Mass and Width of the A_1†

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We determine the mass and width of the A_1 meson using a unitary and analytic isobar parametrization.

We have analyzed the $I = 1$, $J^P = 1^+$ and 0^- 3π systems using a properly unitarized isobar model to describe the diffractive process $\pi^- p \to (3\pi)p$ and $\pi^- p \to (K^*\bar{K})p$. This model includes consistently Deck background and resonance (A_1) production plus all possible rescattering of the three pions in the final state. $K^*\bar{K}$ intermediate states are included *perturbatively*; the possible importance of coupling to this channel has recently been emphasized by Berger and Basdevant.[1] The four production diagrams being considered are shown in Fig. 1. Rescattering, which occurs through the amplitude X, affects not only the Deck background [Fig. 1(b)] but also the decay vertices of the A_1 and the A_1 propagator [which appear in Figs. 1(c) and 1(d)] as shown in Fig. 2. The parametrized solutions of the above model are then fitted to the 3π results of Ascoli *et al.*[2] and those of Antipov *et al.*[3] (for more reliable phase information) and to the $K^*\bar{K}$ results of Otter *et al.*[4] We find that an $I = 1$, $J^P = 1^+$ 3π resonance at ~1450 MeV with width ~350 MeV is required to describe the above data.

(1) Three-body problem.—We decompose the 3π interaction into long- and short-range components. The long-range mechanism is taken as one-pion exchange [Fig. 2(a)]; the short-range mechanism is introduced only in the 1^+ channel as a direct coupling at a primitive vertex of $\rho\pi$, $\epsilon\pi$, and $K^*\bar{K}$ ($\bar{K}^*K$) states to a heavy particle, which we call the *bare* A_1 [Fig. 2(b)] and which presumably summarizes quark interactions, etc. The full amplitude is obtained by first summing all pion-exchange diagrams into the amplitude X using relativistic three-body integral equations [Fig. 2(c)],[3,5] and then including all possible insertions of the *bare* A_1 in a manner first suggested by Bronzan.[6] The resulting renormalized vertex functions and physical propagator (which appeared in Fig. 1) are shown schematically in Figs. 2(d) and 2(e). If a *physical* A_1 exists, it will appear as a zero of the propagator function. We do not prejudge its existence in the data. For example, the fit could easily place the physical A_1 at energies far above the data with a large width, and it would then be interpreted as a smooth background. Unknown parameters that enter our theory are the coupling constants $g_{A_1\rho\pi}$, $g_{A_1\epsilon\pi}$, and $g_{A_1K^*\bar{K}}$ and the *bare* A_1 mass $m_A^{(0)}$. In addition, smooth form factors required for con-

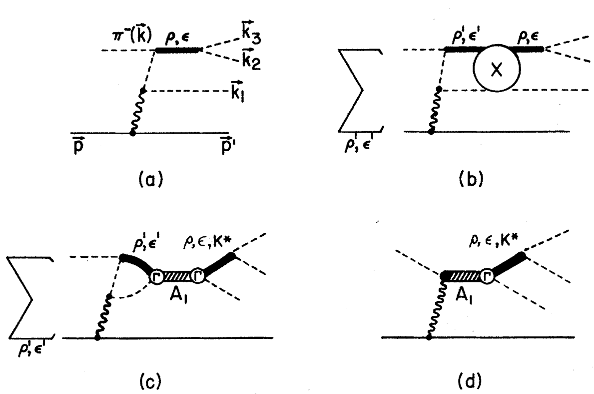

FIG. 1. (a) Deck diagram; (b) Deck plus rescattering through one-pion exchange (OPE); (c) Deck plus A_1 resonance rescattering; (d) direct resonance production.

