COMMENTS

Resolution of an Ambiguity in Alternative Soft-Pion Approaches to $\pi N \to \pi\pi N$ near Threshold*

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A recent observation that the current-commutator and effective-Lagrangian results differ for low-energy pion production is discussed. A simple unitarity argument shows that the commutator result as calculated is incorrect. The present status of $\pi\pi$ scattering lengths determined by threshold pion production is reviewed. The s-wave scattering lengths, in inverse pion masses, are found to be $a_0 = 0.24 \pm 0.03$ and $a_2 = -0.03 \pm 0.015$.

The demonstration[1] that the chiral-symmetry-breaking properties of $\pi N \to \pi\pi N$ near threshold are identical to low-energy $\pi\pi$ scattering makes this process an excellent probe of $\pi\pi$ scattering. Currently a Clinton P. Anderson Meson Physics Facility experiment is being performed for the charge state $\pi^- p \to \pi^- \pi^+ n$ in the threshold region which will explore $\pi^- \pi^+ \to \pi^- \pi^+$ scattering at very low energies.

In a recent Letter, Rockmore[2] has noted that the effective-Lagrangian calculation[1] for pion production does not agree with the current-commutator calculation of Chang.[3] Rockmore pointed out that the only significant difference between the two results is the pion-pole part of the amplitude. According to Rockmore, "one does *not* expect the pion-pole term to be the same in both approaches, since only the *external* pions participate in the Bose symmetrization in the current-commutator theory, while that symmetry of the effective $\pi\pi$ Lagrangian includes the virtual exchanged pion as well." He concludes that because of this ambiguity, measurement of the $\pi N \to \pi\pi N$ process near threshold cannot be uniquely related to $\pi\pi$ scattering. We disagree with this conclusion.

A unitarity argument resolves this ambiguity. Using t-channel unitarity applied to the pion pole, we know that the residue must factorize[4] into the product of the πNN vertex function and the on-mass-shell $\pi\pi$ scattering amplitude (with one pion crossed). This condition must be satisfied by any pion-production model with pion exchange whatever its origin. While this requirement is met for the effective-Lagrangian result[1,2] it is not in the current-commutator calculation.[3,2] The apparent reason for this difficulty in the commutator case[3,2] is the neglect of the pion-pole diagrams[4] schematically shown in Fig. 1. In the commutator calcu-

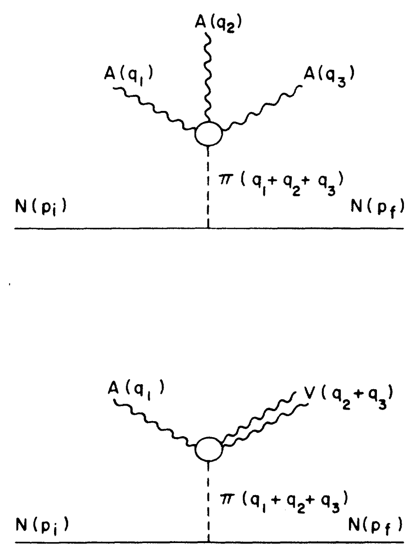

FIG. 1. Neglected pion-pole diagrams in the commutator calculation (Refs. 2 and 3).

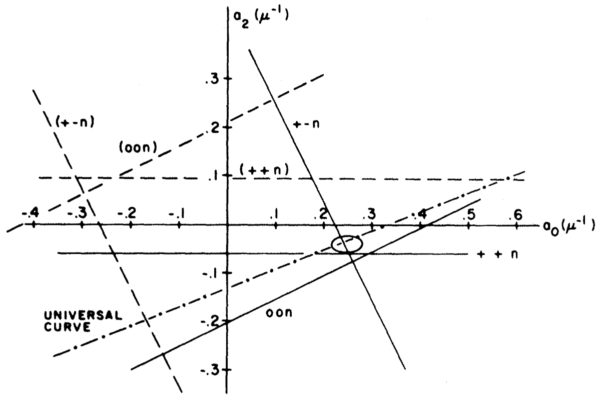

FIG. 2. $\pi\pi$ s-wave isospin scattering lengths a_0 and a_2 determined by threshold pion-production data. The solid lines correspond to the initial sign choices in Eq. (4) while the sign choices in parentheses generate the dashed lines. The dash-dotted line is the "universal curve" of Eq. (5). A consistent solution is found in the region $a_0 \simeq 0.24$, $a_2 \simeq -0.03$ indicated by the ellipse.

lation[3,2] only pion-pole contributions from the double-commutator terms were considered. When all diagrams have been considered the two methods of calculation are expected to agree.[4,5] The neglect of these diagrams is analogous to the well-known "factor-of-2" error noticed in early electroproduction calculations.[6]

Our conclusion is that the effective-Lagrangian result[1] is correct and may be used to determine low-energy $\pi\pi$ scattering unambiguously.

