1051 (1966).

[6]J. B. Bronzan, Phys. Rev. 139, B751 (1965).

[7]R. L. Schult and H. W. Wyld, University of Illinois Report No. ILL-(TH)-77-2 (unpublished).

[8]R. Aaron, R. D. Amado, and J. E. Young, Phys. Rev. 174, 2022 (1968).

[9]L. Stodolsky, Phys. Rev. Lett. 18, 973 (1967).

[10]M. G. Bowler, M. A. V. Game, and I. J. R. Aitchison, unpublished. These authors first emphasized the importance of the production of the A_1 through rescattering.

[11]G. W. Brandenburg *et al.*, Phys. Rev. Lett. 36, 1239 (1976).

[12]G. Ascoli *et al.*, Phys. Rev. D 8, 3894 (1973).

[13]E. Berger, ANL Report No. HEP-PR-75-06 (unpublished).

[14]R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968); G. F. Chew and A. Pignotti, Phys. Rev. Lett. 20, 1078 (1968).

[15]D. Morgan, in Proceedings of the International Conference on High Energy Physics, Palermo, Italy, 1975 (unpublished).

Reaction μ^- + Nucleus → e^- + Nucleus in Gauge Theories*

W. J. Marciano and A. I. Sanda
Department of Physics, The Rockefeller University, New York, New York 10021

(Received 1 March 1977)

The process μ^- + nucleus → e^- + nucleus is examined within the framework of gauge theories. We find that the rate for this reaction is much larger than what one might naively expect for a large class of models which allow the decay $\mu \to e+\gamma$. Further experimental search for this rare process is strongly urged.

The search for separate muon- and electron-number nonconservation has led to rather severe bounds on the following reactions: (1) $\mu^+ \to e^+ + \gamma$,[1] (2) $\mu^+ \to e^+ + e^- + e^+$,[2] and (3) $\mu^- + N \to e^- + N$.[3,4] Presently, the published bounds on these processes are

$$R_{e\gamma} \equiv \Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\bar{\nu}\nu) \leq 2.2\times 10^{-8}, \tag{1}$$

$$R_{3e} \equiv \Gamma(\mu \to 3e)/\Gamma(\mu \to e\bar{\nu}\nu) \leq 1.9\times 10^{-9}, \tag{2}$$

$$R_{eN} \equiv \frac{\omega(\mu^- + N \to e^- + N)}{\omega(\mu^- + N_Z \to \nu_\mu + N_{Z-1})} \leq 1.6\times 10^{-8}. \tag{3}$$

Interest in these exotic processes has recently been aroused by experimental[5] and theoretical developments. On the theoretical side, the advent of gauge theories now allows finite computations of the ratios in Eqs. (1)–(3)—a feature missing from earlier calculations which depended on ambiguous cutoffs.

In this Letter we will discuss these processes[6] in the context of a class of gauge theories which allow their occurrence through one-loop diagrams involving intermediate leptons (see, for example, Fig. 1). Our primary purpose is to point out that the models that we consider predict a much larger ratio for $R_{eN}/R_{e\gamma}$ than the naive estimate $[O(\alpha)]$ that one might have made. For comparison and completeness, we discuss all three ratios, $R_{e\gamma}$, R_{3e}, and R_{eN} in these models. Our theme will be that the reaction $\mu^- + N \to e^- + N$ is a sensitive test of muon-number nonconservation, which should be more precisely measured.

Following the notation of Marciano and Sanda,[7] we assume that these exotic reactions are induced by lepton flavor-changing currents of the form

$$\mathfrak{L}_I = -g\sum_j\{W^\alpha(\bar{\mu}\gamma_\alpha c_{1j}\gamma_\pm L_j + \bar{e}\gamma_\alpha c_{2j}\gamma_\pm L_j) + (1/M_W)S[\bar{\mu}c_{1j}(m_\mu\gamma_\pm - m_{L_j}\gamma_\mp) + \bar{e}c_{2j}(m_e\gamma_\pm - m_{L_j}\gamma_\mp)]L_j\} + \text{H.c.}, \tag{4}$$

where $\gamma_\pm \equiv \frac{1}{2}(1\pm\gamma_5)$; the leptons L_j and the intermediate vector boson W carry electric charges $Q'e$ and $(1-Q')e$, respectively; the couplings of the unphysical Higgs boson S have been added to insure gauge invariance; and g is the usual weak coupling $(g^2/2\sqrt{2}M_W^2 \equiv G_F)$.[8] In (4) we have limited ourselves to either right- or left-handed currents but not a mixture of both. (However, some results for theories with right-left mixing will be given.) We consider only theories which satisfy the leptonic Glashow-

