compares results obtained from ordinary low- and high-temperature series expansions.[8]

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CP Conservation in the Presence of Pseudoparticles*

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We give an explanation of the *CP* conservation of strong interactions which includes the effects of pseudoparticles. We find it is a natural result for any theory where at least one flavor of fermion acquires its mass through a Yukawa coupling to a scalar field which has nonvanishing vacuum expectation value.

It is experimentally obvious that we live in a world where P and CP are good symmetries at the level of strong interactions. In the context of quantum chromodynamics the strong interactions are believed to be due to non-Abelian vector gluons coupled to massive quarks. In such a theory, when the effects of gluon configurations of nonzero pseudoparticle number are included, CP invariance requires a very special choice of parameters. We will show, however, that CP invariance of the strong interactions is, in fact, a natural consequence, provided at least one flavor of quark acquires its mass from a Yukawa coupling to a scalar field which has a nonzero vacuum expectation value, and the Lagrangian originally possesses a U(1) invariance involving all Yukawa couplings.

The physical importance of gauge field configurations with nontrivial topology has been stressed by 't Hooft.[1] He has reminded us that the physics of such theories involves a parameter θ which does not appear in the original Lagrangian.[2] This parameter defines the choice of vacuum[3] among an infinity of possible distinct and generally inequivalent vacua. Each θ represents a possible true vacuum and there are in general an infinity of distinct theories arising from any given Lagrangian.

If all fermions which couple to the non-Abelian gauge fields are massless then the various θ choices give equivalent theories.[1,3] This is most clearly seen by remarking that a change in the effective value of θ can be induced by making an $\exp[i\gamma_5\eta]$ rotation of the fermion fields. We define the effective Euclidean action in the qth sector to be

$$S_{\text{eff}}{}^q = \int d^4x\mathcal{L} + i\theta q, \tag{1}$$

where

$$q = (g^2/32\pi^2)\int d^4x\, F_{\mu\nu}{}^a \tilde{F}^{a\mu\nu}. \tag{2}$$

The rotation of a fermion field by $\exp[i\gamma_5\eta]$ induces a change in the effective action given by

$$\delta S_{\text{eff}}{}^q = -i\int(\partial^\mu j_\mu{}^5)\eta = -2iq\eta \tag{3}$$

since

$$\partial^\mu j_\mu{}^5 = (g^2/16\pi^2) F_{\mu\nu}{}^a \tilde{F}_\mu{}^{\mu\nu a}. \tag{4}$$

Thus in such a theory the net effect of such a rotation is

$$\theta \to \theta' = \theta - 2\eta. \tag{5}$$

If, however, all fermions are massive such a rotation will also change the fermion mass term in $\mathcal{L}$. Hence one can define inequivalent theories which have the same mass term and various choices of θ. There are of course also classes of equivalent theories related by $\exp[i\gamma_5\eta]$ transformations. Only one such class of theories, those in which $\theta \to 0$ when all fermion masses have been made real by a suitable $\mathrm{ex}[i\gamma_5\eta]$ rotation, yield a CP- and P-invariant theory of the strong interactions.[1,3] We will present a mechanism which explains why we live in such a world.

We consider theories in which at least one fermion flavor acquires its mass from a Yukawa coupling to a color-singlet scalar field which has a nonzero vacuum expectation value. The effective potential for the scalar fields is θ dependent and does not have the same symmetry as the scalar polynomial $U(\varphi)$ appearing in the Lagrangian. This reduction in symmetry has been used by 't Hooft[1] in explaining away the U(1) problem. Here it means that the minimum of the potential corresponds to a particular choice of phases for the various scalar vacuum expectation values. These phases appear in the fermion mass terms and we find they are always such that when all fermion masses are made real by $\exp[i\gamma_5\eta]$ rotations of the fermion fields the resulting θ is zero.

We will illustrate this result first for a toy model of the strong interactions in which there is only a single fermion flavor and a single color-singlet complex scalar field. Introducing additional fermion flavors and scalar multiplets will not change our result. Finally we will discuss the effect of adding weak interactions to the theory.

Let us begin by examining our toy model. The Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu}{}^{a} F^{\mu\nu a} + i\bar{\psi} D_\mu \gamma^\mu \psi + \bar{\psi}[G\varphi \tfrac{1}{2}(1+\gamma_5) + G^*\varphi^* \tfrac{1}{2}(1-\gamma_5)]\psi - |\partial_\mu \varphi|^2 - \mu^2|\varphi|^2 - h|\varphi|^4; \quad \mu^2 < 0. \tag{6}$$

We are here thinking of, for example, a color SU(3) gauge theory where the fermion belongs to the fundamental triplet representation. We can write the generating functional for Green's functions in the θ vacuum as

$$Z_\theta(J,J^*) = \textstyle\sum_q e^{i\theta q} \int (dA_\mu)_q \int d\psi \int d\bar{\psi} \int d\varphi \int \varphi^* \exp[\int d^4x\, \mathcal{L}(\varphi,\psi,A) + J^*\varphi^*]. \tag{7}$$

The scalar vacuum expectation value is then defined by

$$\left.\frac{1}{Z_\theta}\frac{\delta Z_\theta}{\delta J}\right|_{J=J^*=0} = \langle\varphi\rangle = \lambda e^{i\beta}, \tag{8}$$

where λ and β are real constants to be determined.

