Stabilization of Resistive Kink Modes in the Tokamak*

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Optimized current profiles are shown to be capable of providing simultaneous stability against all resistive kink modes in the tokamak.

The efficient utilization of the magnetic field in a tokamak increases with the ratio B_θ/B_φ of poloidal to toroidal field strength. In order to avoid unstable helical magnetohydrodynamics (MHD) perturbations (kink modes) of the form $\exp i(m\theta - n\varphi)$, the "safety factor" $q(r) \equiv 2\pi/\iota(r) = rB_\varphi/RB_\theta$ must, however, be restricted.[1] If there is a radial range wherein $q(r) < 1$, then the fundamental mode with $m = 1$ and $n = 1$ is unstable,[1] whether or not the plasma is perfectly conducting at the point where $q(r) = 1$. Higher modes (with $m > 1$) can be unstable only as resistive kink, or tearing, modes[2] for which the singular point, where $q(s) = m/n$, falls into a resistive region.

The object of this Letter is to demonstrate, within a constant-resistivity, cylindrical theory, the existence of q profiles that provide simultaneous stability against all the low-m kink modes, while minimizing the limiter value $q_a \equiv q(a)$. The principle is contained in a comparison theorem[2] that states the following: For two profiles of the rotational transform ι having the same shear $(d\iota/dr)_s$ and the same transform ι_s at the singular point of a given mode, if the two profiles everywhere satisfy $|\iota_1(r) - \iota_s| > |\iota_2(r) - \iota_s|$, then $\iota_1(r)$ is more stable against the given mode than $\iota_2(r)$. Resistive kink instabilities can also be eliminated[3] (in sufficiently hot plasmas) by a local pressure gradient at s due to favorable average toroidal curvature,[4] or by the proximity of a perfectly conducting exterior shell.

To illustrate optimum profiles, we will consider two cases: case A, a profile with $q_a > 2$, giving stability against all finite-m modes, without need of a conducting shell or of toroidal-curvature effects; and case B, a similar profile, but adding a conducting shell to achieve stability at $q_a < 2$.

We consider first a straight cylindrical configuration, and neglect pressure-gradient effects. The magnetic perturbations outside the resistive layer satisfy the equation for a marginal MHD mode, namely,[1,2]

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) - \frac{m^2}{r^2}\psi = \frac{m}{rF}\frac{dj}{dr}\psi, \tag{1}$$

where $\psi = irB_r/m$ is the perturbed poloidal flux function, $j = d(rB_\theta)/rdr$ is the equilibrium longitudinal current density, and

$$F = \vec{k}\cdot\vec{B} = (m - nq)B_\theta/r. \tag{2}$$

For a *marginal* resistive mode, Eq. (1) is satisfied everywhere, i.e., there is no discontinuity at the singular surface.

Analytic solutions of Eq. (1) can readily be found for the model of $j(r)$ shown in Fig. 1. In this model, there is a central current channel of radius $r = c$ with uniform current density $j(r) = j_1$, surrounding by a "pedestal" of radius $r = a$ with uniform, but lower, current density $j(r) = j_2$. For $r > a$, the current density vanishes, so that the limiter could be placed just outside $r = a$, with a conducting shell at $r = b$.

Within $0 < r < c$, the solution of Eq. (1) is given by

$$\psi/\psi_c = (r/c)^m. \tag{3}$$

Across $r = c$, the matching conditions are that $\psi = \psi_c$ be continuous, and that

$$\frac{[\psi']_c}{\psi_c} = \frac{m}{cF_c}[j]_c = -\frac{2m(1-p)}{c(m - nq_c)}, \tag{4}$$

where the square brackets $[\,]_c$ denote the discontinuity of the variable across $r = c$, and $p = j_2/j_1$ is a factor describing the height of the pedestal. (Given q_c and q_a, values of p are possible within

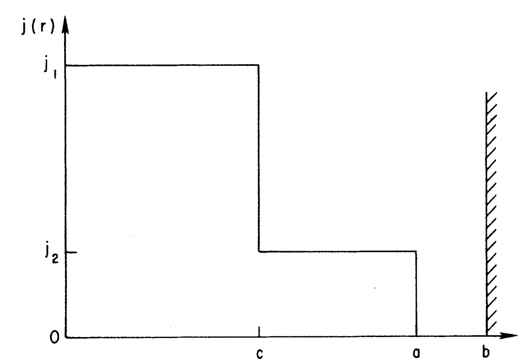

FIG. 1. Simple model of $j(r)$ used in the analytic calculation.

