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High-Momentum-Transfer Electron Scattering from ^{208}Pb

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^{208}Pb elastic electron scattering data have been extended to large momentum transfer ($q = 3.7$ fm^{-1}). The present data combined with previous electromagnetic data allow a precise determination of the charge density. It shows a small central depression and density fluctuations much less pronounced than theoretically predicted.

Recent electron-scattering experiments have determined the density fluctuations of nuclear charge densities $\rho(r)$. Such measurements are one of the most stringent tests of theoretical microscopic wave functions. Furthermore, they are very sensitive to the details of the N-N force. For the only two well-investigated cases,[1-3] ^{48}Ca and ^{58}Ni, the experimental densities show much less structure than the best available theories. The most ideal nucleus for a quantitative comparison with theory is ^{208}Pb; its density is hardly influenced by long-range correlations,[4] and Hartree-Fock (HF) theory is most directly applicable to this heavy, doubly magic nucleus. The very large number of theoretical calculations available for ^{208}Pb offers an optimal opportunity to test present theories. Furthermore, as discussed by Friar and Negele,[5] the amount of structure predicted by different calculations varies strongly with the relative phase of the oscillations of neutron and proton densities. In Ca and Ni, these oscillations are in phase, while they are completely out of phase for ^{208}Pb. A measurement for Pb then would provide a valuable complementary information on the lack of structure previously observed.

Because of the maximum momentum transfer $q_{max} = 2.7$ fm^{-1} previously reached,[6] the fluctuations of $\rho(r)$ have not been determined to the precision necessary [$\delta\rho(0) = \pm 7\%$, as discussed below]. Moreover, previous analyses[6-8] provide conflicting values for $r < 4$ fm; this concerns in particular the possible existence of the much-disputed central depression.[6-9] The high-q experiment presented in this Letter was carried out in order to yield a precise determination of $\rho(r)$ at small radii.

The experiment was performed at the Saclay linear accelerator using the HE 1 end station.[10] The electron energy of 502 MeV was determined

to ± 0.05% by the field maps of the SP900 spectrometer. Scattering angles were checked to be accurate to ± 0.05°. The incident beam current was measured by ferrite monitors and a Faraday cup. The scattered electrons were detected using the standard focal-plane equipment.[3] Special attention was paid to long-term stability which was found to be better than ± 2%. The overall detection efficiency was obtained by normalizing the angular distribution measured to the Stanford Linear Accelerator Center[6] (SLAC) and the University of Mainz[7] measurements of ^{208}Pb cross sections at 1.7 fm^{-1}, where both sets of data closely agree. The normalization has been determined to ± 3%; it was verified by measuring ^{12}C cross sections[11] at low momentum transfers.

The target of 217 ± 2 mg/cm^2 ^{208}Pb (99.14%) was held between two aluminum foils. Water circulating between the aluminum foils cooled the target, and allowed the use of an average beam intensity of 20 μA necessary to measure cross sections down to 10^{-10} mb/sr. Aluminum and oxygen contributions were separated by recoil energy difference. Background was absent.

The experimental results are shown in Fig. 1, together with previous 502-MeV data taken at SLAC.[6] The data now span 12 decades.

The data analysis has been performed according to Sick.[12] The density is expanded on a basis of a sum of Gaussians, the amplitudes of which are fitted to the data. The limitation to full model independence comes in through the use of Gaussians of finite width. This restricts the amplitudes of unmeasured high-frequency Fourier components of $\rho(r)$. According to present theoretical understanding the amplitudes of such components are expected to be severely limited; this is due to the Schrödinger equation that strongly couples second derivatives of nucleon wave functions to known energy eigenvalues. The width parameter used, $\gamma = 1.388$ fm, allows one to reproduce a number of theoretical ^{208}Pb densities[13-15] with less than 0.1% deviation and therefore provides enough flexibility to reproduce any fine structure in $\rho(r)$ occurring in presently existing theoretical densities.

The error bars on the resulting density are hence expected to include a realistic estimate for the completeness error (due to the finite q_{max}).