To proceed further we need to know some properties of the fermion eigenmodes in the $q=n$ sector. The essential points for our argument are the following: (i) The number of orthogonal eigenmodes with eigenvalue zero is n. (This is true when the fermions are in the fundamental representation of the gauge group.[4]) (ii) The zero eigenmodes for $n>0$ are purely right handed.[1] (iii) The zero eigenmodes for $q=-n$ are of opposite chirality to those for $q=+n$ but otherwise identical. (iv) The nonzero eigenmodes are the same for $q=n$ and $q=-n$,[5] and are admixtures of both left- and right-handed parts.

These four facts allow us to schematically express $Z_\theta(J,J^*)$ as it appears after integrating out both the vector and the fermion fields. We introduce new scalar variables ρ and σ by the definition

$$\varphi = e^{i\beta}(\lambda + \rho + i\sigma). \tag{9}$$

Then we find

$$Z(J,J^*) = \int d\rho \int d\sigma \{A_0(\rho,\sigma^2) + \textstyle\sum_n A_n(\varphi\varphi^*)[Ge^{i\theta}\varphi]^n + A_n^*(\varphi\varphi^*)[G^*e^{-i\theta}\varphi^*]^n\} \times \exp[Je^{i\beta}(\lambda+\rho+i\sigma) + J^*e^{-i\beta}(\lambda+\rho-i\sigma)]. \tag{10}$$

In this equation the A_n are polynomials of the form

$$A_n(\varphi\varphi^*) = \sum_m \prod_{i=1}^{m} [\int dx_i \int dy_i\, \varphi(x_i)\varphi^*(y_i)]\, c_m{}^n(x_i,y_i). \tag{11}$$

The $c_m{}^n$ are real functions which depend on $|G|^2$, μ, h, and λ but are β independent. The terms proportional to G^n arise from the fermion zero eigenmodes in the $q=n$ sector, while those with $(G^*)^n$ are from the $q=-n$ sector. Now we define

$$\alpha = \arg[Ge^{i(\theta+\beta)}]$$

and rewrite (10) as

$$Z(J,J^*)=\int d\rho\int d\sigma\{A_0(\rho,\sigma^2)+\sum_n[F_n(\rho,\sigma^2)\cos n\alpha-\sigma G_n(\rho,\sigma^2)\sin n\alpha]\}\times\exp[Je^{i\beta}(\lambda+\rho+i\sigma)+J^*e^{-i\beta}(\lambda+\rho-i\sigma)],\qquad(12)$$

where F_n and σG_n are the real and imaginary parts of $A_n|G|^n(\lambda+\rho+i\sigma)^n$, respectively. We note that F_n and G_n are even functions of σ and that $\int d\sigma\,\sigma^{2n+1}$ vanishes identically. Then by definition λ and β must be chosen so that

$$\langle\rho\rangle=0=\int d\rho\int d\sigma\,\rho[A_0+\sum_n F_n\cos n\alpha],\qquad \langle\sigma\rangle=0=\int d\rho\int d\sigma\,\sigma^2\sum_n G_n\sin n\alpha.\qquad(13)$$

The first of these equations is satisfied for arbitrary α by appropriately choosing $\lambda=\lambda(\alpha)$. The second equation can then in general only be satisfied for $\alpha=0,\ \pi$. These values are both stationary points of the potential. To find which is the true minimum we must examine $V(\varphi)$. This we can only do to leading order in G and h, for which we find

$$V(\varphi)=\mu^2\varphi^*\varphi+h(\varphi^*\varphi)^2-K|\varphi|\cos\alpha,$$

where K is a real positive constant. Thus if G and h are sufficiently small the minimum occurs at

$$\alpha=0.\qquad(14)$$

In terms of the shifted scalar variables the fermions acquire a mass term

$$\lambda\bar\psi[Ge^{i\beta}\tfrac{1}{2}(1+\gamma_5)+G^*e^{-i\beta}\tfrac{1}{2}(1-\gamma_5)]\psi.\qquad(15)$$

This mass can be made real by rotating the fermion fields by $\exp[i\gamma_5\theta/2]$. Such a rotation gives

$$\theta\to\theta'=\theta-\theta=0\qquad(16)$$

(Q.E.D.).