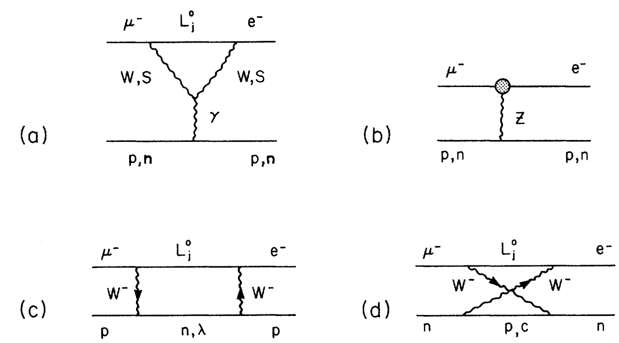

FIG. 1. Diagrams contributing to $\mu^- + N \rightarrow e^- + N$ in $SU(2) \otimes U(1)$ models.

Iliopoulos-Maiani (GIM) mechanism[7,9] condition

$$\sum_j c_{2j} c_{1j} = 0. \tag{5}$$

Our presentation of the results found under these assumptions is divided into two cases, corresponding to the possibility of intermediate leptons with $Q' \neq 0$ and $Q' = 0$, respectively.

(i) $Q' \neq 0$.—This class of theories gives[7]

TABLE I. Estimates of the rates for $\mu^- + N \rightarrow e^- + N$ and $\mu^+ \rightarrow e^+ + e^- + e^+$ relative to the rate for $\mu^+ \rightarrow e^+ + \gamma$ in a variety of models. Values quoted were found using $\sin^2\theta_W = \frac{3}{8}$, $\Delta\,(\mathrm{Cu}) = 0.17$, $\ln(M_W^2/m_L^2) = 5$ for the first three models, and $\ln(M_W^2/m_{L_j}^2) = 3.5$ for model C. Models A, and B, and C are presented in Refs. 11, 12, and 18, respectively.

Models	$R_{eN}/R_{e\gamma}$	$R_{3e}/R_{e\gamma}$
$Q' \neq 0$	$55 \sim 220$	$2.8 \sim 11$
A	54	0.7
B	2	0.05
C	26	0.05

Description of SU(2) x U(1) models with Q' = 0.

A	$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L N_{1_L}\ N_{2_L} \begin{pmatrix}N_1c+N_2s\\ e\end{pmatrix}_R \begin{pmatrix}N_2c-N_1s\\ \mu\end{pmatrix}_R$	$c \equiv \cos\phi$ $s \equiv \sin\phi$
B	$\begin{pmatrix}\nu+(m_e/m_1)N_1c+(m_e/m_2)N_2s\\ e\end{pmatrix}_L \begin{pmatrix}\nu'-(m_\mu/m_1)N_1s+(m_\mu/m_2)N_2c\\ \mu\end{pmatrix}_L$	The rest like A.
C	$\begin{pmatrix}\nu_1\\ e\end{pmatrix}_L \begin{pmatrix}\nu_2\\ \mu\end{pmatrix}_L \begin{pmatrix}\nu_3\\ U\end{pmatrix}_L e_R\ \mu_R\ U_R\ \nu_{U_R}$	$\nu_i \equiv$ orthogonal combinations of ν_e, ν_μ, ν_U

$$R_{e\gamma} = (3\alpha/32\pi)(3Q'-1)^2\Big(\sum_j c_{2j}c_{1j}m_{L_j}^2/M_W^2\Big)^2, \tag{6a}$$

$$R_{3e} = (\alpha^2 Q'^2/12\pi^2)\Big[\sum_j c_{2j}c_{1j}\ln(m_{L_j}^2/M_W^2)\Big]^2. \tag{6b}$$

