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Nonlinear Evolution of Runaway-Electron Distribution and Time-Dependent Synchrotron Emission from Tokamaks*

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(Received 20 May 1976)

Stability of the runaway electrons in tokamaks is analyzed. The distinction between high-density and low-density operation of tokamak discharges is interpreted in terms of the stability condition obtained. In the unstable case, the temporal evolution of the distribution function of the runaway electrons is obtained by solving the quasilinear equation. Time-dependent synchrotron emission from the runaway electrons is calculated.

Recent measurements of cyclotron radiation from low-density tokamak plasmas have shown significantly nonthermal properties[1-3]: The radiation intensity is more than an order of magnitude above the thermal level expected from the measured temperature, with a broad frequency spectrum having minima at the gyroharmonics[4]; the radiation level is nonsteady with sudden ($\lesssim 10\ \mu$sec) increases in intensity, correlated with loop-voltage spikes,[4] with bursts of x rays and of radiation from ω_{pe} to ω_{pi}, and with the ion heating.[5] In this Letter, we attempt to interpret this enhanced radiation as synchrotron emission by the runaway electrons.[6]

A specific distribution function of the runaway electrons is obtained by solving the Fokker-Planck equation in the steady state, and this distribution is then used as the unperturbed runaway distribution for stability analysis. In the unstable region, the time-dependent quasilinear equation is solved analytically to obtain the nonlinear evolution of the runaway distribution. With this time-dependent distribution function of the runaway electrons, we calculate the time evolution of the spectrum of their synchrotron emission, including the effect of reabsorption by the background plasma.

We solve the Fokker-Planck equation in the runaway region where $v_\parallel > v_c = (E_0/E)^{1/2} v_e$, $v_e = (2T_e/m)^{1/2}$, $E_0 = e\ln\Lambda/\lambda_D{}^2$, and $\lambda_D{}^2 = T_e/4\pi n e^2$. Included is a loss term of the form $-v_\parallel f/v_L\tau_0$, as a model to account for the loss of high-energy electrons caused by the imperfect magnetic surfaces,[7] where $\tau_0{}^{-1}$ is the loss rate of particles with $v_\parallel > v_L$. To have a steady state, a source at low energy is introduced to maintain a constant rate, γ_0, of runaway production given by[8] $\gamma_0 = 0.35\nu_0(E/E_0)^{-3/8}\exp\{-[(2E_0/E)^{1/2} + E_0/4E]\}$, where ν_0 is the electron-electron collision frequency. The Fokker-Planck equation in the runaway region[8] is then solved for $v_\parallel \gg v_c$ to obtain the following steady-state solution

$$f_R = 2\Delta n \frac{E}{E_0}\left[\pi^{3/2} v_e{}^2 v_0 \ln\left(\frac{E}{E_0}\frac{v_\parallel{}^2}{v_e{}^2}\right)\right]^{-1} \exp\left\{\frac{E}{E_0} v_\perp{}^2\left[v_e{}^2 \ln\left(\frac{E}{E_0}\frac{v_\parallel{}^2}{v_e{}^2}\right)\right]^{-1} + \frac{v_\parallel{}^2}{v_0{}^2}\right\}. \quad (1)$$

The normalization is so chosen that the loss of the runaways is balanced by the runaway production, i.e., $\Delta n/\tau = n_0\gamma_0$, where $\Delta n/n_0$ is the density ratio of the runaways to the bulk thermal electrons, and τ is the average life time of runaway electrons. Moreover, $v_\perp$ and $v_\parallel$ are the velocity components perpendicular and parallel to the electric field, and $v_0 = [E\nu_0\tau_0/E_0]^{1/2} v_L$ is the effective cutoff velocity. Typically, for low-density discharges, the observed energy of the runaways is cut off at about 200 keV, corresponding to a value of $v_0/v_e \approx 12$ for a bulk electron temperature of 0.7 keV. Note that the effective perpendicular temperature of the runaways is enhanced by a factor $(E_0/E)\ln[Ev_\parallel{}^2/E_0 v_e{}^2]$ with the logarithmic factor typically about 2, and is an order of magnitude less than the parallel temperature.