For completeness we review the present experimental situation concerning threshold pion production and the resulting $\pi\pi$ scattering lengths. As previously[7] discussed, the threshold limit can be parametrized as

$$a(\pi\pi N)\xrightarrow[\text{threshold}]{} \pm\left(\frac{2\sigma(\pi N\to\pi\pi N)}{Q^2\times(\text{phase space})}\right)^{1/2}, \qquad (1)$$

where the factor of 2 in Eq. (1) is present only if the two final pions have the same charge. The theoretical expression for $a(\pi\pi N)$ is defined[1] by

$$A(\pi N\to\pi\pi N)=a(\pi\pi N)\varphi_f^{+}\vec{Q}\cdot\vec{\sigma}\varphi_i; \qquad (2)$$

and in the threshold limit[1,8] we find

$$\begin{aligned} a_{+-}&=0.048a(\pi^+\pi^-n)-0.007,\\ a_{++}&=0.048a(\pi^+\pi^+n)+0.014, \qquad (3)\\ a_{00}&=0.048a(\pi^0\pi^0n). \end{aligned}$$

Here a_{+-}, a_{++}, and a_{00} are the $\pi^+\pi^-$, $\pi^+\pi^+$, and $\pi^0\pi^0$ scattering lengths, respectively, in inverse pion masses.

Using Eqs. (1) and (3) and the experimental data[9] closest to threshold we find

$$\begin{aligned} 6a_{+-}&=2a_0+a_2=0.45\pm0.06 \text{ (or } -0.53\pm0.06),\\ a_{++}&=a_2=-0.06\pm0.04 \text{ (or } 0.09\pm0.04), \qquad (4)\\ 3a_{00}&=2a_2-a_0=-0.42\pm0.06 \text{ (or } 0.42\pm0.06), \end{aligned}$$

where a_0 and a_2 are the s-wave isospin scattering lengths. The values given in parentheses reflect the sign ambiguity of Eq. (1). We also include the uniquely predicted combination of scattering lengths

$$2a_0-5a_2=0.58\pm0.06 \qquad (5)$$

determined by the "universal curve" using data and analyticity.[10,11] That there is only one consistent solution to these scattering-length relations can be seen from Fig. 2. This solution corresponds to the initial sign choices (not in parentheses) in Eq. (4). A statistical fit to Eqs. (4) and (5) yields[12]

$$\begin{aligned} a_0&=0.24\pm0.03,\\ a_2&=-0.03\pm0.015. \end{aligned} \qquad (6)$$

We see that low-energy pion production provides a consistent and unique set of s-wave $\pi\pi$ scattering lengths. To improve our knowledge of these parameters, more accurate $\pi N\to\pi\pi N$ data are required especially for the $\pi^+p\to\pi^+\pi^+n$ charge state.

The above results are independent of chiral-symmetry breaking (or the isospin structure of the sigma commutator). If we wish to investigate symmetry breaking, the ξ parameter of Ref. 1 can be evaluated from the s-wave scattering lengths:

$$\xi=-\frac{2a_0+7a_2}{a_0-\frac{5}{2}a_2}=-0.80\pm0.40. \qquad (7)$$

The assigned error includes statistical correlations between a_0 and a_2. This value is similar to that previously obtained[7] and is somewhat inconsistent with the $\xi=0$ value of the sigma model. The next simplest $SU(2)\otimes SU(2)$ representation ($N=2$) corresponds[13] to $\xi=-2$.

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[1]M. G. Olsson and Leaf Turner, Phys. Rev. Lett. 20, 1127 (1968).

[2]Ronald Rockmore, Phys. Rev. Lett. 35, 1408 (1975).

[3]Lay-Nam Chang, Phys. Rev. 162, 1497 (1967).

[4]Roger Dashen and Marvin Weinstein, Phys. Rev. 183, 1261 (1969).

