the polarization vector. The opposite behavior of the p_x and p_z levels for $\vec{A}$ in the x-z plane is in contradiction to the plane-wave final-state picture. In this limit, the cross section is proportional to $(\vec{k}_f \cdot \vec{A})^2$ regardless of the symmetry of the initial state, i.e., curve b should nearly coincide with curve c. These results demonstrate that both the initial and the final states need to be treated in a consistent manner.

The adsorption of the oxygen monolayer also results in pronounced modifications of the emission from the Ni d bands. These will be reported together with the extension to a $c(2\times2)$ structure in a separate publication.[4]

Uncertainties in the potential near the surface, relaxation effects, and the spatial dependence of $\vec{A}$, which have not been considered here, might limit one's ability to fit experimental spectra quantitatively. Nevertheless, the characteristic variation of position, shape, and intensity of adsorbate-induced resonances, which the above proposed scheme predicts, should provide a means of identifying orbital symmetries and of studying the adsorbate-substrate interaction.

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Spontaneous Magnetization at Marginal Dimensionality in $LiTbF_4$ †

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The spontaneous magnetization in the critical region of the uniaxial, dipolar-coupled ferromagnet $LiTbF_4$ has been measured using sensitive optical techniques. These measurements provide a quantitative comparison between the predicted logarithmic corrections to mean-field theory and the experimentally observed magnetization of a ferromagnet at marginal dimensionality. We conclude that the spontaneous magnetization in $LiTbF_4$ is described by the relation $M_c(T)/M_c(T=0) = B|t|^{1/2}|\ln|t_0/t||^{1/3}$ over the range $0.002 \leq t \leq 0.09$ with $B = 1.77 \pm 0.06$, $T_c = 2.8700 \pm 0.002$ K, and $t_0 = 0.53 \pm 0.06$.

In this Letter we present measurements of the spontaneous magnetization of the uniaxial dipolar-coupled ferromagnet $LiTbF_4$ which are sufficiently precise to enable a quantitative comparison between the predicted logarithmic corrections to mean-field theory and the observed behavior. We show the magnetization predicted by renormalization-group calculations is quantitatively consistent with our experimental results and provides a better fit over a wider temperature range than any simple power law.

These results are important because they pro-

vide a direct test of a central result of the renormalization-group theory of phase transitions. For each type of cooperative behavior there is a value d^* of the spatial dimensionality d such that mean-field theory exponents are correct if $d > d^*$ and nonclassical exponents are correct if $d < d^*$. At $d = d^*$ the renormalization-group equations can be solved "exactly," in the sense that no ϵ or $1/n$ expansions are required, yielding mean-field-theory exponents with logarithmic corrections.[1] The marginal dimensionality d^* of a critical point is determined by the nature of the interaction responsible for the cooperative behavior and there are only a few types of critical points for which d^* coincides with a dimensionality that is observable in the laboratory. One such type is the critical point of the uniaxial ($n = 1$), dipolar-coupled ferromagnet for which it has been shown that $d^* = 3$.[2]

$LiTbF_4$ is a uniaxial, optically transparent ferromagnet which becomes magnetically ordered at $T_c \sim 2.87$ K with a saturation magnetic moment of $\sim 8.9\ \mu_B$. Previous magnetization,[3] neutron scattering,[4] and NMR[5] data have shown that the magnetic coupling between Tb^{3+} ions is predominantly dipolar in nature. Because of the crystal-field splitting of the ground $^7F_6$ term of the Tb^{3+} ion, the magnetic moments are strongly confined to the c axis resulting in an Ising-like ($n = 1$) magnetic order. Measurements of the magnetic behavior reported here rely on the unusual magneto-optic properties of $LiTbF_4$. The Ising nature of the ground levels of the Tb^{3+} ions along with a strong electronic absorption in the uv produces an unusually large saturation Faraday rotation (8×10^3 deg/cm).[6] The uniaxial anisotropy is also responsible for the structure of the ferromagnetic domains below T_c, which consist of alternating cylindrical domains confined to the c axis in a structure previously found in $CrBr_3$.[7] This domain structure along with the large circular birefringence produces an unusually large elastic, forward-direction scattering of a laser beam propagating parallel to the c axis. A crystal ~ 4 mm thick at $T = 1.5$ K scatters 70% of the incident beam at $\lambda = 6328$ Å. The determination of the coexistence curve in $LiTbF_4$ reported here was accomplished by precisely measuring the onset of such scattering as a function of temperature and external magnetic field. These measurements were accomplished using a double-beam optical arrangement, in which the intensity of the beam transmitted by the crystal was compared with the intensity of a reference beam using polarization