Let us illustrate the source of the large enhancement[7,10] in R_{3e} relative to $R_{e\gamma}$ in terms of the form factors used by Weinberg and Feinberg.[4] We have

$$\langle e | J_\lambda^{\mathrm{el}}(0) | \mu\rangle = -ie(2\pi)^{-3}\bar{u}_e\{[f_{E0}(q^2)+\gamma_5 f_{M0}(q^2)]\gamma^\alpha(g_{\alpha\lambda}-q_\alpha q_\lambda/q^2) + [f_{M1}(q^2)+\gamma_5 f_{E1}(q^2)](i\sigma_{\lambda\alpha}q^\alpha/m_\mu)\}u_\mu, \quad q = p_\mu - p_e. \tag{7}$$

Whereas only f_{M1} and f_{E1} can contribute to the decay $\mu \rightarrow e + \gamma$, all four form factors contribute to the decay $\mu \rightarrow 3e$. For the $Q' \neq 0$ models considered here, $f_{E0}/f_{M1} = f_{M0}/f_{M1} \approx O(M_W^2/m_L^2)$ at $q^2 = -m_\mu^2$—a realization of the early suggestion[4] that f_{E0} and f_{M0} might dominate processes involving the exchange of a virtual photon, thereby enhancing R_{3e} relative to $R_{e\gamma}$.

The induced form factors in (7) also give rise to the reaction $\mu^- + N \rightarrow e^- + N$. Weinberg and Feinberg found[4]

$$R_{eN} \simeq \frac{|f_{E0}(-m_\mu^2)+f_{M1}(-m_\mu^2)|^2 + |f_{M0}(-m_\mu^2)+f_{E1}(-m_\mu^2)|^2}{8G_F^2 m_\mu^4}, \tag{8}$$

for nuclei with mass number $A \geqslant 60$. Using this formula and the dominating f_{M0} and f_{E0} (see Ref. 7) that led to (6b), we find

$$R_{eN} \simeq (Q'^2/1152\pi^4)\Big[\sum_j c_{2j}c_{1j}\ln(m_{L_j}^2/M_W^2)\Big]^2 \simeq 20R_{3e}. \tag{9}$$

(In fact the ratio $R_{eN}/R_{3e} \approx 20$ is true for a much larger class of models than considered here.) This result tells us that for these $Q' \neq 0$ models, the present experimental bound on R_{eN} [Eq. (3)] is a more severe constraint than the bound on R_{3e} [Eq. (2)].

As a further illustration of the enhancement of R_{eN}, consider the case of only two charged heavy leptons with $m_{L_1} > m_{L_2}$. Then comparing (9) with (6a), we find

$$R_{eN} \geqslant 0.04[Q'^2/(3Q'-1)^2](M_W/m_{L_1})^4 R_{e\gamma} \tag{10}$$

(we note that $m_{L_1}^2/M_W^2 \ll 1$). Such a large enhancement of R_{eN} relative to $R_{e\gamma}$ (numerical estimates are given in Table I) suggests that even if experiments fail to detect $\mu\to e+\gamma$ at the level of $R_{e\gamma}\sim 10^{-8}-10^{-10}$, searches for R_{eN} in this range are still very worthwhile, since $\mu^-+N\to e^-+N$ is potentially a more sensitive test of muon-number nonconservation for this class of models.

(ii) $Q'=0$.—We begin our discussion of this second class of models[11] by considering the induced $\mu e\gamma$ vertex that results from (4). (For non-Abelian theories, this term by itself may be gauge dependent; we work in the 't Hooft–Feynman gauge.) From the top half of Fig. 1(a), we find

$$\frac{f_{E0}}{q^2}=\mp\frac{f_{M0}}{q^2}=\frac{4f_{M1}}{m_\mu^2}=\mp\frac{4f_{E1}}{m_\mu^2}=\left(\frac{G_F}{8\sqrt{2}\pi^2}\right)\sum_j\frac{c_{2j}c_{1j}m_{L_j}^2}{M_W^2}, \tag{11}$$

where the upper sign corresponds to a right-handed theory and the lower to a left-handed theory. If used alone, these form factors yield

$$R_{e\gamma}=(3\alpha/32\pi)(\textstyle\sum_j c_{2j}c_{1j}m_{L_j}^2/M_W^2)^2, \tag{12a}$$

$$R_{3e}\simeq 0.006R_{e\gamma},\quad R_{eN}\simeq 0.05R_{e\gamma}. \tag{12b}$$

However, (12b) would be modified in a complete theory. For example, Treiman, Wilczek, and Zee[12] have observed that for the $SU(2)\otimes U(1)$ model illustrated as B in Table I, the box diagrams and induced Z exchange diagram dominate over the photonic contribution to $\mu\to 3e$ (these can be obtained from Fig. 1 by replacing quarks with leptons). These additional contributions enhance R_{3e} (see Table I). Motivated by their observation, we have looked for a similar enhancement of $\mu^-+N\to e^-+N$ and therefore evaluated all diagrams in Fig. 1.