the range $0<p<q_c/q_a$; the relation

$$\frac{c^2}{a^2}=\frac{q_c/q_a-p}{1-p} \tag{5}$$

then determines the ratio c/a.) Applying these matching conditions at $r=c$, the solution of Eq. (1) within $c<r<a$ is given by

$$\frac{\psi}{\psi_c}=\left(\frac{r}{c}\right)^m-\frac{1-p}{m-nq_c}\left[\left(\frac{r}{c}\right)^m-\left(\frac{c}{r}\right)^m\right]. \tag{6}$$

Across $r=a$, the matching conditions are again that $\psi=\psi_a$ be continuous and that

$$\frac{[\psi']_a}{\psi_a}=\frac{m}{aF_a}[j]_c=-\frac{2mq_a p}{aq_c(m-nq_a)}. \tag{7}$$

Applying these matching conditions at $r=a$, and using Eq. (6) at $r=a$ for ψ_a, the solution of Eq. (1) for $r>a$ is given by

$$\frac{\psi}{\psi_c}=\left(\frac{r}{c}\right)^m-\frac{1-p}{m-nq_c}\left[\left(\frac{r}{c}\right)^m-\left(\frac{c}{r}\right)^m\right]-\frac{q_a p}{q_c(m-nq_a)}\left[\left(\frac{r}{a}\right)^m-\left(\frac{a}{r}\right)^m\right]\left\{\left(\frac{a}{c}\right)^m-\frac{1-p}{m-nq_c}\left[\left(\frac{a}{c}\right)^m-\left(\frac{c}{a}\right)^m\right]\right\}. \tag{8}$$

If the conducting shell is absent ($b\to\infty$), the stability condition (i.e., the condition that no marginal mode exists) is that the coefficient of the r^m term be positive, i.e.,

$$1-\frac{1-p}{m-nq_c}>\frac{q_a p}{q_c(m-nq_a)}\left\{1-\frac{1-p}{m-nq_c}\left[1-\left(\frac{c}{a}\right)^{2m}\right]\right\}. \tag{9}$$

Suppose, first, that the pedestal is entirely absent, i.e., $p=0$. In this case, instability occurs if $0<m-nq_c<1$. For the $(m,n)=(2,1)$ mode to be stable, it is clearly necessary to have $q_c<1$, in which case the $(m,n)=(1,1)$ mode is unstable. Moreover, if q_c is just above 1, the entire sequence of modes, (2,1), (3,2), (4,3), etc., is unstable. Even if the $m=2$ mode were stabilized by means of a fairly close conducting shell, the higher-m modes of this sequence would typically remain unstable, since the effect of the shell falls off rapidly with rising m. With the $m=2$ mode stabilized by a conducting shell, one might consider setting q_c just above 1.5, so that the modes, (3,2), (4,3), etc., become stable. However, in this case, the sequence of modes, (5,3), (8,5), etc., would be unstable. It is, thus, of considerable interest to determine whether, in either case, a current profile with a nonzero pedestal can provide simultaneous stability against all modes.

Let us consider case A, in which q_c is just above 1, and q_a is just above 2, with a finite value of p in the range $0<p<0.5$. If q_c is *infinitesimally* above 1, Eq. (9) shows clearly that the "inner" sequence of modes (2,1), (3,2), (4,3), etc., whose singular surfaces fall into the pedestal region, is positively stable, since in each case the left-hand side of Eq. (9) is positive and the right-hand side is negative. We must also, however, demonstrate the stability of the "outer" sequence of modes (3,1), (5,2), (7,3), etc., whose singular surfaces fall outside the pedestal region. A condition stronger than (9) would result from replacing $(c/a)^{2m}$ by $(c/a)^2$; we do this, and substitute Eq. (5) for c/a, to obtain the sufficient stability condition $(1-2p)(m-nq_c-1)>0$, after using $m-nq_a=1$ and $q_a/q_c=2$. Since $p<q_c/q_a=0.5$, this condition is always satisfied by the modes of the "outer" sequence, which have $m-nq_c\geq 2$.

If q_c and q_a exceed 1 and 2, respectively, by small but *finite* increments, a reasonable number of the modes in both the "inner" and "outer" seqences can be made positively stable. This is illustrated in Fig. 2, for the case where $q_c=1.05$ and $q_a=2.1$, and for various values of the pedestal p. We see that, in this case, the optimum value for p, in the sense of stabilizing the greatest range of low m values ($m<8$), is about 0.3.