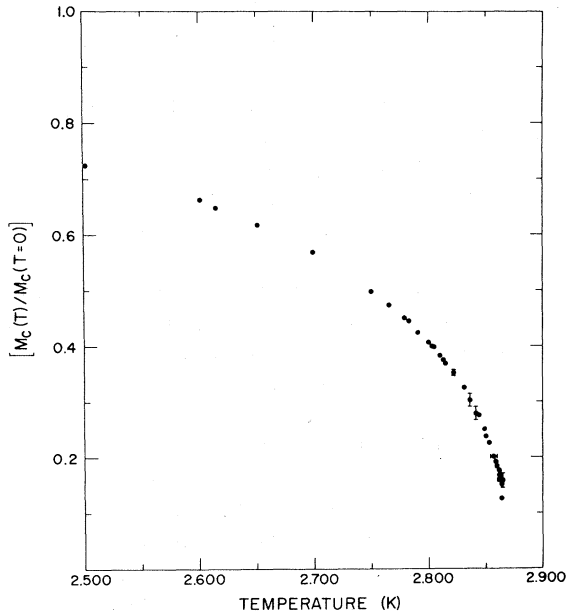

FIG. 1. Normalized spontaneous magentization $M_c(T)/M_c(T=0)$ of $LiTbF_4$ between 2.5 and 2.9 K obtained from domain light scattering.

modulation techniques.[8] As the phase boundary was crossed, from the paramagnetic phase to the ferromagnetic phase for example, the onset of ferromagnetic domains produced an onset of the domain light scattering. In a ferromagnet below T_c the spontaneous magnetization $M_c(T)$ on the coexistence curve is related to the external critical field $H_c(T)$ by $M_c(T) = H_c(T)/N$, where N is the demagnetization factor. By measuring the external magnetic field $H_c(T)$ at which the onset of domain scattering occurs as a function of temperature, the ratio $M_c(T)/M_c(T=0)$ may be determined independently of N.

The $LiTbF_4$ sample used in the experiment was a rectangular plate with dimensions of 1 cm × 1 cm × 0.4 cm. It was masked by a copper plate so that the transmitted beam was limited to the central portion of the sample. The sample was mounted on a copper support where temperature was held constant to within ~ 0.4 mK by a capacitance thermometer controller.[9] The sample temperature was measured by a germanium resistance thermometer calibrated in zero magnetic field. The sample was immersed in $\sim 20\ \mu$m of He^4 exchange gas and the incident power level was $\sim 10\ \mu$W at $\lambda = 6328$ Å.

The results are presented in Figs. 1 and 2. Figure 1 illustrates the normalized spontaneous magnetization $M_c(T)/M_c(T=0)$ versus tempera-

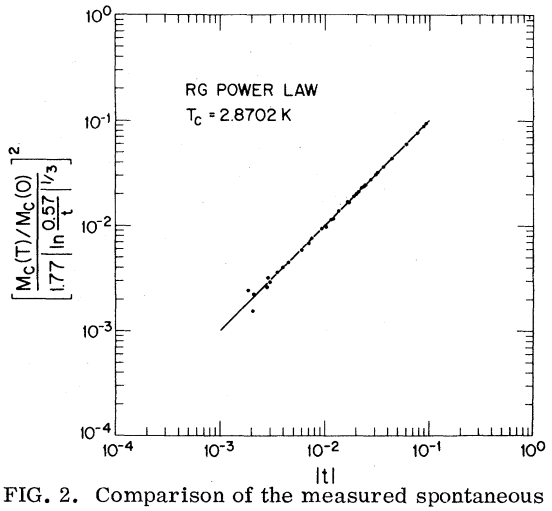

FIG. 2. Comparison of the measured spontaneous magnetization of $LiTbF_4$ to the behavior predicted by the renormalization group. The solid line represents $M_c(T)/M_c(0) = 1.77\,|t|^{1/2}|\ln(0.57/|t|)|^{1/3}$.

ture obtained from domain light scattering using $M_c(T=1.5\text{ K}) = 0.93 M_c(T=0)$. Data points at low temperature ($T \leq 2.75$ K) were obtained by stepwise external-magnetic-field scans at constant temperature from the paramagnetic to the ferromagnetic phase, and the phase boundary could be determined to $\pm 0.1\%$ of $M_c(T)$. Constant temperature scans from the ferromagnetic to paramagnetic phase yield the same phase boundary points. At higher temperatures ($2.85\text{ K} \leq T \leq T_c$) the phase boundary was obtained by stepwise temperature scans at constant external magnetic field from the paramagnetic to ferromagnetic phase. As was the case in field scans the phase boundary points obtained showed no hysteresis. In the intermediate temperature range ($2.75\text{ K} \leq T \leq 2.85$ K) scans of both types were used, and phase boundary points obtained by field scans were experimentally found to coincide with those obtained by temperature scans. The error bars in Fig. 1 reflect the uncertainty in locating the onset of domain scattering, and are considerably smaller than those obtained by isothermal magnetization versus external-magnetic-field measurements in which $M_c(T)$ is determined by a kink which occurs in the magnetization as the phase boundary is crossed.