In order to get the most reliable estimate for $\rho(r)$, we have included in our analysis all data concerning electromagnetic information on ^{208}Pb. The result presented here is based on the most recent data published by different laboratories

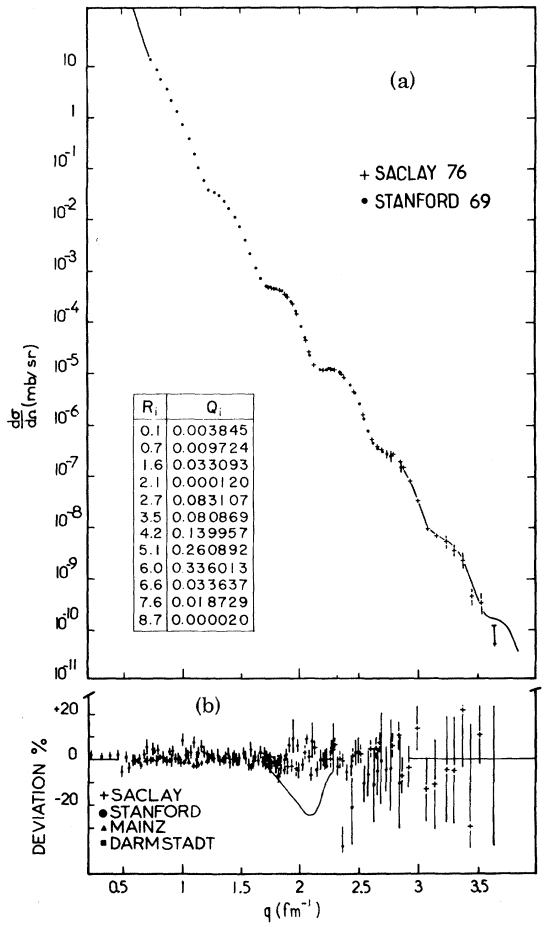

FIG. 1. (a) Cross sections at $E_c = 502$ MeV as a function of effective momentum transfer. The parameters (Ref. 12) of the fit are also given: $\gamma = 1.388$. (b) Deviation between fit and data used; the curve shows the difference to the fit of Ref. 7.

[Fig. 1(b)]. This includes the present electron-scattering data (34 points, $q = 1.7$–3.7 fm^{-1}), SLAC data[6] (87 points, 0.5–2.7 fm^{-1}), the University of Mainz data[7] (17 points, 0.6 – 1.8 fm^{-1}), and the Technical University of Darmstadt data[16] (12 points, 0.3–0.8 fm^{-1}). We have also taken into account the five muonic x-ray transition energies[17-19] that provide additional information on $\rho(r)$. However, for the present fit, we have discarded the 289-MeV data points measured recently at the University of Mainz[7] between 1.8 and 2.3 fm^{-1}. These points strongly disagree (Fig. 1) with both the present and SLAC data. (The discrepancy observed can probably be assigned to a difference in energy calibration. The steep diffraction minimum causes a strong energy dependence in the ^{12}C cross sections[20] relative to which the University of Mainz Pb data[7] have been nor-

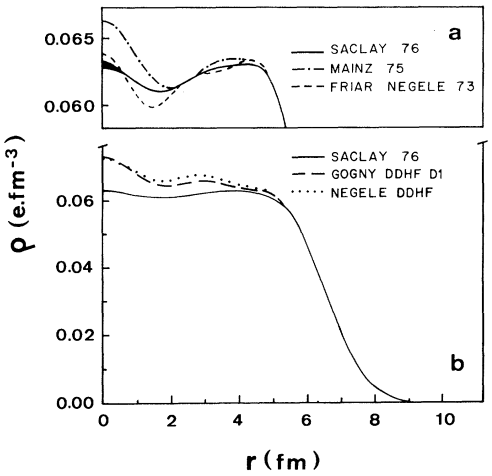

FIG. 2. (a) Charge density determined by present analysis and Refs. 7 and 8. The systematical uncertainties of the data allow an overall shift of $\pm 0.8\%$ for ($r < 4$ fm) without sizeable influence on the details of the structure. (b) Present density compared to density-dependent HF densities of Refs. 13 and 22. The scale of (a) is expanded by a factor of 4.

