COMMENTS

Induced Cosmological Constant Expected above the Phase Transition Restoring the Broken Symmetry

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The vacuum energy density now is small or zero but must have been prodigious if the universe was once hotter than $T_c \sim 10^{15}$ K and if elementary-particle symmetry is spontaneously broken by a Higgs mechanism. If symmetry is broken nondynamically, in the hot disordered phase the huge vacuum energy density is nevertheless negligible, compared to the energy density of ultrarelativistic particles. Because the broken symmetry is non-Abelian, the long-range forces arising on symmetry restoration need not lead back to an anisotropic, inhomogeneous, or domain-structure universe.

Recent developments in quantum field theory have reemphasized an old problem and seemingly led to a new problem concerning the gravitational consequences of the self-energy of the elementary-particle vacuum. In recent unified theories of the weak and electromagnetic interactions, symmetry is broken by the appearance of a nonvanishing vacuum expectation $\langle\varphi_i\rangle$ for some component of a scalar field φ. The vacuum expectation $\langle T_{\mu\nu}\rangle$ of this field is only observable gravitationally since, in present elementary-particle theories, all physical amplitudes are renormalized by dividing out the vacuum-vacuum amplitude. In fact, by the introduction of a suitable mass counter term, $\langle T_{\mu\nu}\rangle$ may be given any observable value.

If elementary-particle summetry is spontaneosly broken by a Higgs mechanism, but not if it is dynamically broken, then above a critical temperature T_c, a symmetry-restoring phase transition takes place. This leads to a tremendous energy-density difference[1] betweeen the hot ($T > T_c$) disordered and present cold ($T = 0$) ordered universes from which any possible mass counter term cancels out, so that this energy *difference* between the two vacua is observable in principle. If this energy difference could be observed cosmologically, this would dramatically demonstrate the physical reality of the Higgs symmetry-breaking mechanism. We will show, however, that the thermal energy density $\mathfrak{N}aT^4$ of the $\mathfrak{N}$ elementary-particle species in the early hot universe is so large as to smooth out entirely any consequences of the (admittedly tremendous) energy-density change at any phase transition. This means, unfortunately, that there is no way cosmologically to discriminate among theories in which the symmetry is spontaneously broken, dynamically broken, or formally identical and unbroken.

(1) In any quantum field theory, vacuum polarization leads to an expectation value $\langle T_{\mu\nu}\rangle = \lambda\kappa^{-1}g_{\mu\nu}$ for the vacuum stress energy. Required by Lorentz invariance, this form of $\langle T_{\mu\nu}\rangle$ is that of an ideal fluid with energy density and pressure $\epsilon = -p = \lambda/\kappa$.

When gravity is considered (even on the classical or tree-approximation level), any nonvanishing vacuum stress energy would have prodigious effects even on the static or Newtonian level. Laboratory, solar system and galactic observations already show $|\lambda/\kappa c^2| < 10^{-8}$ g cm^{-3} or 10^{-24} g cm^{-3}, respectively ($|\Lambda| < 2\times 10^{-35}$ or 2×10^{-51} cm^{-2}), but the most sensitive limits are set cosmologically.[3] The present deceleration

$$q_0 \equiv -(R\ddot{R}/\dot{R}^2)_0 = \frac{\rho + 3pc^{-2}}{2\rho_{cr}} - \frac{\Lambda c^2}{3H_0^{\,2}} \tag{1}$$

(for a homogeneous isotropic universe) is observed to be less than 1 in absolute value, while $\rho + 3p c^{-2} \ll \rho_{cr} \equiv 3H_0^2/8\pi G$. Thus

$$|\Lambda/\kappa c^2| < \rho_{cr} = 5\times 10^{-30}\text{ g cm}^{-3},$$
$$|\Lambda| < 3(H_0/c)^2 = 10^{-56}\text{ cm}^{-2}, \qquad (2)$$

for a Hubble constant $H_0 = 50$ km s^{-1} Mpc^{-1} = 5 $\times 10^{-29} c$ cm^{-1} (1 Mpc = 10^6 parsecs).

