PHYSICAL REVIEW LETTERS

VOLUME 38 14 FEBRUARY 1977 NUMBER 7

Equivalence Principles and Electromagnetism*

Wei-Tou Ni
Department of Physics, Montana State University, Bozeman, Montana 59715, and Department of Physics, National Tsing Hua University, Hsinchu, Taiwan, Republic of China†
(Received 16 June 1976)

The implications of the weak equivalence principles are investigated in detail for electromagnetic systems in a general framework. In particular, I show that the universality of free-fall trajectories {Galileo weak equivalence principle (WEP[I])} does *not* imply the validity of the Einstein equivalence principle (EEP). However, WEP[I] plus the universality of free-fall rotation states (WEP[II]) *does* imply EEP. To test WEP[II] and EEP, I suggest that Eötvös-type experiments on polarized bodies be performed.

The Einstein equivalence principle (EEP), i.e., the minimal coupling principle, is the cornerstone of the gravitational coupling of matter and nongravitational fields in general relativity and metric theories of gravity. For such an important principle, it is crucial that its empirical foundations be analyzed in depth.

Empirical evidence supporting EEP comes from Eötvös experiments[1-3] and gravitational-redshift experiments.[4,5] Eötvös experiments provide direct tests of Galileo weak equivalence principle (WEP[I]). Braginsky and Panov[3] demonstrated that the accelerations of an aluminum test body and a platinum test body, placed in the sun's gravitational field at the location of Earth, agree to 1 part in 10^{12}. Redshift experiments provide tests of EEP. Pound, Rebka, and Snider[4,5] demonstrated that, to 1% accuracy, the gravitational redshifts of photons are metric (i.e., those predicted by EEP). These experiments show that the local Lorentz frames determined by test bodies and by photons agree to 1% accuracy.

In 1960, Schiff[6] argued as follows: "The Eötvös experiments show with considerable accuracy that the gravitational and inertial masses of normal matter are equal. This means that the ground-state eigenvalue of the Hamiltonian for this matter appears equally in the inertial mass and in the interaction of this mass with a gravitational field. It would be quite remarkable if this could occur without the entire Hamiltonian being involved in the same way, in which case a clock composed of atoms whose motions are determined by this Hamiltonian would have its rate affected in the expected manner by a gravitational field." He suggested that EEP and, hence, the metric gravitational redshift are consequences of WEP[I]. If this conjecture holds, then since Eötvös experiments verify WEP[I] to high precision, EEP is also verified to high precision. Therefore, the scope of validity of Schiff's conjecture has a direct bearing on the analysis of the empirical foundations of EEP.

Recently, as experimental tests of relativistic gravity have improved, interest in Schiff's conjecture has revived. Thorne, Lee, and Lightman[7] have analyzed in detail the fundamental concepts and terms involved and have given a plausibility argument supporting Schiff's conjecture. Lightman and Lee[8] have proven Schiff's conjecture for electromagnetically interacting systems in a static, spherically symmetric gravitational field using a particular mathematical formalism known as the T-H-ϵ-μ formalism.

In this Letter, I use a general framework (in which the T-H-ϵ-μ formalism is a special case) of gravitational coupling of electromagnetism to study the scope of validity of Schiff's conjecture. I find that although the universality of free-fall trajectories (WEP[I]) constrains the gravitational coupling severely, it does *not* imply the validity of the EEP. In the counterexample (to Schiff's conjecture) which obeys WEP[I] but violates EEP, there is an anomalous torque on a polarized test body so that the test body will change its rotation state. Since the motion of a macroscopic test body is determined not only by its trajectory but also by its rotation state, the motion of polarized test bodies will not be the same. I, therefore, propose the following stronger weak equivalence principle (WEP[II]) to be tested by experiments, which states that in a gravitational field, the motion of a test body with a given initial motion state is independent of its internal structure and composition (universality of free-fall motion). In our general framework, the imposition of WEP[II] guarantees that EEP is valid. Therefore we have, in this framework, the following relations among the equivalence principles:

$$\mathrm{WEP[I]} \not\Leftarrow \mathrm{WEP[II]} \Leftrightarrow \mathrm{EEP}.$$

To test WEP[II] and, hence also EEP, Eötvös-type experiments on electromagnetic-energy-polarized body are proposed.[9] In the following, I will derive these results.[10]