To apply the results of our computations, we adopt the following procedure: All amplitudes from the diagrams in Fig. 1 are added *coherently* and thereby yield an effective interaction Lagrangian of the form

$$\mathcal{L}_{\rm eff}=\bar{e}\gamma_\lambda\gamma_\pm\mu(A\bar{p}\gamma^\lambda p+B\bar{n}\gamma^\lambda n), \tag{13}$$

where p and n are *quark* fields and only the hadronic vector current is retained since it dominates the coherent process. When $\mathcal{L}_{\rm eff}$ is sandwiched between identical initial and final nuclear states, it acts as a counting operator for the number of proton and neutron *quarks* in the nucleus. This causes the box diagrams in Fig. 1 to dominate completely this process (if they do not cancel) and enhances the coherence effect much more than previously suspected.

The actual hadronic matrix elements are evaluated with use of

$$\langle N|\bar{p}\gamma_\lambda p|N\rangle\simeq(Z+A)\delta_{\lambda 0}F_{NN}(q^2)/(2\pi)^3, \tag{14a}$$

$$\langle N|\bar{n}\gamma_\lambda n|N\rangle\simeq(2A-Z)\delta_{\lambda 0}F_{NN}(q^2)/(2\pi)^3, \tag{14b}$$

where $F_{NN}(q^2)$ is the electromagnetic form factor defined in Ref. 4 by

$$\langle N|J_0^{\rm el}|N\rangle=ZF_{NN}(q^2)/(2\pi)^3. \tag{14c}$$

The meaning of (14c) along with a detailed analysis was given by Weinberg and Feinberg[4] and (14a) and (14b) represent our assumption that the same analysis holds for quark currents. Next, by comparing the amplitude obtained in this way with that which comes from photon exchange alone, we express our final results in terms of "effective" form factors which when substituted into (8) yield R_{eN}. We found[13]

$$f_{M1}=\mp f_{E1}=\frac{G_F m_\mu^2}{32\sqrt{2}\pi^2}\sum c_{2j}c_{1j}\left(\frac{m_{L_j}^2}{M_W^2}\right), \tag{15a}$$

$$f_{E0}=\mp f_{M0}=\frac{G_F m_\mu^2}{32\sqrt{2}\pi^2}\sum_j c_{2j}c_{1j}\left(\frac{m_{L_j}^2}{M_W^2}\right)\left[-4+Y_S\left(4+\frac{\Delta}{\sin^2\theta_W}\right)\left(3-\ln\frac{M_W^2}{m_{L_j}^2}\right)-\frac{1}{\sin^2\theta_W}\begin{pmatrix}7\Delta+9\\-2\Delta-9\end{pmatrix}\left(1-\ln\frac{M_W^2}{m_{L_j}^2}\right)\right], \tag{15b}$$

where $\Delta\equiv A/Z-2$, $Y_S\equiv$ weak U(1) hypercharge of S^+, θ_W is the Weinberg angle, and the upper terms correspond to a right-handed theory, the lower to a left-handed theory. Our analysis has not taken the strong interactions into account; however, the leading contributions to (15b) [$\ln(M_W^2/m_{L_j}^2)$ terms] should be basically unchanged even in a more rigorous derivation as long as m_{L_j} is at least a few GeV.

Finally, it has been pointed out by Bjorken, Lane, and Weinberg[14] that model A of our table can be easily modified to accomodate ν_L mixing with N_L in a natural manner (thereby becoming model B in Table I). This introduces left-right contributions to the $\mu \to e+\gamma$ transition rate, which have the net effect[14,15] of increasing $R_{e\gamma}$ [Eq. (12a)] by a factor of 25. On the other hand, the effect on R_{eN} and R_{3e} is much smaller; therefore the ratios $R_{eN}/R_{e\gamma}$ are decreased considerably in going from model A to B. [For model B, the right-hand side of (15a) should be multiplied by -5.]