The generalization to multiple fermion flavors and numerous scalars follows the same pattern. The pseudoparticles now have a zero mode for each fermion flavor. Provided the original scalar Lagrangian possesses a U(1) invariance, which is violated by the additional terms arising from the fermion determinant. there will always exist a range of parameters for which the CP-conserving phase choices correspond to the true minimum of the scalar potential. The requirement $\alpha=0$ generalizes in these cases to

$$\arg\left\{\prod_{i=1}^{m}(G_i\exp[i\beta_{j_i}])\exp[i\theta]\right\}=0,\qquad(17)$$

where G_i is the Yukawa coupling of the ith fermion flavor to the j_ith neutral scalar. Just as in the single-flavor case the resulting fermion masses can all be made real by a rotation the ith fermion field by $\gamma_5\eta_i$ where, from (17),

$$\sum_i\eta_i=\theta/2.\qquad(18)$$

The resulting theory again has $\theta_{\rm eff}=0$ when all fermion masses are real. This point will be discussed, with examples, in a subsequent paper.[6]

If some fermion masses appear in the original Lagrangian the expression (17) will be modified by $e^{i\theta}\to e^{i\theta}\Pi_k m_k$. When all masses including the m_k are made real this gives $\theta_{\rm eff}=0$.

The introduction of electromagnetic and weak interactions in the form of gauge fields coupling to fermion flavor can be treated as a perturbative correction to this theory. P and CP noninvariance can be introduced at the level of the weak interactions (or CP noninvariance may occur in the scalar self-couplings). There are two features of such a theory which could invalidate our earlier arguments. The first is the fact that the there are also $q_{\rm weak}\neq0$ sectors in the integration over weak-field configurations. 't Hooft has argued[1] that the contribution of these sectors is extremely small, being approximately of order $\exp[-(2\pi/\alpha)q]$. The remaining possible problem for standard phenomenology is the occurrence of parity-nonconserving order-α corrections to the fermion masses.[7] These do not alter our conclusions. In the presence of such terms it is important to recognize that the quantity G appearing in (16) must be corrected to include all orders of weak interactions—it is the Yukawa coupling renormalized at zero scalar four-momentum. The argument then proceeds as before. The vacuum expectation value of the scalar field picks up a phase such that the effective θ will be zero when the fermion fields are redefined so that all renormalized fermion masses are real.

We have thus shown that the observed CP invariance of the strong interactions is a natural feature of a theory such as quantum chromodynamics provided only that at least one fermion flavor acquires its mass through a Yukawa coupling to a scalar field which has a nonvanishing vacuum expectation value. The physical mechanism that forces this result is the same asymmetry of the scalar potential in the θ vacuum which was used by 't Hooft to explain away the U(1) problem.

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Study of Double-Scattering Effects in $\bar{p}d$ Annihilation Processes*

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The characteristics of the surviving nucleon in $\bar{p}d$ annihilation are analyzed in terms of a double-scattering model with only two free parameters. Evidence is presented that the surviving nucleons not only may be very different dynamically from the true-spectator nucleon in the annihilation process, but also may have a different identity through charge exchange in the double-scattering process.

For experiments with a deuterium target, the phenomenon of excess in high-energy spectator nucleons has been known for some time. It has been observed in reactions ranging from simple deuteron breakup with no additional particles produced[1,2] to antiproton-deuteron annihilation processes.[3] The need for a better understanding of the source of these excesses is obvious. Due to the nonexistence of a free-neutron source, deuterons frequently serve as a source of quasi-free neutrons. A good understanding of the role of the specator nucleon is essential to our ability to extract information from a free-neutron target. Furthermore, since deuterons are the simplest of available nuclear targets, a good description of the role of the spectator nucleons in the scattering processes may shed light on the subject of particle-production cross-section dependency on the atomic weight of the nuclear target.

In this Letter, characteristics of the recoiled proton from $\bar{p}d$ annihilation processes are studied in terms of a simple double-scattering model. This model, constructed in the spirit of the Glauber model, is of great importance for studies involving a deuteron target. The presence of a spectator proton in the final state is often taken as a signature for a neutron target. It will be shown that the double-scattering effect can modify not only the expected characteristics but also the identity of the spectator nucleon.

Data for this analysis come from an exposure of the Brookhaven National Laboratory 31-in. deuterium bubble chamber by a separate $\bar{p}$ beam. A total of 150 000 triads were taken, with 64 000 triads at 1.31 GeV/c and the remaining roughly equally divided among 1.09, 1.19, and 1.43 GeV/c. In this work, events from the reactions

$$\bar{p}+d\rightarrow p+2\pi^-+\pi^++j\pi^0, \quad j\geq 0, \tag{1}$$

$$\bar{p}+d\rightarrow p+3\pi^-+2\pi^++j\pi^0, \quad j\geq 0, \tag{2}$$

will be analyzed.

Figures 1(a) and 1(b) show the spectator-proton momentum distribution for Reactions (1) and (2), respectively. A usual depletion of events is seen below 100 MeV/c. Spectator protons for those events are not energetic enough to leave a visible track in the bubble chamber; therefore these events appear in the odd-prong topologies and are not used in this work. On the high-momentum side, an obvious excess is seen near 300 MeV/c. This excess amounts to almost 80%