The Lagrangian density.—Start with the following interaction Lagrangian density[11] $\mathfrak{L}_{\mathrm{I}}$ for an electromagnetically interacting system in a gravitational field

$$\mathfrak{L}_{\mathrm{I}} = -\left(\frac{1}{16\pi}\right)\chi^{ijkl}F_{ij}F_{kl} - A_k j^k(-g)^{1/2} - \sum_I m_I \frac{ds_I}{dt}\delta(\vec{x}-\vec{x}_I), \tag{1}$$

where χ^{ijkl} is a tensor density of the gravitational fields (e.g., g_{ij}, φ, etc.), and j^k, $F_{ij} \equiv A_{j,i} - A_{i,j}$ have the usual meaning. This is the most general interaction Lagrangian density with the following conditions: (i) uncharged particles following geodesics of a Riemannian metric; (ii) electric charge being conserved; (iii) only gravitational fields (potentials), not their gradients, being involved in $\mathfrak{L}_{\mathrm{I}}$; (iv) quadratic in the gradient of the electromagnetic potential and no masslike terms involved in $\mathfrak{L}_{\mathrm{I}}^{(\mathrm{EM})}$. In the absence of gravity, $\mathfrak{L}_{\mathrm{I}}$ is assumed to reduce to the special-relativistic Lagrangian. For test bodies, the gravitational fields can be treated as external fields.

Constitutive tensor density.—The constitutive tensor density χ^{ijkl} has the symmetries $\chi^{ijkl} = \chi^{klij} = -\chi^{klji}$. In general, it has 21 independent components. For a metric theory, χ^{ijkl} is determined by the metric g^{ij} and equals $(-g)^{1/2}(\frac{1}{2}g^{ik}g^{jl} - \frac{1}{2}g^{il}g^{kj})$.

Stress-energy tensor density, four-momentum, and center of mass.—In conformity with the definitions for the standard Lagrangian formulation and for dielectric materials, I define the electromagnetic stress-energy tensor density as

$$\mathcal{T}_i^{k(\mathrm{EM})} = A_{l,i}(\partial\mathfrak{L}^{(\mathrm{EM})}/A_{l,k}) - \delta_i^{\,k}\,\mathfrak{L}^{(\mathrm{EM})} = (1/4\pi)(-\chi^{klmn}A_{l,i}F_{mn} + \tfrac{1}{4}\chi^{jlmn}F_{jl}F_{mn}\delta_i^{\,k}). \tag{2}$$

The total stress-energy tensor density is $\mathcal{T}_i^{\,k} = \mathcal{T}_i^{\,k(\mathrm{EM})} + \mathcal{T}_i^{\,k(\mathrm{P})}$, where $\mathcal{T}_i^{\,k(\mathrm{P})}$ is the usual stress-energy tensor density of particles. The four-momentum vector of a test body is $P_i = \int \mathcal{T}_i^{\,0}\,d^3x$. Defining the center of mass as $X^i = \int x^i \mathcal{T}_0^{\,0}\,d^3x/P_0$, then one can readily show that

$$\dot{X}^i = P^i/P^0 \tag{3}$$

for a test body.

Matter-response equation.—From the Euler-Lagrange equations, I derive the matter-response equation

$$\mathcal{T}_{i,k}^{\;\;k} = -\partial\mathfrak{L}_{\mathrm{I}}/\partial x^k. \tag{4}$$

From Eq. (4), one can show that

$$\dot{P}_m = (1/16\pi)\chi^{ijkl}{}_{,m}\int F_{ij}F_{kl}\,d^3x + \tfrac{1}{2}g_{kl,m}\int \mathcal{T}^{kl(\mathrm{P})}\,d^3x. \tag{5}$$

I now impose the condition of WEP[I]. Since a nonelectromagnetically interacting test body follows a geodesic in the metric g^{ij}, any other test body will follow such a geodesic too. Choose a Fermi-normal coordinate system such that the test body is at rest in the system and the Christoffel symbols vanish along the geodesic. I then have $\dot{P}_m = d(P^0\dot{X}_m)/dt = 0$. Compare this with Eq. (5), I conclude that

$$\chi^{ijkl}{}_{,m}\int F_{ij}F_{kl}\,d^3x = 0. \tag{6}$$

Lemma.—In a fixed coordinate system, Eq. (6) holds for every test body if and only if $\chi^{ijkl}{}_{,m}$

$=\varphi_{,m}e^{ijkl}$, *where*

$$e^{ijkl}=\begin{cases}1, & \text{if } (ijkl) \text{ is an even permutation of } (0123),\\ -1, & \text{if } (ijkl) \text{ is an odd permutation of } (0123),\\ 0, & \text{otherwise.}\end{cases}$$