Any observed (possibly zero) value for the cosmological constant $|\Lambda|$ can be obtained from field theory by addition of suitable counter terms. The observed value is a problem only if one takes the attitude that it should be derivable from other fundamental constants in particle physics. Clearly the observed limit (2) is far below the (divergent) zero-point energy density

$$\tfrac{1}{2}N_i(2\pi)^{-3}\int(p^2+m_i^2)^{1/2}d^3p \qquad (3)$$

of any free field of mass m_i and N_i independent one-particle states; N_i is counted positive for bosons, negative for fermions.[4] If the boson and fermion fields were put together in supermultiplets of precisely degenerate mass suitably combined, the terms (3) would sum to zero over the multiplet. Zumino has shown[4] that then all vacuum diagrms sum precisely to zero whether or not the zero-point energy density of free fields is subtracted off. This follows immediately from the basic anticommutation relation characteristic of supersymmetry $\{Q_\alpha, J_{\nu\beta}(x)\} = f_{\alpha\beta}{}^{\mu}T_{\mu\nu}(x)$ between Fermi supercharges Q_α and supercurrents $J_{\nu\beta}(x)$, provided the vacuum is nondegenerate. Unfortunately, boson-fermion masses are not degenerate and supersymmetry probably disagrees with what we know about fermion conservation and the quark model. As soon as supersymmetry is broken, even spontaneously, the delicate cancellation of vacuum self-energies or induced cosmological term breaks down.

The small or zero value observed for the cosmological constant may suggest some supersymmetry or new gauge-invariance principle to be discovered in some future supergravity theory or may simply be a fundamental constant. To this old problem (or pseudo-problem), neither broken symmetry[5] nor we have anything to add.

(2) In Weinberg-Salam theories, symmetry is broken through the appearance of a nonvanishing vacuum expectation $\lambda = \langle\varphi_i\rangle$ for some scalar field φ in the symmetric Lagrangian. Weinberg[6] has shown that a phase transition restoring symmetry takes place at a critical temperature $\theta_c = kT_c$, if the symmetry is broken by such a Higgs mechanism, but not if it is dynamically broken. He calculated the nonanalytic or Landau free energy $F(\lambda) = -\theta\Omega^{-1}\ln\operatorname{Tr}\exp(-H/\theta)$ as the minimum of $P_{\text{eff}}(\varphi) = \frac{1}{2}\mathfrak{M}^2(\theta)\varphi^2 + \frac{1}{4}e^2\varphi^4$, where $\mathfrak{M}^2(\theta) = \mathfrak{M}^2(0) + e''^2\theta^2$; e and $e'' \ll 1$ are small coupling constants and $\mathfrak{M}^2(0) < 0$. Thus, for $\theta < \theta_c \equiv |\mathfrak{M}(0)|/e''$, we have

$$F(\lambda) = -\tfrac{1}{4}\mathfrak{M}^2(\theta)\lambda^2,$$
$$\lambda(\theta) = |\mathfrak{M}(\theta)|/e = (e''/e)(\theta_c^2 - \theta^2)^{1/2}, \qquad (4)$$

while for $\theta > \theta_c$, $\lambda = 0 = F$. Since $F[\lambda(\theta)]$ and the other thermodynamic variables $p = -F(\lambda)$, $s = -\partial F/\partial\theta$, $\epsilon = F - \theta\partial F/\partial\theta$ are continuous and $c_v = \theta\partial^2 F/\partial\theta^2$ is discontinuous at $\theta = \theta_c$, the transition from ordered to disordered state is a second-order phase transition with at least some broken symmetry restored at temperatures above T_c. To the singular free energy (4) should be added a background free energy $F_0(\theta)$ that is less singular than the terms calculated by Weinberg. $F_0(\theta)$ is continuous at θ_c but contributes to the entropy and energy amounts $s_0(\theta)$ and $\epsilon_0(\theta)$.

