wave analyses of the 3π data[11] show $\sigma_T(J^P=1^+,N) \lesssim \sigma_T(\pi, N)$, whereas $\sigma_T(J^P=0^-,N) \sim 2\sigma_T(\pi, N)$. It should be noted that the fits for σ_T depend substantially on the ratio of the N^* yields from heavy nuclei to light nuclei, since the attenuation of the signal depends dominantly on $\sigma_T(N^*N)$. Thus, for example, fits to the data with fixed $\alpha(N^*N) = -0.2$ yielded results little different from those in Fig. 3(d).

Qualitatively the behavior of $\sigma_T(N^*N)$ with $M(N^*)$ is remarkably similar to the t slope data for $pp \to pN^*$. Fäldt[12] has discussed possible refinements to the Glauber model for multiple N^* scattering and helicity flip which would give rise to an "apparent" decrease in $\sigma_T(N^*N)$ with $M(N^*)$.

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[1]E. J. Makuchowski, Ph.D. thesis, Carnegie-Mellon University, 1976 (unpublished).

[2]K. J. Foley *et al.*, Nucl. Instrum. Methods 108, 33 (1973).

[3]R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Britten *et al.* (Interscience, New York, 1959), Vol. 1.

[4]L. Strawczynski, Ph.D. thesis, University of Rochester, 1974 (unpublished).

[5]Data include events which are either one-constraint or zero-constraint under the coherent assumption.

[6]V. Blobel *et al.*, Nucl. Phys. B97, 201 (1975). pp data at 12 and 24 GeV are included.

[7]*Landolt-Börnstein Nuclear Radii*, edited by H. Schopper (Springer, Berlin, 1967), Vol. 2.

[8]K. Foley *et al.*, Phys. Rev. Lett. 19, 857 (1967), and Phys. Rev. 181, 1775 (1969).

[9]See, for example, H. Johnstad *et al.*, Nucl. Phys. B42, 558 (1972).

[10]C. Bemporad, Nucl. Phys. B33, 397 (1971), and B42, 627 (1972).

[11]P. Muhlemann *et al.*, Nucl. Phys. B59, 106 (1973); U. Kruse *et al.*, to be published.

[12]G. Fäldt, in Proceedings of the Topical Meeting on High Energy Collisions Involving Nuclei, International Center for Theoretical Physics, June 1976 (to be published).

Extracting Hadron-Neutron Scattering Amplitudes from Hadron-Proton and Hadron-Deuteron Measurements*

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I present a method to extract hadron-neutron scattering amplitudes from hadron-proton and hadron-deuteron measurements within the framework of the Glauber approximation. This method, which involves the solution of a linear integral equation, is applied to pn collisions between 15 and 275 GeV/c, and effects arising from inelastic intermediate states are estimated.

One of the earliest applications of the Glauber approximation[1] was to the scattering of hadrons by deuterium.[2] It was sufficient, at that time, to assume identical black-sphere interactions for the free hadron-neutron (xn) and hadron-proton (xp) collisions in order to calculate the cross-section defect $\delta\sigma \equiv \sigma_n+\sigma_p-\sigma_d$, i.e., the difference between the free xn plus xp total cross sections and the hadron-deuteron (xd) total cross section. An expression for the xd scattering amplitude, f_d, in terms of the free xn and xp elastic scattering amplitudes, f_n and f_p, and the deuteron form factor, S, was later presented.[3,4] Since neutron targets are unavailable, this result has often been used in attempts to extract the xn total cross section, elastic scattering amplitude, or other information regarding xn scattering from a knowledge of the xp and xd amplitudes and the deuteron form factor, these latter quantities being easier to measure directly. However, in order to extract this information some assumptions are made regarding the xn amplitude, i.e., regarding the function about which information is desired. I present here a solution to the problem of extracting f_n from f_p, f_d, and S, within the framework of the Glauber approximation, which requires no explicit assumption regarding f_n. I then apply these results to recent pd measurements[5] to extract xn cross sections and angular distributions and to estimate the magnitude and energy dependence of the contribution to these cross sections arising from inelastic intermediate states.[6,7]