For the stability analysis, we choose, for simplicity, the unperturbed distribution to be a composite of a bulk of Maxwellian electrons $v_\parallel \leq v_c = v_e(E_0/E)^{1/2}$ and a runaway tail given by Eq. (1) for $v_\parallel > v_c$ in a magnetic field aligned with the inductive electric field. The anisotropy of the runaway distribution can then drive the plasma wave unstable through the anomalous cyclotron resonance $\omega_k + \Omega = k_\parallel v_\parallel$.[9] This instability was first qualitatively examined by Kadomtsev and Pogutse and its quasilinear effects have been qualitatively studied by Shapiro and Shevchenko.[9] Here we use the explicit runaway distribution given by (1) to obtain a definite stability boundary and the quasilinear evolution of the distribution needed for the calculation of the synchrotron radiation. The frequency of the plasma wave in a strong magnetic field is $\omega_k = k_\parallel \omega_p / k \ll \Omega$, where $\omega_p = (4\pi n e^2/m)^{1/2}$ and $\Omega = eB/mc$. The growth rate is given in terms of the imaginary part of the dielectric function given by[10]

$$\gamma_k = \frac{\omega_k}{2} \sum_{n=0,-1} \frac{2\pi^2 \omega_p^{\,2}}{k^2 |k_\parallel|} \int_0^\infty v_\perp dv_\perp J_n^{\,2}\left(\frac{k_\perp v_\perp}{\Omega}\right)\left(\frac{n\Omega}{v_\perp}\frac{\partial}{\partial v_\perp} + k_\parallel \frac{\partial}{\partial v_\parallel}\right) f_0 \Big|_{v_\parallel = \omega_k - n\Omega/k_\parallel}, \quad (2)$$

where only $n = 0, -1$ terms are necessary for $\omega_k + \Omega \lesssim k_\parallel v_0 \ll \omega_k + 2\Omega$. Note that the growth term ($n = -1$) is significant only if $(\omega_k + \Omega)/k_\parallel \lesssim v_0$ or equivalently $k\lambda_D \gtrsim (v_e/v_0)(\Omega/\omega_k)$. The Landau damping term ($n = 0$) depends strongly on the distribution (i.e., Maxwellian or runaway) at the phase velocity $\omega_k/k_\parallel$, typically in the runaway region for $E/E_0 > 0.1$. The stability boundary shown in Fig. 1 is obtained for a given resonant velocity $v_R = (\omega_k + \Omega)/k_\parallel \lesssim v_0$ by setting $\gamma_k = 0$. In tokamaks, $E = \eta j = c\eta(\nabla \times B_p)/4\pi$, where η is the resistivity and B_p is the poloidal magnetic field. Thus, in terms of tokamak parameters, $E/E_0 = 0.38(\eta/\eta_S)(1/Rq)(c^2/v_e)(\Omega/\omega_p^2)$, where η_S is the Spitzer resistivity, $q = rB_T/RB_p$ is the safety factor, typically about unity near the axis, and r and R are the minor and major radii, respectively. The variation of E/E_0 with density traces out a curve of tokamak operation with changing density but fixed magnetic field as shown also in Fig. 1. Interestingly it intersects the stability boundary twice. In the high-density region ($1 > \omega_p/\Omega > 0.4$, $E/E_0 < 0.03$) it is a weak instability due to the very small number of the runaway electrons at low E/E_0, and therefore it has little effect on the low-harmonic cyclotron radiation and macroscopic plasma behavior. The excited plasma waves, however, can be transformed into electromagnetic waves by a variety of mechanisms and could therefore be relevant to the observed peak at ω_p. In the low-density, high-E-field region ($\omega_p/\Omega < 0.3$, $E/E_0 > 0.1$) the runaways carry a substantial portion of the energy and current with a few percent concentration, $\Delta n/n$. The instability is a strong one with growth rate typically $(\Delta n/n)(\omega_p/\Omega)^2\omega_p$. The turbulent pitch-angle scattering of the runaway electrons by the unstable waves then enhances their perpendicular energy at the expense of their parallel energy, thereby causing enhanced synchrotron radiation.

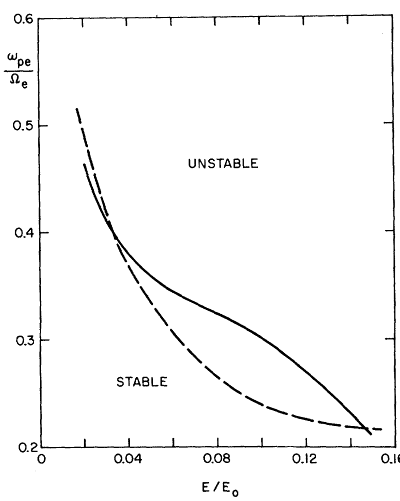

FIG. 1. Stability boundary and tokamak operation curve. The solid line is the stability boundary for waves resonating with electrons of velocity $v_\parallel = 11 v_e$ (the most unstable mode) in a plasma with a temperature of 0.6 keV. The broken line is the tokamak operation curve with varying density but constant magnetic field $B = 40$ kG, major radius $R = 60$ cm and $q = 1$. The resistivity of the plasma is taken to be Spitzer resistivity. The runaway lifetime is taken to be the free-streaming time from v_e to the cutoff velocity $v_0 = 13 v_e$. The fraction of the runaways in the high- and low-density unstable regions, $\Delta n/n$, is of the order 10^{-4} and 10^{-1}, respectively.