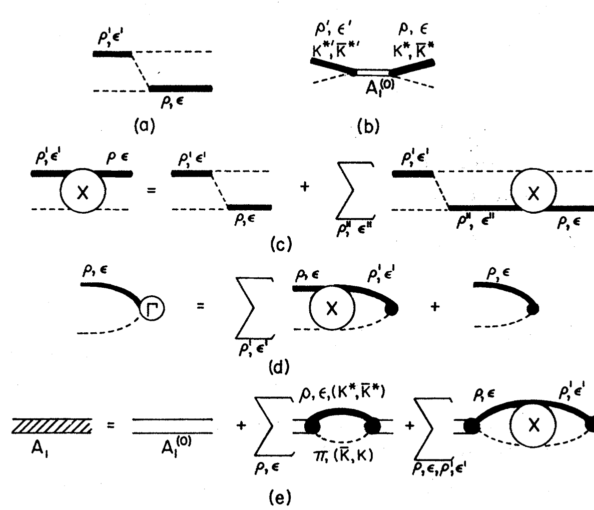

FIG. 2. (a) Long-range mechanism (OPE); (b) short-range mechanism; (c) unitary sum of OPE diagrams; (d) vertex function; (e) A_1 propagator.

vergence are introduced at the primitive vertices with momentum cutoffs β^2 ranging from 1 to 4 $(\mathrm{GeV}/c)^2$, physical results being independent of these cutoff parameters. We find that the long-range forces that sum to the amplitude X are much too weak to produce a resonance,[7] and a physical A_1 must be the result of the short-range forces. On the other hand, rescattering through X can produce a considerable change in some of the renormalized vertex functions.

(2) Deck background.—From Fig. 1(a) we write schematically,

$$(T_\rho^{\mathrm{D}} \text{ or } T_\epsilon^{\mathrm{D}}) \propto i\boldsymbol{B}(V_\lambda \text{ or } 1), \quad (1)$$

$$B = s_{\pi N}(\mu^2 - t_R)^{-1}, \quad (2)$$

$$V_\lambda = k_\lambda + \alpha(\vec{k}^2, \vec{k}_1^2)k_{1\lambda}, \quad (3)$$

where $t_R = (k - k_1)^2$ and T_ρ^{D} and T_ϵ^{D} are the Deck amplitudes for ρ and ϵ production with the factors describing propagation and decay of the ρ or ϵ divided out. In (2) $s_{\pi N}$ is the Pomeron propagator and V_λ a (in general, off-shell) polarization vector defined by Aaron, Amado, and Young[8]; the momenta in Eq. (3) are defined in Fig. 1. If Eq. (1) described pure pion exchange, the Deck background could be combined neatly with our three-body amplitudes. Unfortunately, for $s_{\pi N} = (p' + k_1)^2$ large and $t = (p' - p)^2$ near its minimum, the very kinematical region under consideration, one may write[9]

$$s_{\pi N} \approx M(\mu^2 - t_R)/(s_A - \mu^2), \quad (4)$$

with $s_A = (k_1 + k_2 + k_3)^2$, and thus the pion pole in (1) is largely canceled. Rather than attempt to describe this complicated situation theoretically, we choose to write

$$B = (B_0 + 3B_1 z)/4\pi, \quad (5)$$

where B_0 and B_1 are parametrized functions of $\vec{k}^2$ and $\vec{k}_1^2$ and where $z = \hat{k}\cdot\hat{k}_1$. In terms of the above quantities

$$\begin{aligned} T^{\mathrm{D}}(0^-) &= B_0 \text{ for } \epsilon\pi, \\ T^{\mathrm{D}}(1^+) &= B_1 \text{ for } \epsilon\pi, \\ T^{\mathrm{D}}(0^-) &= \boldsymbol{k}B_1 + \boldsymbol{k}_1(\alpha B)_0 \text{ for } \rho\pi, \\ T^{\mathrm{D}}(1^+) &= \boldsymbol{k}B_0 + \boldsymbol{k}_1(\alpha B)_1 \text{ for } \rho\pi, \end{aligned} \quad (6)$$

where $()_j$ refers to the jth-partial-wave projection and α is defined in Eq. (3).

