/N}-\int dx\, j\varphi] \tag{9}$$

converges in a circle $|b|<\frac{1}{8}d(d-2)S_d^{d/2}$ with a singularity coming from the vicinity of the functions given in (8). This yields for the positive $C_k(j)$ the estimate

$$C_k(j)\sim\frac{(Nk)!}{k!}\left[\frac{8}{d(d-2)}S_d^{-d/2}\right]^{kN}, \tag{10}$$

up to yet unknown powers of k. To proceed further, one notices that the saturation of the Sobolev bound implies a variational problem which coincides with a variation of the original action A_0+gA_1 for a *negative effective coupling constant*

$$g_c=-\tfrac{1}{8}d^2S_d^{2/d}[A_1(\varphi)]^{-2/d}. \tag{11}$$

Therefore, one can understand the use of the steepest-descent method in Refs. 1 and 2 to compute, in either form, the original functional integral

$$C_k(j)=\frac{1}{k!}\int D(\varphi)\exp[-A_0(\varphi)-k\ln A_1-\int dx\,j\varphi]=\int D(\varphi)\,\frac{dg}{2\pi i g}\exp[-A_0(\varphi)-gA_1(\varphi)-k\ln(-g)-\int dx\,j\varphi], \tag{12}$$

both leading to the same equation, up to a scale on φ, as the saturation of the Sobolev bound.

This method enables one to compute the next terms in (10) by integration over the fluctuations around the classical solution (8). Care has to be taken of translation and dilatation invariance, using collective coordinates. It is of course necessary to reintroduce the renormalization counterterms in the original action, and one is led to a new perturbation theory to extract the full asymptotic series in k, up to standard calculational difficulties. It should be noticed that the very divergent character of the initial perturbation series allows one to obtain rather easily estimates of one-particle irreducible Green's functions. We refer to the original works for an appreciation of how surprisingly accurate these estimates are and how they allow an improved numerical treatment of perturbation theory.

Although the Sobolev inequalities offer a compact mathematical means to understand the structure of Lipatov's result, physical insight is increased by realizing that one has naturally been led to wander in the coupling-constant complex plane, until one has reached a point of vacuum instability. This point of view is in agreement with an early argument of Dyson,[7] who pointed out that perturbation theory is presumably only asymptotic, and also with Langer's[8] calculation of the exponentially small vacuum decay probability for small negative coupling constant.

To implement the above program for QED we face new difficulties. Either the potential A_μ is coupled to a Fermi field and we have to treat statistics correctly, or it is coupled to a scalar complex field φ and the coupling constant appears both linearly and quadratically in the interaction, which does not *a priori* possess positivity properties. Furthermore, gauge freedom complicates the task. In any case, we might expect cancellations among sets of gauge-invariant diagrams. We choose to discuss the scalar case and omit mass terms and four-scalar particle coupling at first. Therefore, we write in Euclidean variables and with a gauge-fixing term

$$\begin{aligned}\exp G(J_\mu,j)&=\int D(A,\varphi,\varphi^*)\exp\{-\int d^4x[\tfrac{1}{4}F^2+\tfrac{1}{2}\lambda(\partial\cdot A)^2+(\partial+ieA)\varphi^*\cdot(\partial-ieA)\varphi+j\varphi^*+j^*\varphi+J\cdot A]\}\\ &=\sum_k e^k\int D(A,\varphi,\varphi^*)\exp\{-\int d^4x[\tfrac{1}{4}F^2+\tfrac{1}{2}\lambda(\partial\cdot A)^2+\partial\varphi^*\partial\varphi+j\varphi^*+j^*\varphi+J\cdot A]\}\\ &\qquad\times\frac{(\int d^4x\,A^2\varphi\varphi^*)^{k/2}}{k!}H_k\left(\frac{\int d^4x\,(1/i)\varphi^*\overleftrightarrow{\partial}\varphi\cdot A}{\{\int d^4x\,A^2\varphi^*\varphi\}^{1/2}}\right).\end{aligned} \tag{13}$$

