around T_D, which will be responsible for the tiny anomaly of ϵ_c.

The intervention of the extra short-range interaction between the phonon modes represented by the piezo-optic coupling may also be responsible for the unexplained high-temperature tail in the heat capacity data in $DyVO_4$ and $DyAsO_4$ around T_D,[14] and also for the discrepancy between the observed T_D values and the theoretical ones predicted by the molecular-field theory.

In conclusion, the dielectric anomaly observed at the Jahn-Teller transition temperature in $DyVO_4$ may be understood as a direct consequence of the onset of an antiferroelectric ordering[8] which is driven by the softening B_{1g} strain mode, i.e., as an optical-mode condensation without its softening. By following and extending the conventional terminology,[15] this phenomenon may suitably be called as the improper (or indirect) antiferroelectricity.

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[8]Throughout this Letter, the terms "staggered" and "antiferroelectricity" are referred to as the state in which the displacement alternates its direction from one sublattice to another *within a unit cell;* i.e., it is associated, contrary to the convention, with a Fourier component at zero wave vector, not with that at the zone boundary.

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COMMENTS

$p_\perp$ Dependence of Heavy-Particle Production

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I emphasize the close similiarity in the inclusive production of genuine heavy resonances compared to nonresonant hadron pairs. For small x and not too big $p_\perp$ a simple relation describes both with proper mass dependence of the $p_\perp$ slope. Data on J-production support this picture.

In a recent measurement by Aubert *et al.*[1] at Brookhaven National Laboratory all pair combinations of $\pi^\pm$, $K^\pm$, and $p^\pm$ were detected with a double-arm spectrometer.[2] The cross sections for the simultaneously measured nine neutral two-body final states $(\pi^+\pi^-, \pi^+K^-, \dots)$ show a band structure[1] when plotted against their invariant mass, m. For production at rest in the c.m. system, i.e., $x = 2p_\parallel^*/\sqrt{s} \simeq 0$ and $p_\perp \simeq 0$, cross sections in all channels decrease like $\exp(-5m)$ over the measured range of $1.5 \leq m \leq 5.5$ GeV/c^2. No simple relations such as the frequently[3] used $\exp(-a \cdot p_\perp)$ or $\exp(-b \cdot p_\perp^2)$ describe these data because a or b will depend on m. However, all nine channels give decreases with $p_\perp$. For the measurement[1] with $0 \le p_\perp \le 1.2$ GeV/c and $|x| < 0.1$, the following *Ansatz* was found to be appropriate:

$$E d^3\sigma/dp^3 = C\,e^{-5E} = C\,e^{-5m}e^{-5T} \quad (1)$$

with $T = (m^2+p^2)^{1/2} - m$. In this way the observed $\exp(-5m)$ is accounted for. Since x is small I ignore lingering effects from leading particles[4] and assume isotropic emission, such that

$$\langle p^2\rangle = \tfrac{3}{2}\langle p_\perp^2\rangle. \quad (2)$$

Then, from Eq. (1) I anticipate the $p_\perp$ behavior

$$f(p_\perp) = \exp\{-5[(m^2 + \tfrac{3}{2}p_\perp^2)^{1/2} - m]\}. \quad (3)$$

This expression was verified[1] to describe the nonresonant pairs produced in 28.5-GeV proton-nucleon collisions. I consider Eqs. (1) and (3) mainly as a phenomenological description of the copious pair data rather than a derivation. Formulas similar to (3) have been quoted[3] and more elaborate models[5] confirmed the features of (3) later.

Most remarkably, the measured $p_\perp$ behavior of genuine resonances follows this pattern of the nonresonant pairs too. In Fig. 1 data on resonance production[6] from 24–28.5-GeV incident protons are compared to the predictions of Eq. (3) at the relevant mass values. The Massachusetts Institute of Technology–Brookhaven National Laboratory (MIT-BNL) inclusive π^- data are taken at 90° c.m. and cover $0.7 < p_\perp < 1.6$ GeV/c. They show the well-known[3] $\exp(-6p_\perp)$ decrease in accordance with Eq. (3) for $m \simeq 0$.

However, at extremely low $p_\perp$ (< 0.4 GeV/c) even for the light π meson the deviation from this exponential gets visible from very recent CERN intersecting storage ring (ISR) data—in Fig. 1 the only ones taken at higher energies ($\sqrt{s} = 23$ GeV). The CERN measurements of inclusive ρ and ω production[6] agree well with Eq. (3), too.

A crucial test of Eq. (3) is the production of a very heavy particle. This is provided by the J data[6] from the MIT-BNL experiment. As seen in Fig. 1 their shape differs drastically from π^- production, yet is still in good agreement with the prediction of Eq. (3).