Fitting procedure.—Under the assumption of diffractive production via Pomeron exchange, the full $(3\pi)p$ production amplitude is obtained by summing the diagrams of Fig. 1. Parameters entering the calculation are $m_A^{(0)}$, $g_{A_1\rho\pi}$, $g_{A_1\epsilon\pi}$, and $g_{A_1K^*\bar{K}}$ mentioned earlier, $g_{\mathrm{P}A_1\pi}$ describing coupling of the Pomeron to the πA_1 system, and six additional ones which give the $|\vec{k}|$ dependence of B_0 and B_1. In addition, we allow ourselves certain freedom to adjust the subenergy dependence of the latter quantities. We than fit our theoretical amplitudes to the data of Refs. 2–4 which include 0^- and 1^+ "ρ" and "ϵ" cross sections, 1^+ $K^*\bar{K}$ cross sections, and relative phases of the isobar amplitudes averaged over phase space at c.m. energies ranging from 950 to 1700 MeV. Fitting the model to the data is an enormous technical task; for example, calculation of the A_1 propagator involves a two-dimensional integral over the fully off-shell amplitude X which itself is obtained by solving a complicated integral equation. Fortunately none of the fitting parameters enter the integral equation because its kernel depends only on the one-pion-exchange mechanism. Our best fit to the cross section and phases is shown in Figs. 3 and 4 (solution I), where we obtain $\chi^2 = 64$ for 37 degrees of freedom. Except for the 1^+ ϵ, the partial-wave amplitudes [Eq. (6)] in this solution have some subenergy dependence and the corresponding amplitudes of Ascoli (defined as subenergy independent) should

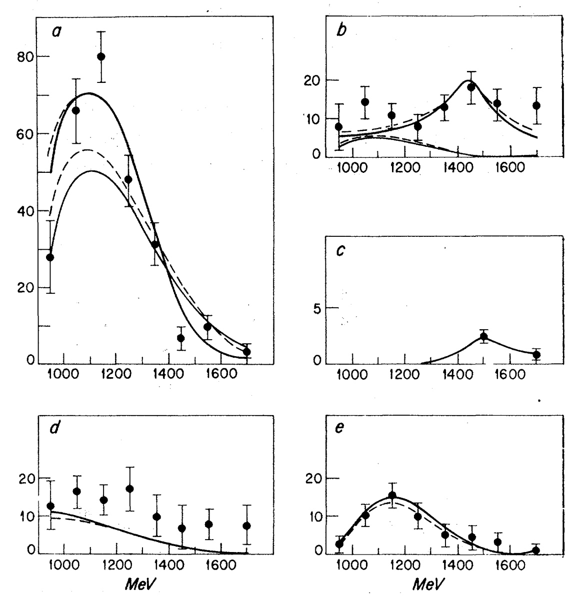

FIG. 3. Solution I (solid line) and Solution II (dashed line): (a) 1^+ $\rho\pi$ cross section in arbitrary units vs 3π c.m. energy. Lower curves are corresponding Deck cross sections. (b) 1^+ $\epsilon\pi$ cross sections (with Deck). (c) 1^+ $K^*\bar{K}$ cross section. (d) 0^- $\epsilon\pi$ cross section. (e) 0^- $\rho\pi$ cross section.