Call C_K the coefficient of e^k. The Hermite polynomials are of course not positive definite. However, the inequalities,

$$(\int d^4x\,A^2\varphi^*\varphi)<(3/16\pi^2)\int d^4x\,\partial\varphi\partial\varphi^*\int d^4x\,\tfrac{1}{2}\textstyle\sum_\mu(\partial A_\mu)^2, \tag{14a}$$

$$\frac{[\int d^4x\,(1/i)\varphi\overleftrightarrow{\partial}\varphi^*\cdot A]}{[\int d^4x\,A^2|\varphi|^2]^{1/2}}<2[\int d^4x\,|\partial\varphi|^2]^{1/2}, \tag{14b}$$

$$|H_n(x)|\leq 2^{n/2-E(n/2)}[n!/E(n/2)!]\exp\{2|x|[E(n/2)]^{1/2}\}, \tag{14c}$$

enable one to show that a constant A exists, such that

$$|C_{2k}(J,j)|\sim|C_{2k+1}(J,j)|\lesssim[(2k)!/k!]A^k.$$

To obtain the best value of A and the oscillation in sign which allows the series to be asymptotic for $e^2>0$, we can proceed as in the scalar case. We try to find solutions for the classical equations of motion, with a finite action. In order to investigate the possibility of a saddle point involving real fields we have to set $i\varphi^*\overleftrightarrow{\partial}\varphi=0$, i.e., φ real, and try to fulfill the following constraints, obtained when using the analog of (12):

$$(\partial^2 - e^2A^2)\varphi = 0, \tag{15a}$$

$$(\partial A + A\partial)\varphi = 0, \tag{15b}$$

$$-\partial^2 A_\mu + (1-\lambda)\partial_\mu \partial\cdot A + 2e^2 A_\mu \varphi^2 = 0, \tag{15c}$$

$$k + e^2 \int d^4x\, A^2\varphi^2 = 0. \tag{15d}$$

Clearly the saddle-point value e^2 is negative, and we propose the following *Ansatz*:

$$(-e_c^{\,2})^{1/2} A_\mu = M_{\mu\nu} x^{\nu-2} a(x^2), \tag{16a}$$

$$(-e_c^{\,2})^{1/2}\varphi = (2/x^2)^{1/2} f(x^2), \tag{16b}$$

$$-e_c^{\,2} = (2\pi^2/k)\int_0^\infty dx^2\, x^{-2} a^2 f^2, \tag{16c}$$

with $M_{\mu\nu}$ a nonsingular 4×4 antisymmetric matrix such that

$$M = -M^T, \quad M^2 = -1. \tag{17}$$

The above parametrization ensures that

$$\partial A = 0, \quad x\cdot A = 0, \tag{18}$$

in such a way that the gauge-fixing parameter λ will disappear from the solution and (15b) will be automatically fulfilled. By use of $t = \ln x^2$ as a variable, the original action takes the form

$$\int d^4x[\tfrac{1}{4}F^2 + \lambda\tfrac{1}{2}(\partial\cdot A)^2 + (\partial + ieA)\varphi^*\cdot(\partial - ie)\varphi] = [2\pi^2/(-e_c^{\,2})]\int dt\{(\dot a^2 + a^2) + 4\dot f^2 + f^2 - a^2 f^2\}, \tag{19}$$

with

$$\ddot a = a(1-f^2), \quad \ddot f = \tfrac{1}{4}f(1-a^2), \tag{20}$$

corresponding to (15a) and (15c). The solution is required to make the integrals

$$I = \int_{-\infty}^{+\infty} dt\, a^2 f^2 = \int_{-\infty}^{+\infty} dt(\dot a^2 + a^2) = \int_{-\infty}^{+\infty} dt(4\dot f^2 + f^2) \tag{21}$$

well defined. We have looked for a numerical solution of (20) using the observation that one can search for symmetric functions with respect to an arbitrary origin in t chosen to be zero. We follow the tra-

