High–Momentum-Transfer Elastic *e-d* Scattering

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We have calculated the deuteron electromagnetic form factor to all orders of q^2/M^2 in the impulse approximation. Our results are compared to the data for selected deuteron wave functions. We also extract the ultrahigh-q^2 limit of our results, and obtain most naturally the same q^{-10} falloff predicted by the quark model.

Recent measurements[1] have made it necessary to calculate electron-deuteron elastic scattering without making nonrelativistic approximations or q^2/M^2 expansions. Here we report on a relativistic calculation of the deuteron electromagnetic form factors in the impulse approximation (RIA), retaining terms to all orders in q^2/M^2.[2] Two effects are included in this relativistic treatment. First, relativistic kinematics is used throughout. Second, the two nucleons in the deuteron cannot both be on shell. We have included the most important consequence of the latter by allowing the interacting nucleon to be off shell, which requires that all four invariants (or, equivalently, four wave functions) be retained in the deuteron-nucleon vertex.[3] We obtain the three deuteron form factors as functionals of the four deuteron wave functions.

We shall present two aspects of our results in this Letter. We first examine the ultrahigh-q^2 limit of our results, discussing its implications, and then compare numerical results for selected deuteron wave functions with the recent data at high q^2.

The key to understanding the high-q^2 behavior of the form factor lies in examining the q^2 dependence of the generic overlap integral

$$I=\int d^3p\, u(k_1^2)u'(k_2^2), \tag{1}$$

where u and u' are any two of the deuteron wave functions. The arguments are the magnitudes of the relative momenta of the incoming and outgoing deuterons evaluated in their respective rest frames. This is related to the three-momentum of the on-mass-shell spectator, $\vec{p}$, and the momentum transfer $\vec{q}$ in the Breit or brick-wall frame by

$$k_{1,2}^2=p_\perp^2+[(D_0 p_\parallel \pm \tfrac{1}{2}qE_p)/M_d]^2, \tag{2}$$

where $p_\perp$ and $p_\parallel$ are components perpendicular and parallel to $\vec{q}$, $D_0=(M_d^2+q^2/4)^{1/2}$, and $E_p=(M^2+p^2)^{1/2}$. If we expand the wave functions in a series of Hulthén-like functions

$$u(p)=\textstyle\sum_i c_i(p^2+\beta_i^2)^{-1}, \tag{3}$$

we find that for very large momentum transfer

$$I(q)\sim\begin{cases} K_1/q^3 \text{ if } \sum_i c_i\neq 0, & \text{(4a)}\\ K_2/q^7 \text{ if } \sum_i c_i=0, \text{ but } \sum_i c_i\beta_i\neq 0, & \text{(4b)}\end{cases}$$

where

$$K_1=8\pi^2 M_d^3(\textstyle\sum_i c_i)(\sum_j c_j\gamma_j^{-1}\arctan(\gamma_j/\beta_j)],$$

$$K_2=32\pi^2(M_d^7/M^4)(\textstyle\sum_i c_i\beta_i^2)(\sum_j c_j\beta_j), \tag{5}$$

and

$$\gamma_i=(M^2-\beta_i^2)^{1/2}.$$

It may be seen that the second result is the one which is natural for the deuteron. The high-momentum behavior of the vertex functions or wave functions may be determined by studying a covariant wave equation obtained by restricting one particle to the mass shell.[4] If the binding is due to one-boson exchanges and if each *BNN* vertex has a form factor which goes like[5] $(t+p_1^2+p_2^2)^{-1}$, where t is the momentum transfer through the boson and p_1^2 and p_2^2 are the square of the nucleon four-momenta, then the momentum-space wave functions used in Eq. (1) must fall like $1/k_i^4$ and may be arbitrarily well approximated by a sum of Hulthén functions with $\sum_i c_i=0$. If the nucleons and bosons were themselves elementary particles, there should be no *BNN* form factor, the wave functions would fall like $1/k_i^2$, and we would get $\sum_i c_i \neq 0$.

The e-d differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\bigg|_{\text{Mott}} [A(q^2) + B(q^2)\tan^2\tfrac{1}{2}\theta] = \frac{d\sigma}{d\Omega}\bigg|_{\text{Mott}} F_d^2(q^2,\theta), \quad (6)$$

and an examination of the detailed formulas relating A and B to integrals like Eq. (1) yields

$$F_d(q^2,\theta) \underset{\theta\text{ fixed}}{\sim} \begin{cases} q^{-2}F_N(q^2) & \sum c_i \neq 0, & (7a) \\ q^{-6}F_N(q^2) & \sum c_i = 0, & (7b) \end{cases}$$

where F_N is the nucleon isoscalar form factor. The first result is consistent with work[6] which showed that for a system composed of n elementary constituents the electromagnetic form factor should behave as $(q^2)^{1-n}$. The second result shows that the RIA for the deuteron electromagnetic form factor falls like q^{-10}, provided F_N falls like q^{-4}. This is the same result predicted[6,7] on the basis of the quark model.[8]

We note an interesting consequence of the above result concerning the relative size of the RIA and meson-exchange contributions to $F_d(q^2,\theta)$. Since the quark model and the RIA both fall like q^{-10}, their difference—presumably the meson-exchange effects—must fall like q^{-10} (or faster). While the constants multiplying the falloffs of the RIA and meson-exchange effects remain to be determined, there is no *a priori* reason to expect one to dominate the other.