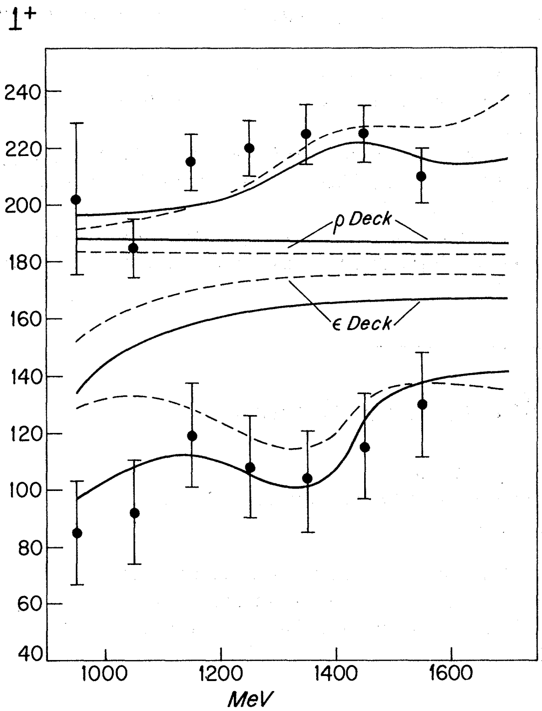

FIG. 4. 1^+ $\rho\pi$ and 1^+ $\epsilon\pi$ phases relative to 0^- $\epsilon\pi$; phases of Deck plus rescattering are also shown. Solution I (solid line) and Solution II (dashed line) are shown.

therefore be interpreted as average ones. Eventually the question of subenergy dependence must be answered by applying our model to the raw data. We tried to obtain solutions more consistent with the standard isobar model by choosing B_0 and B_1 to be relatively constant as a function of subenergy. In the latter case the 1^+ ρ amplitude is also reasonably constant, but the 0^- ρ amplitude has a rapid variation. For this solution (solution II), we obtain $\chi^2 = 79$ for 37 degrees of freedom.

Examination of the A_1 propagator, shown schematically in Fig. 2(e) shows the presence of a *well-behaved Breit-Wigner resonance.* For the mass M_{A_1}, width Γ_{A_1}, and partial widths $\Gamma_{\rho\pi}$, $\Gamma_{\epsilon\pi}$, and $\Gamma_{K^*\bar{K}}$ in units of MeV, solution I (II) yields $M_{A_1} = 1460$ (1450), $\Gamma_{A_1} = 325$ (380), $\Gamma_{\rho\pi} = 85$ (130), $\Gamma_{\epsilon\pi} = 145$ (150), and $\Gamma_{K^*\bar{K}} = 47$ (50). We find the presence of a 1^+ resonance with the above mass and width an essential ingredient in fitting the data, *particularly the phase information.* It is the very phase behavior interpreted by previous authors as showing that there is no A_1, which, we find, demands the A_1. Another interesting feature of the fit is the near equality of the coupling of the Pomeron to the A_1 and the rescattering diagram, Fig. 1(c).[10] Finally, a parametrized Deck amplitude is obtained from the fit which may be used in other contexts.[11] The pure Deck cross sections for solutions I and II are shown in Fig. 3. Our liberally parametrized Deck amplitude can give a good fit to the cross sections, *but even with rescattering it cannot fit the phases* (see Fig. 4).

In summary, we feel that we have performed a calculation which compels one to accept the reality of the A_1. Because the A_1 is seen through its interference with a much larger Deck background, the full complexities of a three-body theory incorporating unitarity are necessary to extract resonance parameters. For example, the inclusion of rescattering, although a relatively small effect, is necessary because its neglect results in a poorer fit yielding a much wider A_1 ($M_{A_1} \approx 1450$, $\Gamma \approx 550$). Also, at lower energies, it is the rescattering which helps keep the ρ and ϵ phases apart as shown in Fig. 4. The Reggeized Deck calculations of Ascoli *et al.*[12] and (more recently) similar work in the $K\pi\pi$ system by Berger[13] give important insight into the physics, but any resonance information is presumably being obtained through duality and cannot be of a detailed nature. Dolen-Horn-Schmid[14] duality implies that Reggeized exchanges might yield an average description of direct channel resonance, but that a more complete picture requires a dynamical theory (such as ours?) in which these resonances appear as complex poles in the scattering amplitudes. It is interesting to note that in Ref. 12 it is the Regge signature factor which causes the narrowing of the Deck cross section and the required behavior of the 1^+ ρ and 1^+ ϵ amplitudes, while in our case it is the presence of the A_1 resonance.