[If one expands F_{ij} in powers of χ^{ijkl} and $\chi^{ijkl}{}_{,m}$, and substitutes it into Eq. (6), every order in the expansion of (6) must vanish.] From the vanishing of the first-order expansion,

$$\chi^{ijkl}{}_{,m}\int F_{ij}{}^{(0)}F_{kl}{}^{(0)}\,d^3x=0, \qquad (7)$$

where $F_{ij}{}^{(0)}$ is the solution of Maxwell's equations in special relativity. Using Eq. (7) and considering test bodies consisting of a parallel-plate capacitor and a solenoidal coil of current, one can derive (i) $\chi^{0123}{}_{,m}=\chi^{0231}{}_{,m}=\chi^{0312}{}_{,m}$, and (ii) all other components of $\chi^{ijkl}{}_{,m}$ not related to the components in (i) by the symmetry property vanish. Therefore $\chi^{ijkl}{}_{,m}=\varphi_{,m}e^{ijkl}$, where $\varphi\equiv\chi^{0123}$ and I prove the "only if" part of the Lemma. If $\chi^{ijkl}{}_{,m}=\varphi_{,m}e^{ijkl}$, we have

$$\int\chi^{ijkl}{}_{,m}F_{ij}F_{kl}\,d^3x=4\int\varphi_{,mj}e^{ijkl}A_iA_{k,l}\,d^3x=0 \qquad (8)$$

because the second derivatives of gravitational fields can be neglected for test bodies. In the derivation of the first equality in Eq. (8), an average over a dynamical time scale for the body has been performed in order to make the surface terms vanish. This is the standard virial-theorem technique used in treating a macroscopic body. Q.E.D.

Theorem I.—*For a system whose Lagrangian density is given by (1), WEP[I] holds if and only if*

$$\chi^{ijkl}=(-g)^{1/2}(\tfrac{1}{2}g^{ik}g^{jl}-\tfrac{1}{2}g^{il}g^{kj}+\varphi\epsilon^{ijkl}), \qquad (9)$$

where φ is a scalar function of the gravitational fields and $\epsilon^{ijkl}=(-g)^{1/2}e^{ijkl}$.—In a Fermi-normal coordinate system of a test-body geodesic, by the Lemma, WEP[I] holds if and only if

$$\chi^{ijkl}{}_{,m}=\varphi_{,m}e^{ijkl}. \qquad (10)$$

In a region R where gravitational effects are negligible, $\chi^{ijkl}=\frac{1}{2}\eta^{ik}\eta^{jl}-\frac{1}{2}\eta^{il}\eta^{kj}$ (where η^{ij} is the Minkowski metric). Starting from this region, integrating Eq. (10) along the geodesic, and transforming to an arbitrary coordinate system, I get Eq. (9). On the assumption that there exists a geodesic network connecting every point in space-time to such a region R, Eq. (9) then holds for every space-time point. Now it is easy to see that φ is a scalar function of the gravitational fields. Moreover, if Eq. (9) holds, then Eq. (6) holds in the Fermi-normal frame, and hence WEP[I] holds. Q.E.D. If $\varphi\neq0$ in (9), the gravity coupling to electromagnetism is not minimal and EEP is violated. Therefore in Theorem I, we have shown that WEP[I] does *not* imply the validity of EEP.

From Theorem I, one can easily prove the results of Lightman and Lee[8]: *Corollary.*—*For an electromagnetically interacting system in a static spherical-symmetric gravitational field whose Lagrangian density is given by*

$$\mathcal{L}_I=(1/8\pi)(\epsilon E^2-B^2/\mu)-A_kj^k(-g)^{1/2}-\textstyle\sum_I m_I(ds_I/dt)\delta(\vec{x}-\vec{x}_I),$$

where ϵ and μ are functions of the gravitational fields, WEP[I] implies EEP.—For the above Lagrangian density $\chi^{1234}=0$; hence, $\varphi=0$. Q.E.D.

In the theory with χ^{ijkl} given by (9), it can be shown with some calculations that there are anomalous torques on electromagnetic-energy–polarized bodies unless $\varphi=0$. For $\varphi=0$ in (9), the theory reduced to metric theory and EEP holds. Therefore we arrive at the following theorm: *Theorem II.*—*For the Lagrangian (1), WEP[II] implies EEP.*

Eötvös-type experiments on test body.—To test WEP[II], it is crucial to perform Eötvös-type experiments on polarized bodies. In view of Theorem II, this is also an excellent test for EEP.

The nonmetric theory given by Eq. (9) may be related to the existence of parity-nonconserving field or spontaneously broken symmetry. Since these concepts are quite fruitful in weak interactions and in obtaining unified weak and electromagnetic theories, serious efforts deserve to be spent in this direction for gravitation. Detailed analyses of this nonmetric theory and the Eötvös-type experiments on polarized bodies will be presented elsewhere.

*Work supported in part by the National Aeronautics and Space Administration Montana State University Contract No. NGR27-001-040 and by the National Science Council of the Repubic of China.

†Present address.

[1]R. V. Eötvös, D. Pekar, and E. Fekete, Ann. Phys. (Leipzig) 68, 11 (1922); also R. V. Eötvös, Math. u. naturw. Ber. Ungam. 8, 65 (1890).