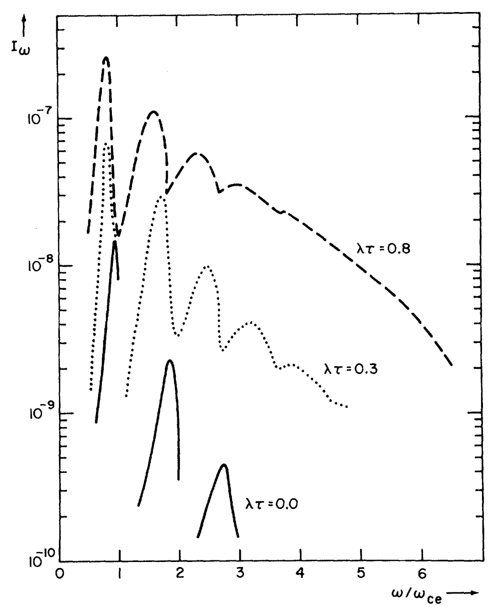

FIG. 2. Spectrum of synchrotron radiation by the unstable runaway electrons. Radiation is generated by both background electrons and runaways, which concentrate within 5 cm near the minor axis. Absorption is due to background only. $\lambda\tau$ is a natural time scale of evolution of the distribution as discussed in the text. $I\omega$ is in CGS units. Observation is made vertically. $B = 40$ kG, $n_e = 5 \times 10^{12}$ cm^3, $E/E_0 = 0.13$, $Z_{\rm eff} = 1$.

To obtain the temporal evolution of the distribution function of the runaway electrons, we solve the time-dependent quasilinear equation analytically, using the unperturbed distribution Eq. (1) as the initial function. The quasilinear equation for the runaways is approximately[11]

$$\frac{\partial f(v_\parallel, v_\perp, t)}{\partial t} = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left(D\frac{1}{v_\perp}\frac{\partial f}{\partial v_\perp}\right), \tag{3}$$

where

$$D = 8\pi^2 e^2 \Omega^2 m^{-2} \int d^3k\, \epsilon_k k^{-2} \delta(\omega + \Omega - k_\parallel v_\parallel) J_1^{\,2}[k_\perp v_\perp/\Omega],$$

$\epsilon_k = |E_k|^2/8\pi$ is the electrostatic energy, and we have neglected terms involving the parallel derivative because $\partial f_0/\partial v_\parallel \ll \partial f_0/\partial v_\perp$ for the most part of the initial runaway distribution. We consider the case of strong magnetic field so that the unstable waves have $b_1 = k_\perp^{\,2}\lambda_D^{\,2}(\omega_p/\Omega)^2(E_0/E)\ln(E_0 v_\parallel^{\,2}/E v_e^{\,2}) \ll 1$. The diffusion coefficient can then be written as

$$D = (2\pi^2 e^2 v_\perp^{\,2}/m^2)\int d^3k\, (k_\perp^{\,2}/k^2)\epsilon_k(t)\delta(\omega + \Omega - k_\parallel v_\parallel),$$

upon expanding the Bessel function. Because of the explicit dependence of D on $v_\perp^{\,2}$, we may introduce a new time variable τ defined as

$$\tau = (2\pi^2 e^2/m^2)\int_0^t dt' \int d^3k (k_\perp^{\,2}/k^2)\epsilon_k(t')\delta(\omega_k + \Omega - k_\parallel v_\parallel),$$

so that Eq. (3) becomes simply a diffusion equation: $\partial f/\partial\tau = v_\perp^{-1}(\partial/\partial v_\perp)(v_\perp \partial f/\partial v_\perp)$, with initial value given by Eq. (1). The solution to the above equation can be readily obtained, for $v_\parallel > (\Omega/\omega_p)(E_0/E)^{1/2}v_e$, as

$$f(v_\parallel, v_\perp, \tau) = \frac{f_0}{(4\lambda\tau + 1)}\,\frac{\exp[-\lambda v_\perp^{\,2}(4\lambda\tau + 1)^{-1} - v_\parallel^{\,2}/v_0^{\,2}]}{\ln(E v_\parallel^{\,2})}, \tag{4}$$