The hadron-deuteron elastic scattering amplitude is given in the Glauber approximation by[3,4]

$$f_d(q)=f_n(q)S(\tfrac{1}{2}q)+f_p(q)S(\tfrac{1}{2}q)+\tfrac{1}{2}i\pi^{-1}\int S(\vec{q}\,'-\tfrac{1}{2}\vec{q})f_p(\vec{q}-\vec{q}\,')f_n(q')d^2q', \tag{1}$$

where $\hbar\vec{q}$ is the momentum transfer and our normalization is such that the xn, xp, and xd total cross sections are

$$\sigma_j=4\pi\,\mathrm{Im}f_j(0),\quad j=n,p,d, \tag{2}$$

and the differential cross sections are

$$d\sigma_j/dt=\pi\hbar^{-2}|f_j|^2, \tag{3}$$

where $-t=\hbar^2q^2$ is the square of the four-momentum transfer. (I neglect complications due to double charge exchange, spin dependence, Fermi motion, and the deuteron d state, which contribute negligibly at high energies and small momentum transfers. d-state contributions can be included without great difficulty in the present formalism.) One may rewirte Eq. (1) as a linear integral equation for $f_n(q)$ in the form

$$f_n(q)=h(q)+\int K(q,q')f_n(q')dq', \tag{4}$$

where

$$h(q)=f_d(q)S^{-1}(\tfrac{1}{2}q)-f_p(q), \tag{5}$$

$$K(q,q')=-\tfrac{1}{2}i\pi^{-1}q'S^{-1}(\tfrac{1}{2}q)\int_0^{2\pi}S(\vec{q}\,'-\tfrac{1}{2}\vec{q})f_p(\vec{q}-\vec{q}\,')d\varphi, \tag{6}$$

and in which $\cos\varphi=\vec{q}\cdot\vec{q}\,'/qq'$.

In the past the xn amplitude, f_n, has been extracted from Eq. (1) by assuming that it is Gaussian in q with the same slope parameter as f_p, and that it has the same ratio $\mathrm{Re}f_n(0)/\mathrm{Im}f_n(0)$ as does f_p. Equation (1) then becomes a simple linear algebraic equation for the remaining xn parameter, σ_n. However, these assumptions are not necessary.

The solution of Eq. (4) depends, of course, on $K(q,q')$ and $h(q)$; i.e., it depends on $f_d(q)$, $f_p(q)$, and $S(q)$. An accurate representation for S may be obtained by writing

$$S(q)=\sum_{i=1}^{N}\mu_i\exp(-R_i^2q^2), \tag{7}$$

where μ and R_i^2 are parameters. At high energies and over a sufficiently large range of momentum transfer, f_p is well described by the Gaussian form

$$f_p(q)=c_pe^{-aq^2/2},\quad c_p\equiv\sigma_p(i+\rho_p)/4\pi. \tag{8}$$

In contrast to xp scattering, the angular distribution for xd elastic scattering cannot always be well described by a simple Gaussian.[5] It can, however, be fitted by an amplitude of the form

$$f_d(q)=c_d\sum_{i=1}^{M}\lambda_i\exp(-\tfrac{1}{2}d_iq^2),\quad c_d\equiv\sigma_d(i+\rho_d)/4\pi. \tag{9}$$

With these forms for S, f_p, and f_d, if one defines

$$p_{l,i}(q)\equiv-\frac{ic_p\mu_i^{1/2}\exp[-(\tfrac{1}{2}a+\tfrac{1}{4}R_i^2)q^2]q^{2l}[\tfrac{1}{2}(a+R_i^2)]^l}{l!\sum_i\mu_i\exp(-\tfrac{1}{4}R_i^2q^2)}, \tag{10}$$

$$b_{l,i}(q)\equiv(l!)^{-1}\mu_i^{1/2}[\tfrac{1}{2}(a+R_i^2)]^lq^{2l+1}\exp[-(\tfrac{1}{2}a+\tfrac{1}{4}R_i^2)q^2], \tag{11}$$

one obtains

$$h(q)=\frac{c_d\sum_i\lambda_i\exp(-\frac{1}{2}d_iq^2)}{\sum_i\mu_i\exp(-\frac{1}{4}R_i^2q^2)}-c_pe^{-aq^2/2}, \tag{12}$$

$$K(q,q')=\sum_{k=0}^{\infty}p_{[k/N],k-N[k/N]}(q)b_{[k/N],k-N[k/N]}(q'), \tag{13}$$

$$\equiv\sum_{k=0}^{\infty}\bar{p}_{k,N}(q)\bar{b}_{k,N}(q'), \tag{14}$$

where $[x]$ denotes the largest integer $\leq x$.