[2]P. G. Roll, R. Krotkov, and R. H. Dicke, Ann. Phys. (N.Y.) 26, 442 (1964).

[3]V. B. Braginsky and V. I. Panov, Zh. Eksp. Teor. Fiz. 61, 873 (1971) [Sov. Phys. JETP 34, 463 (1972)].
[4]R. V. Pound and G. A. Rebka, Phys. Rev. Lett. 4, 337 (1960).
[5]R. V. Pound and J. L. Snider, Phys. Rev. B 140, 788 (1965).
[6]L. I. Schiff, Am. J. Phys. 28, 340 (1960).
[7]K. S. Thorne, D. L. Lee, and A. P. Lightman, Phys. Rev. D 7, 3563 (1973).
[8]A. P. Lightman and D. L. Lee, Phys. Rev. D 8, 364 (1973).
[9]This experiment is first proposed by K. Nordtvedt, Jr., to test WEP[I] for polarized bodies (private communication).
[10]These results have been presented in W.-T. Ni, Bull. Am. Phys. Soc. 19, 655 (1974).
[11]This follows the terminology used in W.-T. Ni, Astrophys. J. 176, 769 (1972). Thorne, Lee, and Lightman (Ref. 7) used the term "nongravitational Lagrangian."

Are the *D* Mesons Diquarks?

Berthold Stech
Institut für Theoretische Physik der Universität Heidelberg, Heidelberg, West Germany
(Received 15 November 1976)

The newly discovered mesons with mass near 1.87 GeV are speculatively identified with diquark states of integrally charged Han-Nambu quarks. The weak decays of diquarks are found to be similar but not identical to the decays of charmed mesons.

The SU(3) color forces in quantum chromodynamics are such that the lowest energy states are color singlets. If colored states exist at all, their mass scale is set by the mass of free quarks or by the mass of the lowest colored composite state. In the present Letter I will speculatively identify the newly discovered charged and neutral mesons[1] of mass around 1.87 GeV with the color triplet diquark states $(qq)_3$ formed by integer-charged Han-Nambu quarks.[2] These color-triplet states are the lowest bound colored states to be expected.[2] I will also assume that quarks and diquarks have roughly the same mass. With this input, Nambu's mass formula[2,3] leads to small masses for color-singlet mesons and baryons, and gives for the mass of color octet mesons

$$m((q\bar{q})_8) \approx \tfrac{9}{4} m((qq)_3) \approx 4 \text{ GeV}.$$

Hence, in this picture one may identify the J/ψ family and in particular the states around 4 GeV seen in e^+e^- annihilation with $(q\bar{q})_8$ states as was done with some success in previous work.[4] If energetically allowed, these color-octet states will now partly decay into pairs of quarks and diquarks, and partly—via color-octet symmetry breaking—directly into usual hadrons.

The assumed existence of mesonic diquarks requires special assignments for the baryon numbers of quarks or the violation of baryon number in diquark decays. An interesting possibility is the baryon number assignment[2] $B=0,\ 0,$ and 1 for the three colors red, white, and blue, denoted here by 1, 2, and 3. In the following this case shall serve as an illustrative example. It gives special significance to the SU(2)(color) subgroup (the color isospin group) which affects red and white quarks only. These quarks will eventually decay into an odd number of leptons and thus must carry lepton number. The choice $L=-1,\ 1,$ and 0 gives zero lepton number to usual baryons and allows for the existence of mesonic diquarks with lepton number zero, a case discussed already by Nambu and Han[5] for a somewhat different purpose. With the strangeness quantum number $S=0,\ 0,$ and -1 for u, d, and s quarks, the Gell-Mann–Nishijima formula may be written as

$$Q=(I_3+\tfrac{1}{2}S)_{\text{flavor}}+\tfrac{1}{2}(B+L). \quad (1)$$

According to the Pauli principle the color-triplet diquarks of spin zero and orbital angular momentum zero form a $(\underline{3}^*, \underline{3})$ multiplet of particles.[5] Out of these nine states six have simultaneously baryon and lepton numbers different from zero and $I(\text{color})=\frac{1}{2}$. The remaining $\underline{3}^*$ SU(3) flavor multiplet has $I(\text{color})=0$. It consists of the mesons

$$F^0=d^1u^2-d^2u^1,$$
$$D^0=u^1s^2-u^2s^1, \quad (2)$$
$$D^-=d^1s^2-d^2s^1.$$

The isospin-singlet meson F^0 has zero charge and is expected to be lighter than the isospin doublet $D^{0,-}$, both in contrast to the case of the charmed F meson.[6,7] Another difference with the charm picture appears for the spin-1 mesons with orbital angular momentum zero: The spin-1 diquarks are in a $\underline{6}$ representation of SU(3) flavor