To illustrate our results, especially for the $Q'=0$ case, we have exhibited in Table I estimates of $R_{eN}/R_{e\gamma}$ and $R_{3e}/R_{e\gamma}$ in a variety of models.[16] The first ratio was obtained using (8), (9), (15), and the formulas for $R_{e\gamma}$ that we have given. We should re-emphasize that a significant part of the large estimates for $R_{eN}/R_{e\gamma}$ in the $Q'=0$ models comes about because of a *novel* feature in this process, the domination of the box diagrams due to the coherence of quarks.[17]

For model A, model C, and the $Q'\neq 0$ case, our results indicate that because of the bound in (3), these models would require larger lepton masses than we have assumed in order to accomodate a $R_{e\gamma}$ of order 10^{-9}. On the other hand, if $\mu \to e+\gamma$ events are not observed, then $\mu^- + N \to e^- + N$ assumes a different role in these models; it becomes the best bet for detecting muon-number nonconservation. For model B, the estimate of $R_{eN}/R_{e\gamma}$ is perhaps not as dramatic; however, it is large enough to demonstrate the unquestionable worth of the search for $\mu^- + N \to e^- + N$.

In summary, a glance at Table I indicates that, for the large class of models that we have considered, R_{eN} is a very sensitive test of muon-number nonconservation. Since we believe that present-day meson facilities are easily capable of measuring R_{eN} to the level of 10^{-10}, we therefore strongly urge a careful search for this exotic reaction.

*Work supported in part by the U. S. Energy Research and Development Administration under Contract Grant No. EY-76-C-02-2232B.*000.

[1]S. Parker, H. Anderson, and C. Rey, Phys. Rev. 133, B768 (1964).

[2]S. M. Korenchenko *et al.*, Zh. Eksp. Teor. Fiz. 70, 3 (1976) [Sov. Phys. JETP 43, 1 (1976)].

[3]D. A. Bryman, M. Blecher, K. Gotow, and R. J. Powers, Phys. Rev. Lett. 28, 1469 (1972).

[4]S. Weinberg and G. Feinberg, Phys. Rev. Lett. 3, 111, 244(E) (1959).

[5]Searches for these rare reactions are currently under way at several meson facilities.

[6]For a review of the experimental and pre–gauge-theory theoretical status of these rare processes, see S. Frankel, in *Muon Physics*, edited by C. S. Wu and V. W. Hughes (Academic, New York, 1975), Vol. II, p. 83, and references therein. See also P. Rosen, Nuovo Cimento 15, 7 (1960).

[7]W. J. Marciano and A. I. Sanda, Phys. Lett. 67B, 303 (1977).

[8]The c_{ij} in (4) are left arbitrary; however, they would be fixed by the choice of a particular model.

[9]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[10]F. Wilczek and A. Zee, Phys. Rev. Lett. 38, 531 (1977).

[11]For an example of this kind of model, see T. P. Cheng, and L.-F. Li, Phys. Rev. Lett. 38, 381 (1977); and S. M. Bilenky, S. T. Petcov, and B. Pontecorvo, Phys. Lett. 67B, 309 (1977). This is model A in Table I.

[12]S. B. Treiman, F. Wilczek, and A. Zee, to be published.

[13]Most of the ingredients of this calculation can be found in M. K. Gaillard, B. W. Lee, and R. Shrock, Phys. Rev. D 13, 2674 (1976). Our calculation assumes an SU(2) doublet structure for the hadronic sector; however, the dependence on Y_S allows for arbitrary lepton representations.

[14]J. Bjorken, K. Lane, and S. Weinberg, unpublished; see also Ref. 12 and T. P. Cheng and L.-F. Li, University of Missouri at St. Louis Report No. UMSL-77-2, 1977 (to be published).

[15]See Cheng and Li, Ref. 14.

[16]Models A, B, and C in Table I all ha e $Y_S=1$. Our estimates for R_{3e} include logarithmic as well as nonlogarithmic contributions.

[17]We have greatly benefitted from discussions with Professor M. A. B. Bég on this point.

[18]After completing this work, we received a preprint in which this model is analyzed in detail; it has now appeared in print: B. W. Lee, S. Pakvasa, R. E. Shrock, and H. Sugawara, Phys. Rev. Lett. 38, 937, 1230(E) (1977).