We have recently been informed of a previous theoretical work by D. Morgan[15] which supports a 1450-MeV A_1. Fitting only the ρ cross section with a non-Reggeized (π-exchange) Deck background plus Figs. 1(c) and 1(d), Morgan predicts a 1450-MeV A_1 with a 200-MeV width.

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Reaction μ^- + Nucleus → e^- + Nucleus in Gauge Theories*

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The process μ^- + nucleus → e^- + nucleus is examined within the framework of gauge theories. We find that the rate for this reaction is much larger than what one might naively expect for a large class of models which allow the decay $\mu \to e + \gamma$. Further experimental search for this rare process is strongly urged.

The search for separate muon- and electron-number nonconservation has led to rather severe bounds on the following reactions: (1) $\mu^+ \to e^+ + \gamma$,[1] (2) $\mu^+ \to e^+ + e^- + e^+$,[2] and (3) $\mu^- + N \to e^- + N$.[3,4] Presently, the published bounds on these processes are

$$R_{e\gamma} \equiv \Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\bar\nu\nu) \leq 2.2\times 10^{-8}, \tag{1}$$

$$R_{3e} \equiv \Gamma(\mu \to 3e)/\Gamma(\mu \to e\bar\nu\nu) \leq 1.9\times 10^{-9}, \tag{2}$$

$$R_{eN} \equiv \frac{\omega(\mu^- + N \to e^- + N)}{\omega(\mu^- + N_Z \to \nu_\mu + N_{Z-1})} \leq 1.6\times 10^{-8}. \tag{3}$$

Interest in these exotic processes has recently been aroused by experimental[5] and theoretical developments. On the theoretical side, the advent of gauge theories now allows finite computations of the ratios in Eqs. (1)–(3)—a feature missing from earlier calculations which depended on ambiguous cutoffs.

In this Letter we will discuss these processes[6] in the context of a class of gauge theories which allow their occurrence through one-loop diagrams involving intermediate leptons (see, for example, Fig. 1). Our primary purpose is to point out that the models that we consider predict a much larger ratio for $R_{eN}/R_{e\gamma}$ than the naive estimate $[O(\alpha)]$ that one might have made. For comparison and completeness, we discuss all three ratios, $R_{e\gamma}$, R_{3e}, and R_{eN} in these models. Our theme will be that the reaction $\mu^- + N \to e^- + N$ is a sensitive test of muon-number nonconservation, which should be more precisely measured.

Following the notation of Marciano and Sanda,[7] we assume that these exotic reactions are induced by lepton flavor-changing currents of the form

$$\mathcal{L}_I = -g\sum_j\{W^\alpha(\bar\mu\gamma_\alpha c_{1j}\gamma_\pm L_j + \bar e\gamma_\alpha c_{2j}\gamma_\pm L_j) + (1/M_W)S[\bar\mu c_{1j}(m_\mu\gamma_\pm - m_{L_j}\gamma_\mp) + \bar e c_{2j}(m_e\gamma_\pm - m_{L_j}\gamma_\mp)]L_j\} + \text{H.c.}, \tag{4}$$

where $\gamma_\pm \equiv \frac{1}{2}(1\pm\gamma_5)$; the leptons L_j and the intermediate vector boson W carry electric charges $Q'e$ and $(1-Q')e$, respectively; the couplings of the unphysical Higgs boson S have been added to insure gauge invariance; and g is the usual weak coupling $(g^2/2\sqrt{2}M_W^2 \equiv G_F)$.[8] In (4) we have limited ourselves to either right- or left-handed currents but not a mixture of both. (However, some results for theories with right-left mixing will be given.) We consider only theories which satisfy the leptonic Glashow-