For the data in Fig. 1, the following normalizations C were used: 80 for CERN "ρ," 150 for ISR "π^-," 120 for MIT-BNL "π^-," and 2 mb GeV^{-2} c^3/ nucleon for the MIT-BNL "J" data. The π^- values at these vastly different energies allow at most for a very weak energy dependence[7] other than the expected relativistic rise.[6] The C value for "J" production in fact rises strongly with energy as seen in Fig. 2. However, if we assume the cross sections[8] after suppression by phase space to reach a plateau at energies of a few hun-

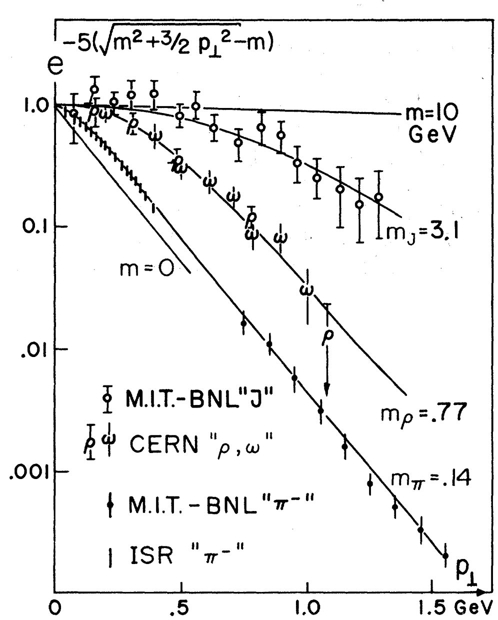

FIG. 1. The points are measured cross sections of resonance (J, ρ, ω, π^-) production from Ref. 6. The solid lines represent nonresonant hadron pairs via Eq. (3), evaluated at the mass value indicated.

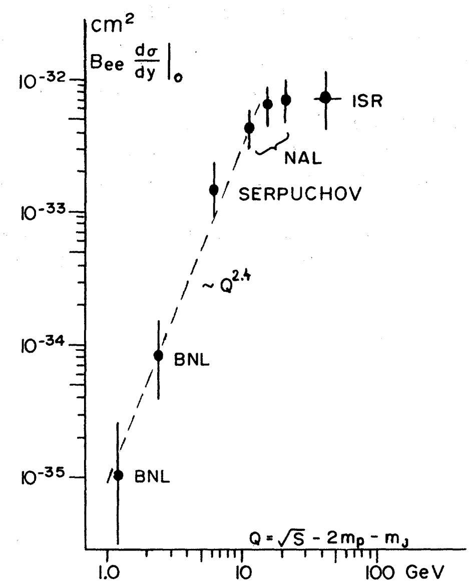

FIG. 2. Excitation curve of J production from proton-nucleon collisions. The errors are mostly systematic; for a discussion see Anderson *et al.*, Ref. 8. B_{ee} is the branching ratio $J \to ee/J \to$ all $\simeq 7\%$.

dred GeV,[9] the unobstructed C value will be much bigger and agree to order of magnitude with those for the other strongly interacting particles.

In Fig. 2, the cross sections[8] are plotted against the Q value of the reaction with the assumed final state ppJ. Considering the big errors, the observed rise with Q is not inconsistent with a three-body (Q^2) or four-body ($Q^{3.5}$) final-state phase-space behavior, if $m_4 \ll 1$ GeV/c^2. This together with the observation at Fermilab of isotropic emission of J's in the c.m. system[10] supports the assumptions in Eqs. (1) and (2).

From Eq. (3) I can easily calculate the average transverse momentum of the produced particles, with the result 350 MeV/c for pions, 490 MeV/c for ρ's, and 850 MeV/c for J's produced at 28.5 GeV. Although Eq. (3) may not hold at large $p_\perp$, this agrees well with present measurements.[3,6,10]

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[4]For $x \neq 0$ usually a superposition of all $p_\parallel$ with empirical distribution has been assumed and integrated over, leaving the $p_\perp$ falloff unchanged. See R. Hagedorn, CERN Report No. 71-12, 1971 (unpublished), p. 108.

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[7]Theoretical speculations based on scaling in $x_\perp = 2p_\perp/\sqrt{s}$ at high $p_\perp$ expect energy dependence. At the $p_\perp$ considered here, s scaling is not reached; see Ref. 3. Also C. Michael, Phys. Lett. 63B, 301 (1976), anticipates only a weak s depencence, if any.

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[10]A. J. S. Smith, in Proceedings of the Eighteenth International Conference in High Energy Physics, Tbilisi, U. S. S. R., 15–21 July 1976 (to be published). The data described with $\exp(-0.9p_\perp^2)$ are well in accordance with Eq. (3), too. No structure in the c.m. production angle was seen. The muon pair spectrum behaved like $\exp(-5.3m)$.