If temperatures above T_c were realized in the early universe and if physical consequences of symmetry restoration were observable, this would discriminate against dynamical symmetry breaking in favor of spontaneous symmetry breaking by the Higgs scalar-meson mechanism.[6]

When a broken gauge symmetry is restored, the vector mesons which are massive in the cold phase become massless in the disordered phase, leading to lines of force between the conserved charges coupled to these massless vector mesons.[1] If they permeated the early hot universe, these lines of force would make it anisotropic and inhomogeneous, would effect any initial singularity, and might condense the universe into domains of different broken-symmetry direction. The long-range forces between generalized conserved charges that arise on symmetry restoration need not lead back to an anisotropic, inhomogeneous, or domain-structure universe if either our universe is overall neutral with respect to the generalized charges or if the lines of force close onto 't Hooft monopole singularities.[2] In fact, the present ordered universe should not be expected to separate into domains with different broken-symmetry direction. Ferromagnetic domains are separated by two-dimensional domain walls across which the magnetization changes, only because the order parameter is one-dimensional. In the elementary-particle situation, on the contrary, the broken symmetry is non-Abelian: The order parameter (Higgs field) is threedimensional so that its singularities may be zero-dimensional points in space—'t Hooft monopoles, not ferromagnetic domains.

(3) There is no energy discontinuity at the phase transition and the energy density $\epsilon(0)$ of the present unsymmetric $T=0$ universe is apparently very small. Between the hot ($T>T_c$) disordered and cold ($T=0$) ordered universes there is, however, an energy difference

$$\epsilon(\theta)-\epsilon(0)=-\tfrac{1}{4}\mathfrak{M}^2(0)\lambda^2(0) \qquad (5)$$

that is very large, independent of possible counter terms, and observable in principle.

The Fermi coupling constant and Higgs meson mass $M(\theta)$ are related[6] to the coupling parameters in Eq. (4) by

$$\lambda^2(0)=2^{-1/2}G_F^{-1}, \quad M^2(\theta)=-2\mathfrak{M}^2(\theta).$$

The Higgs meson mass $M(0)$ cannot be small without producing unobserved long-range forces in the present $T=0$ universe. In unified gauge theories $M(0)/e\sim G_F^{-1/2}=300$ GeV $\sim\theta_c$. Thus, the Landau energy difference in (5) is $+[e^2/8\sqrt{2}(\hbar c)^4]\times\theta_c^4=3\times10^{45}$ erg cm^{-3}, very large and not cancelled by the background energy difference $\epsilon_0(\theta)-\epsilon_0(0)$ at more than one temperature. If the induced cosmological constant is small in the present cold universe, it must have been prodigious in any early hot universe, provided the symmetry breaking was nondynamical and the early temperature exceeded $T_c=10^{15}$ K.

This huge vacuum energy density in the hot universe is nevertheless negligible compared to the thermal energy density of the $\mathfrak{N}$ elementary particles, all of which are ultrarelativistic and contributing aT^4 per boson and $\frac{7}{8}aT^4$ per four-component fermion. Even counting at $T>T_c$ only photons, leptons, three quarks, and their antifermions, we get $\mathfrak{N}=26$. Then

$$\epsilon(\theta)-\epsilon(0)=\frac{e^2}{8\sqrt{2}(c\hbar)^3}\theta_c^4 \ll \mathfrak{N}aT^4=\frac{\pi^2}{15(c\hbar)^3}\theta^4. \qquad (6)$$

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[3]This interpretation of the cosmological constant in Einstein's equations $R_{\mu\nu}=\kappa T_{\mu\nu}+\Lambda g_{\mu\nu}$ as the invariant vacuum self-energy is due to Ya. B. Zel'dovich, Usp. Fiz. Nauk 95, 209 (1968) [Sov. Phys. Usp. 11, 381 (1968)].

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[5]Our view is opposed to that of J. Dreitlein, Phys. Rev. Lett. 33, 1243 (1974), and M. Veltman, Phys. Rev. Lett. 34, 777 (1975), that the symmetric vacuum must have $\langle T_{\mu\nu}\rangle=0$, making $\langle T_{\mu\nu}\rangle$ for the unsymmetric vacuum huge and negative, in conflict with experiment.

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