[5]H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967); I. S. Gerstein and H. J. Schnitzer, Phys. Rev. 170, 1638 (1968).

[6]Stephen L. Adler and William I. Weisberger, Phys. Rev. 169, 1392 (1968).

[7]M. G. Olsson and Leaf Turner, Phys. Rev. 181, 2141 (1969).

[8]The small additive constants in Eq. (3) are due to the non-pion-pole contributions to the production amplitude.

[9]$\pi^- p \rightarrow \pi^- \pi^+ n$: $T_\pi = 210$, 222 MeV, Yu. A. Batusov *et al.*, Yad. Fiz. 1, 687 (1965) [Sov. J. Nucl. Phys. 1, 492 (1965)]. $\pi^+ p \rightarrow \pi^+ \pi^+ n$: $T_\pi = 230$ MeV, Yu. A. Batusov *et al.*, Yad. Fiz. 18, 86 (1973) [Sov. J. Nucl. Phys. 18, 45 (1974)]; $T_\pi = 357$ MeV, J. Kirz *et al.*, Phys. Rev. 126, 763 (1962). $\pi^- p \rightarrow \pi^0 \pi^0 n$: $T_\pi = 276$ MeV, A. V. Kravtsov *et al.*, Yad. Fiz. 20, 942 (1974) [Sov. J. Nucl. Phys. 20, 500 (1975)].

[10]D. Morgan and G. Shaw, Phys. Rev. D 2, 520 (1970), and Nucl. Phys. B10, 261 (1969).

[11]Current algebra also makes a unique prediction for this combination; S. Weinberg, Phys. Rev. Lett. 17, 616 (1966). With the choice $f_\pi = 94$ MeV we find $2a_0 - 5a_2 = 0.52$.

[12]The results quoted are independent of the value of f_π since the pion-pole term of the production amplitude factors into π-π scattering lengths and the known $NN\pi$ vertex.

[13]Steven Weinberg, Phys. Rev. 166, 1568 (1968); M. G. Olsson and Leaf Turner, Phys. Rev. D 6, 3522 (1972).

ERRATA

TWO-DIMENSIONAL CHARACTER OF THE CONDUCTION BANDS OF d-BAND PEROVSKITES. T. Wolfram [Phys. Rev. Lett. 29, 1383 (1972)].

Equations (2) should read

$$G(E) = (\xi/\pi)|E - \tfrac{1}{2}(E_t + E_\perp)|K(\xi)/(pd\pi)^2,$$
$$\xi = 4/\zeta, \qquad (2)$$
$$\zeta = [(E - E_t)(E - E_\perp) - 4(pd\pi)^2]/(pd\pi)^2.$$

CONTACT INTERACTIONS IN THE EINSTEIN AND EINSTEIN-CARTAN-SCIAMA-KIBBLE (ECSK) THEORIES OF GRAVITATION. R. F. O'Connell [Phys. Rev. Lett. 37, 1653 (1976)].

The following typographical errors should be noted: In the sentence containing Eq. (2), the symbols $\Delta \mathcal{S}_E^{(1)}$ and t_c should be replaced by $\Delta \mathfrak{L}_E^{(1)}$ and λ_c, respectively. Every summation sign which appears should have an upper limit n. On the right-hand side of Eq. (12), one should replace the 3 by a 2 and λ_c by λ_c^2. In Ref. 1, the year 1972 should be replaced by 1922. Reference 2 should read "D. W. Sciama, in ... 1962), p. 415." In Eq. (10), $\delta(\vec{r} - \vec{r}_b)$ should be replaced by $\delta(\vec{r}_a - \vec{r}_b)$.

Lagrangians and Lagrangian densities should have been denoted by L and $\mathfrak{L}$, respectively, whereas $\mathfrak{L}$ has been used to denote both quantities. Thus, it should be noted that only Eqs. (1) and (7) treat Lagrangian densities and that, in particular, Eq. (10) is obtained by integrating Eq. (7) over all space.

RADIATION-INDUCED DIFFUSION OF HYDROGEN AND DEUTERIUM IN MgO. Y. Chen, M. M. Abraham, and H. T. Tohver [Phys. Rev. Lett. 37, 1757 (1976)].

On page 1759, column 1, line 11, "... the relationship $\sigma = (1/\Delta N/N)/\Delta\varphi$ ... " should read "... the relationship $\sigma = (\Delta N/N)/\Delta\varphi$ ... ."