The solution of Eq. (4) with kernel $K(q,q')$ of the separable form (14) is

$$f_n(q)=h(q)+\sum_{k=0}^{\infty}q_k\bar{p}_{k,N}(q), \tag{15}$$

where the g_k are obtained by solving the matrix equation

$$(1-Q)g=u \tag{16}$$

in which

$$Q_{jk}=\int\bar{b}_j(q)\bar{p}_{k,N}(q)dq, \tag{17}$$

$$u_j=\int\bar{b}_j(q)h(q)dq. \tag{18}$$

In general, the quantities Q_{jk} and u_j must be computed numerically. (If $N=1$ they can be obtained analytically.)

I apply Eq. (15) to the extraction of high-energy pn total and differential cross sections from pp and pd measurements. For $S(q)$, I use an accurate fit to realistic deuteron wave functions[8]

$$S(q)=0.34e^{-141.5q^2}+0.58e^{26.1q^2}+0.08e^{-15.5q^2}, \tag{19}$$

with q in units of GeV/c. For σ_p and σ_d, I use global fits to the data.[9] At a given energy there are systematic discrepancies between experiments. A global fit should average out some of these discrepancies.[9] I assume $\rho_d=\rho_p$. The results are insensitive to this choice; for example, choosing ρ_d such that $\rho_n=\rho_p$ would produce a negligible change in the results. The parameters a and ρ_p were obtained from pp data.[10] I fit the pd elastic scattering data[5,11] with Eq. (9) with $M=2$. Typical fits are shown in Fig. 1.

To calculate σ_n to an accuracy of 0.01 mb it is sufficient to calculate only the first twelve rows and columns of Q. The results are shown in Fig. 2. The solid curve is a global fit[9] to the np measurements. The dashed curve is the single-scattering impulse-approximation result, $\sigma_d-\sigma_p$. It is too low by ~3–4.5 mb. The dotted curve is the usual Glauber-approximation result which requires special assumptions for the form of f_n. It represents a great improvement over the dashed curve. It gives results which are in agreement with the measurements at the lower energies, but which are too low by ~0.9–1.4 mb above 50 GeV/c. The crosses are the results of the present calculation. [The fluctuation in these results is due to the fact that the present formalism requires pd elastic scattering data (i.e., λ_1, λ_2, d_1, and d_2) as input. These parameters, as obtained from the pd data, are not very smooth functions of incident energy.] One notes that the results, which employ no adjustable parameters and no assumptions regarding $f_n(q)$, give pn total cross sections which are slightly lower than the already low results of the usual Glauber-approximation

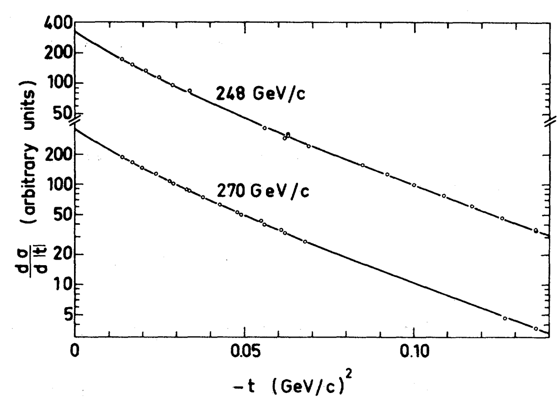

FIG. 1. Fit to pd elastic scattering angular distributions by means of Eq. (9) with $M=2$. The data are from Ref. 5.