Let us now consider case B, which requires a conducting shell to stabilize the $m=2$ mode, but offers the advantage that the limiter q value can be dropped below 2. Here q_c is again just above 1, but q_a is chosen to be just above 1.5, with a value of p in the range $0<p<\frac{2}{3}$. As before, if q_c is *infinitesimally* above 1, Eq. (9) shows that the "inner" sequence of modes, with $(m,n)=(3,2)$, (4,3), etc., is positively stable, for any finite value of p in the above range. We must also, however, demonstrate the stability of the "outer" sequence of modes (2,1), (5,3), (8,5), etc. For q_a infinitesimally above 1.5, we find that the stability condition is never satisfied for the (2,1) mode, but it can be satisfied for the (5,3) mode, and all higher modes of this "outer" sequence, provided $p<0.32$. The (2,1) mode can, however, be stabilized by means of a conducting shell. The requirement on the radius b of the shell can be

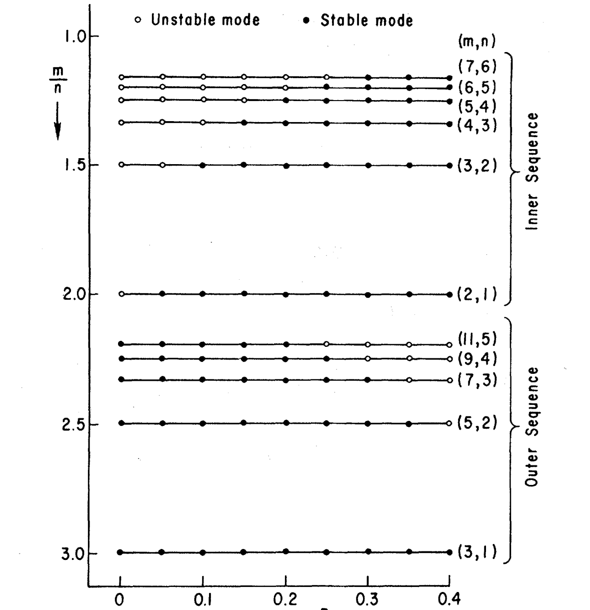

FIG. 2. Stability diagram for modes (m,n) in case A with use of the current profile of Fig. 1, with $q_c = 1.05$, $q_a = 2.1$, $b = \infty$, and various values of the pedestal p.

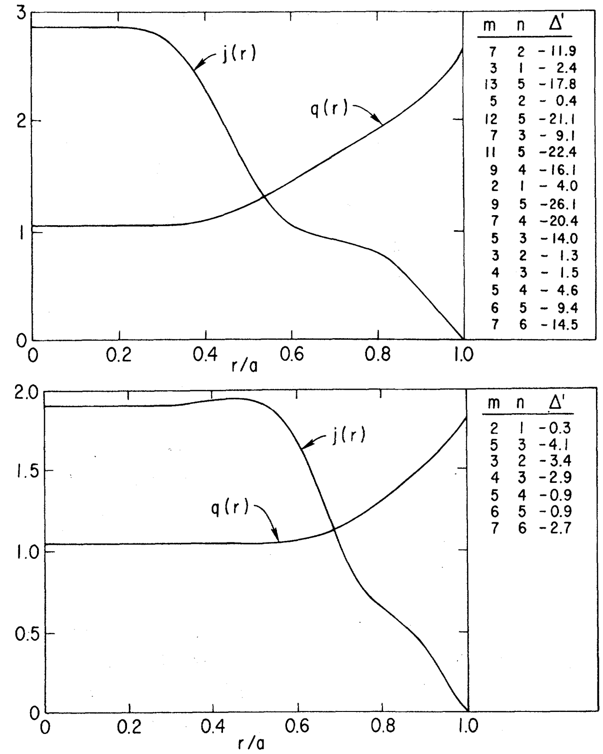

FIG. 3. (a) Example of a stable profile similar to case A, with no conducting shell; the computed values of Δ' show that all the indicated modes are stable. (b) Example of a stable profile similar to case B, with a conducting shell at $r/a = 1.2$.

determined from Eq. (8) by setting $\psi/\psi_c > 0$ at $r = b$. Employing $q_c = 1$, $q_a = 1.5$, Eq. (5) for c/a, and $(m,n) = (2,1)$, we obtain

$$b/a < (4/3p)^{1/4}. \qquad (10)$$

For $p \simeq 0.3$, this gives $b/a < 1.45$, a requirement that could be met rather easily.