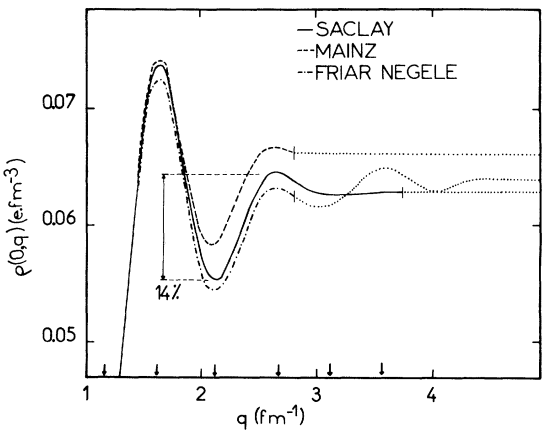

FIG. 3. Central density as a function of maximum momentum transfer for the present fit, and for the fits of Refs. 7 and 8. Vertical bars indicate maximum transfer of data used, arrows indicate diffraction minima.

malized angle by angle.) The total χ^2 of the fit is 194 for 154 data points.

Our result for $\rho(r)$ is shown in Fig. 2; the solid line covers both statistical and completeness errors. The striking features of this density are an almost total lack of fine structure with a peak-to-valley difference of only 3%, and a positive overall average slope of 3% for radii $0 < r < 5$ fm.

The interest in both the possible existence of a central depression and the determination of density fluctuations focuses attention on small radii, i.e., the region where $\rho(r)$ is most difficult to determine. The precision of the measurement at small radii can be discussed in a transparent way by using the sum rule[21]

$$\rho(0, q_{\max}) = \frac{Ze}{2\pi^2}\int_0^{q_{\max}} F(q) q^2 \, dq,$$

where $F(q)$ is the Fourier transform of $\rho(r)$, and $\rho(0) = \rho(0, \infty)$.

In the absence of knowledge on $F(q > q_{\max})$, but with the assumption that $F(q)$ decreases sufficiently fast to damp the oscillations of $\rho(0, q)$, $\rho(0) = \rho(0, \infty)$ will be bounded by the last minimum and maximum value of $\rho(0, q)$. The difference between these two values is the uncertainty $\delta\rho(0)$ of the experimental $\rho(0)$. This gives $\delta\rho(0) = \pm 7\%$ for the SLAC measurement with $q_{\max} = 2.7$ fm^{-1}. Figure 3 indicates that with our extension to $q_{\max} = 3.7$ fm^{-1}, $\delta\rho(0)$ is reduced to 1%. These considerations on $\rho(0)$ are completely independent of the method of analysis used to get the densities of Fig. 2.

The present result for $\rho(r)$ differs in important aspects from analyses of previously available data [Fig. 2(a)]. At small radii, $r < 3$ fm, Friar and Negele[8] obtain a slightly lower average density because they did not include the recent University of Mainz low-q data. In their analysis Friar and Negele have expanded the charge density in two parts $\rho(r) = \rho_0 + \delta\rho$, where ρ_0 is a reasonably good first approximation to $\rho(r)$ and $\delta\rho$ is a Fourier-Bessel expansion. The presence of a large oscillatory component in $\rho(r)$, with the resulting $\rho(0)$ being too large, is due to the model dependence connected with the choice of ρ_0. This leads to the increase of $\rho(0, q)$ in the region not covered by previous data (Fig. 3). The difference between the University of Mainz result for $\rho(0)$ and the present result can be traced to cross-section differences near $q = 2$ fm^{-1} [Fig. 1(b)].

Figure 2(b) is a comparison between our result and two typical Hartree-Fock calculations.[13,22] Both theories use finite range density-dependent effective forces: Negele's density-dependent Hartree-Fock is derived from a realistic two-body interaction while Gogny $D1$ is purely phenomenological. The average slope of the interior density being defined by the percentage change over the region $0 \leq r \leq 5$ fm, the experimental density has a posiitve slope of 3% whereas the theoretical ones exhibit a negative average slope of 10%. This result is typical for most HF calculations and seems to be a systematic shortcoming of the theory.

The amount of structure determined by the present experiment is ~ 3% peak to peak, much smaller than the 9–12% predicted theoretically. In comparing to the previous experiments on Ca and Ni, we find that this reduction is independent of the relative phase of oscillations in the proton and neutron densities. It is a very striking feature to find such a systematic lack of strucutre. Some important ingredients seem to be missing in the Hartree-Fock calculations. We know, for instance, that short-range correlations (SRC) play a very important role in reducing the amplitude of density fluctuations.[23-25] Short-range effects are somehow included in the density dependence of the nuclear force. But although density-dependent HF theories have greatly improved the central density, it could be that SRC are still not properly taken into account. We would also need more complete calculations of the effects of long-range correlations.[4] Therefore a definite explanation of the lack of structure in charge densities requires further developments of the nuclear many-body theory.

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