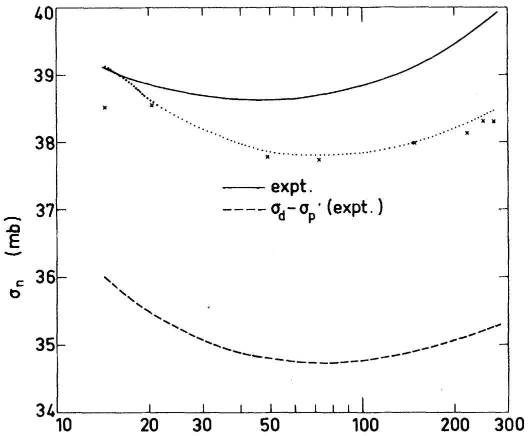

FIG. 2. Proton-neutron total cross sections. The solid curve represents a global fit to the data (Ref. 9). The dashed curve represents the single-scattering impulse approximation. The dotted curve represents the usual Glauber-approximation calculation. The crosses represent the present calculation.

calculation, which is a more approximate calculation. In both calculations, the agreement with experiment gets worse as the incident energy increases. It seems clear that some other effect, such as inelastic intermediate states, is quite significant (~ 1–1.6 mb above ~ 50 GeV/c) in the extraction of σ_n from pp and pd data—in qualitative agreement with Ref. 9 where this effect was estimated to be 0.6–0.97 mb ± 40% for momenta between 50 and 300 GeV/c.

An additional advantage of our calculation is that the hadron-neutron *amplitude* $f_n(q)$ can be extracted from the proton and deuterium data. In Fig. 3 I present the pn elastic scattering angular distributions at 248 and 270 GeV/c calculated from Eq. (15). Note that the angular distributions deviate significantly from a simple exponential in t.

In order to investigate the effect on $f_n(q)$ from terms which are *not* exponential in character for large or small q, I also calculated $f_n(q)$ by truncating Eq. (15) after N terms (in my case, $N=3$). This truncated result would not have any terms that behave like q^2 or q^2 times an exponential in t, and therefore *could* behave like an exponential or a sum of exponentials. These truncated solutions, shown in Fig. 3, are seen to be not at all

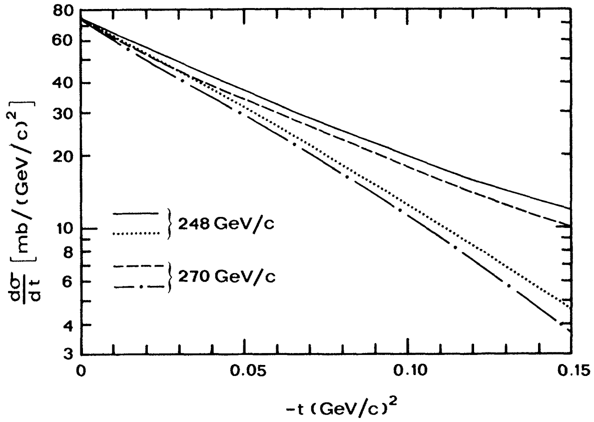

FIG. 3. Calculated angular distributions for pn elastic scattering. The solid and dashed curves are the full solution, Eq. (15). The dotted and dot-dashed curves are solutions obtained by truncating the series in Eq. (15) after the third term.

close to the full solutions, and do not have a simple exponential shape in the t region considered. Consequently, the terms in Eq. (15) that behave like q^2 times an exponential ($k \geq 3$) are significant.

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[1]R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. Brittin *et al.* (Interscience, New York, 1959), Vol. I.

[2]R. J. Glauber, Phys. Rev. 100, 242 (1955).

[3]R. J. Glauber, in *Proceedings of the Conference on Nuclear Forces and the Few Nucleon Problem*, edited by T. C. Griffith and E. A. Power (Pergamon, London, 1960), Vol. I.

[4]V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966).

[5]Y. Akimov *et al.*, Phys. Rev. D 12, 3399 (1975).

[6]J. Pumplin and M. Ross, Phys. Rev. Lett. 21, 1778 (1968).

[7]V. N. Gribov, Zh. Eksp. Teor. Fiz. 56, 892 (1969) [Sov. Phys. JETP 29, 483 (1969)].

[8]V. Franco and G. K. Varma, Phys. Rev. C 12, 225 (1975).

[9]P. V. R. Murthy *et al.*, Nucl. Phys. B92, 269 (1975).

[10]V. Bartenev *et al.*, Phys. Rev. Lett. 31, 1088, 1367 (1973).

[11]G. G. Beznogikh *et al.*, Nucl. Phys. B54, 97 (1973).