where $f_0 = 2\Delta n(E/E_0)/\pi^{3/2}v_e^{\,2}v_0$, E and $v_\parallel, v_\perp$ are dimensionless in units of E_0 and v_e, respectively, and $\lambda = E/\ln(Ev_\parallel^{\,2})$. Note that the mean perpendicular energy increases linearly in τ because of pitch-angle scattering, until near isotropy is reached and the waves become damped. To express f in terms of the real time t, we must solve the following evolutionary equations for the fields. Because of the small radial dimension, L, of the runaway region,[12] the wave convection out of the unstable zone with group velocity v_g is an effective saturation mechanism, yielding $\epsilon_k = \frac{1}{2}T\exp(2\gamma_k L/v_g)$. Substituting this expression into Eq. (4) and noting the relatively narrow band of unstable modes centered around $\overline{K} \equiv 2^{-1/2} \times (\overline{k}\lambda_D)^{-1} \approx 3$, $\langle k_\parallel/k\rangle = \overline{\mu} \approx 0.2$, we may therefore obtain approximately

$$t = \tau\,\frac{n\lambda_D^{\,3}}{\pi^2\omega_p}\,\frac{\overline{K}^4}{\Delta K(1-\overline{\mu}^2)}\exp\left(\frac{-2}{\overline{K}\overline{\mu}(1-\overline{\mu}^2)}\,\frac{L\overline{\gamma}_k}{v_e}\right) \sim \tau\,\frac{n\lambda_D^{\,3}}{\omega_p}\exp\left(-4\times 10^{-3}\,\frac{L}{\lambda_D}\right), \tag{5}$$

where $\Delta K \approx 2$. A simple estimate shows that for $L/\lambda_D = 10^3$, $t \sim 1$–10 μsec for $\tau \sim \lambda^{-1}$, which is the time scale for the perpendicular energy to increase a few times, and also approximately the time scale for saturation of the instability.

Having obtained the runaway distribution and its temporal evolution, the time-dependent synchrotron radiation by the runaway electrons resonantly interacting with plasma waves can be calculated directly from the Schott-Trubnikov formula.[13] The total power emitted into the mth harmonic extraordinary modes perpendicular to the magnetic field is found to be

$$\left(\frac{dp}{d\Omega}\right)_{\text{resonant}} = \delta\,\frac{e^2 m^2\Omega^2}{2\pi c\overline{\lambda}^2}\left(\frac{v_e}{c}\right)^4\frac{\Delta n}{n}\,m^2(4\overline{\lambda}\tau + 1)^2\exp\left(-\frac{m^2(4\overline{\lambda}\tau + 1)}{2\overline{\lambda}c^2/v_e^{\,2}}\right)$$
$$\times\left\{\left(1 + \frac{2\overline{\lambda}c^2/v_e^{\,2}}{m(4\overline{\lambda}\tau + 1)} + \frac{2\overline{\lambda}^2c^4/v_e^{\,4}}{m^2(4\overline{\lambda}\tau + 1)^2}\right)I_m - I_{m-1}\right\}, \tag{6}$$

where I_m, I_{m-1} have argument $m^2(4\overline{\lambda}\tau + 1)(2\overline{\lambda}c^2/v_e^{\,2})^{-1}$, and δ is the fraction of runaways that are resonant, approximately $3v_e/v_0 \approx \frac{1}{4}$ for typical parameters. The temporal behavior of $(dp/d\Omega)_{\text{resonant}}$ can increase a few times in a time scale of the order of microseconds, which was indeed observed experimentally. The spectrum of the radiation is calculated including the effect of reabsorption by the background plasma, as shown in Fig. 2.

*Work supported by the National Science Foundation and the U. S. Energy Research and Development Administration.

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Quantum Tunneling in ^{3}He Monolayers

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Measurements are reported of NMR linewidth, $1/T_2$, for ^{3}He atoms adsorbed on Grafoil at 1 K for various quantities of gas adsorbed. The data show three well-defined regions corresponding to a two-dimensional fluid, a two-dimensional solid, and a region where a second layer of adsorbed atoms is forming. A sharp minimum in T_2 gives a precise indication of monolayer completion. A theory for tunneling in the two-dimensional solid is presented.

Specific-heat and isotherm data for samples of inert gases adsorbed on graphite give evidence of unusual homogeneity of the adatom-substrate potential.[1,2] Researchers from the University of Washington[2] have developed a phase diagram for helium adsorbed on Grafoil[3]—a convenient form of graphite having a large surface area per unit mass. NMR studies of adsorbed ^{3}He atoms are capable of supplementing thermal data by giving new information about the dynamical state of the adsorbate. Previous[4-7] NMR studies of ^{3}He adsorbed on graphite have shown linewidth changes and susceptibility data consistent with the proposed solid-liquid transition on the phase diagram, although there is disagreement over the evidence for the "registry" phase where ^{3}He atoms are located in positions determined by the carbon atoms in the substrate.

We report here detailed ^{3}He NMR linewidth measurements (i.e., $1/T_2$) taken at 1 K as a func-