In Fig. 2 the spontaneous magnetization is compared to the renormalization-group result according to which the spontaneous magnetization should be described by the relation

$$M_c(T)/M_c(0) = B\,|t|^{1/2}\,|\ln|t_0/t||^{1/3}, \tag{1}$$

where the mean-field-theory power law,

$$M_c(T)/M_c(0) = B\,|t|^{1/2}, \tag{2}$$

is corrected by the logarithmic factor $|\ln|t_0/t||^{1/3}$, where $t = (1 - T_c)$ and t_0 is a parametrization of the higher-order lagarithmic corrections.[10] The nonclassical power law normally found to describe magnetic phase transitions is

$$M_c(T)/M_c(0) = B\,|t|^{\beta}, \tag{3}$$

where $\beta \neq \frac{1}{2}$. The data were fitted with each of the above relations and the results of the analysis are quantitatively presented in Table I, which illustrates the range in reduced temperature t that was used, the resultant χ^2, and the best-fit coefficients. In Table I the uncertainties quoted represent approximately a doubling of χ^2. The fit to the mean-field-theory relation (2) is seen to result in coefficients B and T_c that are quite sensitive to the range in reduced temperature t and also yields a value of χ^2 which is relatively large. Equation (2) is ruled out as would be expected. The fit to Eq. (3) is also summarized in Table I. This fit provides an effective exponent β, and

TABLE I. Parameters obtained by fitting spontaneous magnetization to Eqs. (1), (2), and (3) over three ranges in reduced temperature t.

Eq. used in fit	t	B	T_c (K)			χ^2
(1)	$0.002 \leq t \leq 0.04$	1.79 ± 0.02	2.8700 ± 0.002	t_0	0.524 ± 0.047	0.54
	$0.002 \leq t \leq 0.06$	1.78 ± 0.04	2.8702 ± 0.002	t_0	0.553 ± 0.036	0.52
	$0.002 \leq t \leq 0.09$[a]	1.77 ± 0.06	2.8702 ± 0.002	t_0	0.568 ± 0.014	0.54
(2)	$0.007 \leq t \leq 0.06$	2.33 ± 0.024	2.885 ± 0.008		···	12.4
	$0.013 \leq t \leq 0.10$	2.13 ± 0.201	2.902 ± 0.010		···	51.2
(3)	$0.001 \leq t \leq 0.04$	1.71 ± 0.02	2.8686 ± 0.0032	β	0.385 ± 0.003	0.81
	$0.001 \leq t \leq 0.06$	1.66 ± 0.01	2.8680 ± 0.0032	β	0.377 ± 0.002	0.92
	$0.0009 \leq t \leq 0.09$	1.55 ± 0.01	2.8668 ± 0.0034	β	0.357 ± 0.001	2.26

[a] Used in Fig. 2.

such a fit was previously used by Als-Nielsen *et al.*[11] to represent neutron-scattering results over the range $0.001 \leq t \leq 0.03$ with $\beta = 0.45 \pm 0.03$ and $B = 2.13 \pm 0.17$. In Table I the results of the fit of our data to Eq. (3) indicate that $0.81 \leq \chi^2 \leq 2.3$, that 2.8668 K $\leq T_c \leq$ 2.8686 K, that $1.6 \leq B \leq 1.7$, and that $0.36 \leq \beta \leq 0.39$ depending on the range of reduced temperature t used in the fit. Our data are not consistent for any range of t with the effective exponent $\beta = 0.45 \pm 0.03$ obtained from neutron scattering, but we cannot definitely rule out Eq. (3) as a possible description of the observed behavior. Finally, the results of fitting our data by Eq. (1) also appear in Table I. In this fit we find that $0.52 \leq \chi^2 \leq 0.54$, that 2.8700 K $\leq T_c \leq$ 2.8702 K, that $1.77 \leq B \leq 1.79$, and that $0.52 \leq t_0 \leq 0.57$ depending on the range in reduced temperature t that is selected. Equation (1) fits our data with a χ^2 that is not only smaller than that obtained using Eq. (3) but which is also changing by only ~4% over the range in t which produced a change of ~300% in χ^2 using relation (3). Our fit to Eq. (1) indicates that the observed behavior of the spontaneous magnetization and the predicted logarithmic corrections to a mean-field-theory power law are quantitatively consistent, that the χ^2 of the fit obtained is insensitive to the range of t selected, and that the coefficients in the fit are also quite insensitive to the range of t. None of these features occurred in the fit to Eq. (3). The parameter t_0 obtained from (1) is $t_0 = 0.53 \pm 0.06$, and the parameter represents higher-order logarithmic corrections to mean-field theory. Ahlers, Kornblitt, and Guggenheim[12] obtained the value $t_0 = 0.315$ from specific heat measurements over the range $0.001 \leq t \leq 0.01$. Als-Nielsen[13] has subsequently used the value $t_0 = 0.315$ in a test of the ratio between the squared correlation length and specific heat, but did not determine if other values of t_0 were acceptable. At present, we do not understand this discrepancy between our results and those of Ahlers *et al.*

In conclusion, we have utilized the unusually strong magneto-optic features of the uniaxial dipolar ferromagnet $LiTbF_4$ to study the effects of marginal dimensionality on the spontaneous magnetization. These data represent a study of the spontaneous magnetization which allows a quantitative comparison between a simple power law and the logarithmic corrected power law of the renormalization-group theory. These data indicate that a simple power law cannot be ruled out, but that the logarithmic corrections to mean-field theory provide a better fit with the same number of parameters to our data over a wider range in reduced temperature, and one which is insensitive to the range of data selected.

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