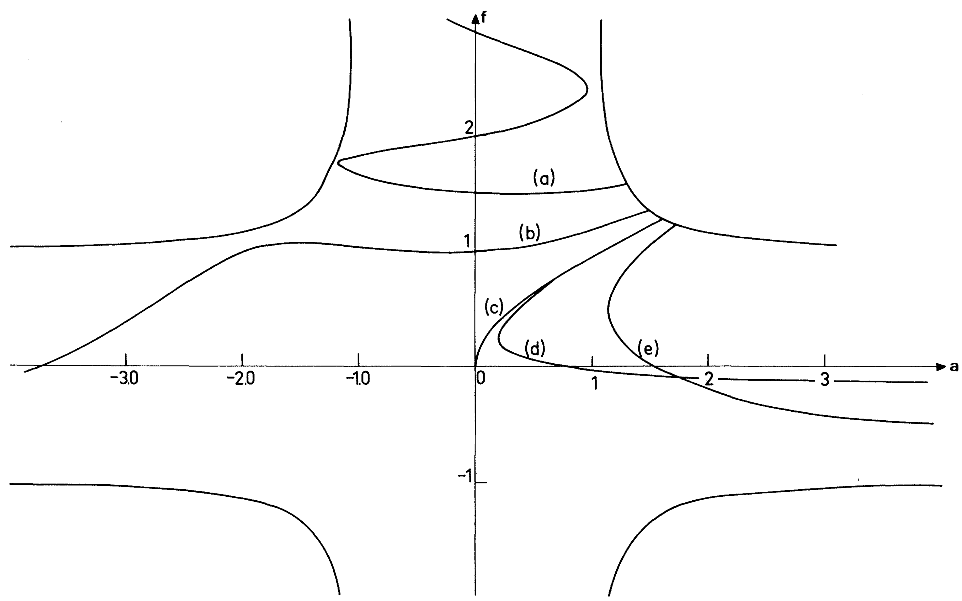

FIG. 1. Trajectories of zero energy in the a,f plane. Curves a, b, c, d, and e correspond to different values of $a(0) = 1.300$, 1.600, 1.6165, 1.620, and 1.732.

jectories leaving with zero velocity the zero-potential curve at $t=0$, as they proceed down the deep valleys except for the unique solution (up to reflections in a and f) which slowly reaches the point $a=f=0$ along the curve $a=f^2$. Figure 1 displays this behavior. We have checked that the virial theorem (21) is satisfied to a great accuracy leading to the value

$$I=7.874. \tag{22}$$

We have of course to presume that our *Ansatz* has picked the nearest singularity of the Borel transform. If this is the case we see that in spite of the new intricacies implied by the gauge field, we have recovered the same situation as in the case of a self-coupled scalar field. After some manipulations we obtain formula (1) with A expressed as

$$A=(2/\pi I) \tag{23}$$

giving the numerical value quoted in (2).

The normalization factors (b_n and c_n, the latter containing all the dependence on external momenta and polarization) can be extracted from the above numerical solution and will involve the renormalization counterterms.

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Precise Measurements of Axial, Magnetron, Cyclotron, and Spin-Cyclotron-Beat Frequencies on an Isolated 1-meV Electron

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A sensitive frequency-shift technique is employed to monitor the magnetic quantum state of a single electron stored in a compensated Penning trap. The electron sees a weak parabolic magnetic pseudopotential in addition to the electric well, which causes the axial oscillation frequency to have a slight magnetic quantum state dependence. Transitions at both the spin-cyclotron-beat (anomaly) frequency and the cyclotron frequency have been measured in the same environment to yield a magnetic anomaly $a_e=(1\,159\,652\,410\pm 200)\times 10^{-12}$.

The first measurement of the electron-spin magnetic moment on *free* (relativistic) electrons was carried out in 1953 by Crane and co-workers[1] giving $|a_e|\leq 5\times 10^{-3}$ for the "anomaly" defined as $a_e\equiv(\nu_s-\nu_c)/\nu_c$. Here, ν_s and ν_c denote spin and cyclotron frequencies, respectively, in the nonrelativistic limit. This study led to the famous University of Michigan "$g-2$" experiments,[1] finally yielding $a_e=(1\,159\,656\,700\pm 3500)\times 10^{-12}$ which was previously the most accurate experimental value. However, the first definite value,[1] $a_e=(1116\pm 40)\times 10^{-6}$, was obtained in 1958 at the