The simple analytic treatment given above has the advantage of clarifying the role of a current pedestal in stabilizing low-m modes. However, as we have seen, the use of a discontinuous function for $j(r)$ has the disadvantage of exciting high-m modes. In order to investigate the possibility of stabilizing *all* kink modes simultaneously, we have employed a computer program that determines the stability of arbitrary *smooth* current profiles by calculating the quantities Δ' that measure the potential-energy perturbations for the various modes.

Figures 3(a) and 3(b) show two examples of "realistic" current profiles resembling the analytic cases A and B. In both cases, we see that the entire spectrum of modes is stabilized ($\Delta' < 0$), the higher-m modes apparently being suppressed by the smoothing of $j(r)$. The corresponding limiter q values are 2.6 and 1.8, respectively. In Fig. 3(b) a fairly close conducting shell was needed ($b/a = 1.2$); alternatively, one could invoke toroidal-curvature stabilization[3] of the weakly unstable higher-m modes of the "inner" sequence, thus permitting a lower pedestal and a larger value of b/a.

Our results are in accord with the experimentally observed destabilizing effects of limiter q values that approach 2, or high levels of impurity influx. In either case, the outer plasma region would be cooled, so that the pedestal on the current profile would tend to be truncated short of the $q(r) = 2$ point. In larger tokamaks, it may be possible to achieve better control over the current distribution, so that profiles resembling Fig. 3(a) could be approximated.

Experiments on conducting-shell stabilization[5] proved successful in suppressing the $(m,n) = (2,1)$ mode, thus obtaining gross stability at $q_a < 2$. There was, however, evidence of a deterioration in confinement, particularly for $q_a \simeq 1.5$. Our results for case B show that this could be explained in terms of the truncation of the pedestal on the current profile short of the point where $q(r) = 1.5$.

We conclude with a word about our approximations. Our calculation is based upon a collisional-fluid treatment of resistive modes in a cylindrical geometry. More detailed studies of these modes have shown that there is no change in the stability criterion due to diamagnetic and gyroviscous effects,[6] and that there is an improvment in stability due to toroidicity.[3] Thus, our results may turn out to be conservative when applied to tokamaks in more collisionless, reactorlike regimes.

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Self-Consistent Calculation of the Electronic Structure at an Abrupt GaAs-Ge Interface

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The potential, charge density, and interface states have been calculated for the ideal interface between intrinsic GaAs, terminated on a (100) Ga plane, and intrinsic Ge. The conduction band is continuous across the interface and negligible interface dipole moment is found. Fractional occupancy of the interface bonds arises via a single partially occupied band of interface states. We find that a long-range potential disturbance must occur unless interface bonds are longer than bulk bonds by about 4%.

The purpose of this Letter is to report the first self-consistent calculation of the potential, charge density, and spectrum of localized states at the interface between two semiconductors. Interface questions of fundamental importance from a carrier-transport point of view are as follows: What are the discontinuities in the valence- and conduction-band edges? What is the spectrum and spatial extent of states whose energy lies in the forbidden gap of both materials? Interface questions of fundamental importance from a chemical point of view are the following: What is the nature of the covalent bond when it contains less than the normal two electrons? What effect does this have on the spatial arrangement of the atoms? The work reported here will address both sets of questions.

The constituent semiconductors chosen for this study were GaAs and Ge, a system of interest for several reasons. Firstly, each material can be grown epitaxially on a substrate of the other, and there is an extensive literature on properties of the heterojunctions formed thereby.[1] Secondly, since both materials have the same lattice constant and lattice structure, it is reasonable to expect continuity of bonding across the interface. Thirdly, problems associated with alloy systems, such as GaAs-$Al_xGa_{1-x}As$ where site occupation (Ga or Al) is random, do not arise. Finally, potentials for Ga, Ge, and As are known with which acceptable self-consistent bulk band structures and self-consistent atomic term values have been calculated. These potentials, with no adjustment, are used in the present work. There is, however, no *a priori* way to know the detailed atomic geometry at the interface; and so in this first work, we shall explore the electronic structure of the simplest plausible geometry, the unreconstructed interface. Within this geometry, we shall consider two limiting cases, one where tetrahedral bond angles are preserved and the other where they are altered to allow the interface bond to lengthen by an amount suggested by the covalent radii of the atoms involved.

The technique used to carry out this calculation is that developed by Appelbaum and Hamann[2] for performing self-consistent calculations of semiconductor surfaces. One feature of the scheme