Since the q^{-10} prediction follows from both the quark model and the (relativistically calculated) conventional n-p bound-state model of the deuteron, it might be regarded as a fairly secure prediction. Nevertheless, one should ask how well it is verified by the present data. The cynic will point out that if one fits the data for $q^2 \geq 0.8$ GeV^2 to a monomial q^{-n}, the best fit has $n = 5.5$. However, in order to fairly compare the q^{-10} prediction to the data, one needs to estimate the nonleading terms, which are important in the q^2 region where the form factor has been measured. For the n-p bound-state model, the numerical evaluations of the complete formulas will be presented shortly, and one will see that while the asymptotic falloff is q^{-10}, the falloff at finite q^2 is slower, and indeed follows the trend of the data. For the quark model, a detailed examination has indicated[7] that the form $(1+q^2/m_0^2)^{-1}F_N^2(q^2/4)$ is appropriate for the deuteron form factor. The data[1,9] divided by this factor, with a scaling mass $m_0^2 = 0.28$ GeV^2, are shown in Fig. 1(a). Note the flatness of the implied curve for $q^2 \gtrsim 1$ GeV^2. The data, thus properly analyzed, certainly do not disagree with the q^{-10} prediction. On the other hand, let us also compare the data to the odd possibility that the nucleons and the bosons that bind them together are elementary, i.e., the *BNN* form factors are unity, but the nucleons still have their measured electromagnetic form factors. This leads to a leading q^{-6} falloff for the deuteron electromagnetic form factor, as given by Eq. (7a). The data divided by $(1+q^2/m_0^2)^{-1}F_N(q^2)$ [the form of the first factor gives the correct normalization at $q^2=0$, but differs from a pure q^{-2} falloff only at low q^2] are plotted in Fig. 1(b). The flatness of the curve above $q^2 \gtrsim 1$ GeV^2 is again striking. We must conclude that, because of the importance of the nonleading terms, the data do not yet distinguish the models, and that we must go to higher q^2 to clearly see the leading falloff.

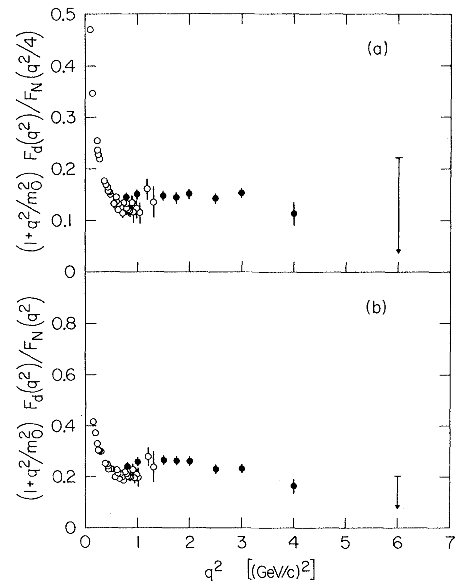

FIG. 1. (a) The deuteron form-factor data compared to $(1+q^2/m_0^2)^{-1}F_N^2(q^2/4)$; this is the quark model prediction of Ref. 6. (b) The data compared to $(1+q^2/m_0^2)^{-1}F_N(q^2)$. The filled circles are data from Ref. 1 and the open circles are data from Ref. 8. In both figures $m_0^2 = 9.28$ (GeV)^2.

This leads us naturally to the question of the predictions for ^{3}He and ^{4}He. While the quark model predicts a falloff of q^{-16} and q^{-22} for these two cases, the actual falloff in the q^2 region of a few GeV^2 would be much slower because of the

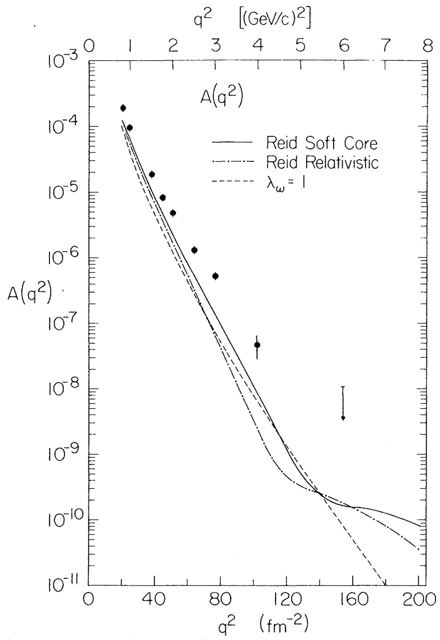

FIG. 2. The data for $A(q^2)$ from Ref. 1 compared to the RIA for selected wave functions described in the text.

large masses which enter the specific scaling prediction of Brodsky and Chertok.[7] If, for example, the quark predictions are compared with the forms $q^{-4}F_N(q^2)$ (for ^{3}He) and $q^{-6}F_N(q^2)$ (for ^{4}He), which follow from the same assumptions leading to (7a), one finds less than a factor of 2 difference in the range 2 $\text{GeV}^2 < q^2 < 8$ GeV^2.

We now turn to the second part of this Letter; we discuss our results for $A(q^2)$ for $q^2 < 6$ GeV^2 as numerically evaluated for several selected wave functions and plotted along with the data in Fig. 2.

There are three theoretical curves in this figure. In each calculation, the nucleon isoscalar form factor was given a dipole form with a squared mass of 0.71 GeV^2. The curve labeled Reid soft core is the nonrelativistic impulse approximation with Reid-soft-core wave functions.[10] The curve labeled Reid relativistic is a calculation with the relativistic formulas using the Reid-soft-core wave functions for usual S and D states, and setting the two additional wave functions to zero. The difference between these two curves is due entirely to treating the kinematics to all orders in q^2/M^2.

The Reid wave functions, however, were obtained from a nonrelativistic Schrödinger equation, and it should be clear that a consistent evaluation of the form factors requires wave functions which were themselves calculated relativistically. There is no consensus on what the best such wave functions are and the curve labeled $\lambda_\omega = 1$ is representative of recent calculations fully described elsewhere.[11]

From the figure we see that the relativistic effects tend to decrease the form factor at $q^2 \approx 2$ GeV^2 and that the calculated form factors can vary by an order of magnitude near $q^2 = 6$ GeV^2. Note also that our calculations fall systematically below the data, so that there is room for other processes, such as meson-exchange corrections[12] or contributions to the impulse approximation with the *spectator* off shell, to make up the difference. Also, further work on the high-momentum components of the wave functions is needed, and this, along with relativistic evaluations of the meson-exchange effects, will clarify the results.

A detailed report on this work will be presented elsewhere.

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[8]It should be noted that both the result (7b) and the q^{-10} prediction of the quark model depend on the assumption that the relativistic wave function is not zero at the origin. Interestingly enough, if one makes the same assumption for the nonrelativistic wave function, and further assumes that there are *BNN* form factors which go like t^{-1}, then one also obtains the result (7b) from the *nonrelativistic* impulse approximation. However, while we believe that it is reasonable to assume that

the relativistic wave functions are finite at the origin, the nonrelativistic wave functions currently in use are zero there because of the repulsive nature of the nonrelativistic potential at short distances, and in this case one would obtain a form factor going like 2 powers of q^2 faster than the result (7b).

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Threshold Pion Production and Multiplicity in Heavy-Ion Collisions*

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Charged pions emerge from roughly 70% of the neon interactions that produce stars in nuclear emulsions at incident energies between 100 and 280 MeV/nucleon. The charged-pion multiplicity averaged 2.8 per pion-producing event. The data are in apparent disagreement with predictions based upon the independent-particle model. Agreement is found with the pion-condensation model of high-density nuclear states as formulated by Kitazoe *et al.*

Nucleus-nucleus interactions are of considerable interest in nuclear and cosmic-ray physics.[1] Of particular importance is the extent to which these interactions differ from predictions of the independent-particle model according to which the interactions are between individual incompressible nucleons in the incident and target nuclei.[2] To search for possible deviations that would be evidence for such effects as collective phenomena, compression of nuclear matter, shock waves, and pion condensation we have been studying pion production using counter techniques[3] and nuclear emulsions.[4]

To the best of our knowledge the emulsion experiment described here (some preliminary results were presented earlier[4]) represents the first study of pions produced in heavy-ion collisions at near threshold incident energies. The number of pion-producing events and the number of pions emerging from the nuclear stars appear to be in substantial disagreement with the predictions of the independent-particle model as formulated by Bertsch[2] and can be interpreted as evidence for pion condensation of the form described by Kitazoe and co-workers.[5]

A stack of 15 Ilford *G*-5 emulsion pellicles were exposed to a beam of 280-MeV/nucleon neon nuclei at the Princeton particle accelerator. Each pellicle had dimensions of 3 in. $\times$4 in. $\times$600 μm. Standard development procedures were followed that normally result in between 15 and 25 blobs along each 100 μm of trajectory of a relativistic, minimum-ionizing, $Z=1$ particle. The beam particles entered the stack parallel to the plane of the pellicle and in the absence of strong interactions came to rest about 25 mm into the emulsion. Beam tracks at entrance into the pellicle were examined under a microscope and followed to the location where the neon nucleus came to rest, interacted or left the pellicle. Roughly 20 m of neon track has been followed yielding 189 events.

Since the primary neon nuclei have energies below 280 MeV/nucleon the tracks of energetic pions emerging from interactions are easily distinguished from those of protons or α's. At these energies it is unlikely that any protons or α's would emerge from either the incident or target