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Irreversible Thermodynamics of Black Holes

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Black holes are shown to obey the principles of irreversible thermodynamics in the form of a fluctuation-dissipation theorem for their zero-point quantum fluctuations. Moreover Hawking radiation is shown to be related to the macroscopic radiation of a nonstationary black hole in accordance with Onsager's principle.

Recent work on black holes, culminating in Hawking's[1] remarkable discovery of their quantum radiance, has shown that they obey the laws of equilibrium thermodynamics.[2] In this Letter we show that they conform also to the principles of *nonequilibrium* and *irreversible* thermodynamics, in the form of a fluctuation-dissipation theorem.[3] The dissipation is associated with the absorption of ordered energy by the black hole and its subsequent reradiation by the Hawking process. It has been shown[4] that Hawking radiation has the same stochastic properties as black-body radiation, and so is completely disordered. A black hole is thus a perfect dissipator.

To help understand this property of black holes, we apply the theory of dissipative processes.[5] This theory is based on the following ideas: (a) A dissipative system D possesses a large number of closely spaced energy levels lying near to the ground state. (b) In consequence, when this system is coupled to another system S, it exerts a force on it which fluctuates in time and is usually initially uncorrelated with the natural fluctuations of S. The cumulative effect of this fluctuating force is to dissipate the excess energy of S by distributing it among the many energy levels of D. (c) The effect of S on D, in linear approximation, is to produce a deviation from its equilibrium state which on average cannot be distinguished from a purely spontaneous fluctuation of D (Onsager's[6] principle). (d) The fluctuating force exerted by D represents a source of noise *power* as well as of dissipation. The fact that S and D can come into equilibrium depends on both the dissipative and exciting aspects of the force. The *rate* at which equilibrium is approached, and therefore the associated impedance function, are determined by the statistical properties of the fluctuations. The formal statements of these relations are the various fluctuation-dissipation theorems. The first version of this theorem was in fact discovered by Einstein[7] in the course of his work on Brownian motion. (e) The fluctuation-dissipation theorem can also be regarded as providing an expression for the energy density residing in the fluctuations. In this expression the (frequency-dependent) impedance takes on a new significance as a quantity proportional to the *density of states* of the dissipative system.

We now apply these ideas to the dissipative action of a black hole. In this Letter, we confine ourselves in the main to the dissipation of gravitational disturbances. A more comprehensive and detailed discussion will be given elsewhere.

It has been known—at least since the work of Callen and Welton[3]—that it is possible to ascribe the radiation damping of the motion of an accelerated charge to a coupling between the radiating charge and the quantum fluctuations of the electromagnetic vacuum. In order to proceed in this same spirit we shall briefly examine the effect of the quantum fluctuations of a scalar field φ in the Minkowski vacuum $|0\rangle$ on a monopole charge which possesses *internal* degrees of freedom and is *uniformly* accelerated. The existence of these internal degrees of freedom enables us to regard the accelerating charge as a *detector* of φ particles. The classical analog of such an accelerating detector is well known.[8] The quantum calculation was first performed by Unruh[7] and has since been elaborated by DeWitt.[8]

The analysis is facilitated by the introduction of accelerated (Rindler) coordinates defined in terms of standard Minkowski coordinates by

$$t=\xi \sinh\tau, \quad x=\xi \cosh\tau,$$

and a worldline of constant ξ is a path of uniform acceleration ξ^{-1}. It is supposed that the coupling of the particle to the scalar field is achieved by an interaction Lagrangian of the form

$$L_{\text{int}}=\epsilon m(x)\varphi(x),$$

where $m(x)$ is a monopole charge and ϵ is a small coupling constant. If we adopt the convention of denoting $\varphi(x)$ evaluated at $x(\tau)=(\tau, \xi, y, z)$ by $\varphi(\tau)$ and similarly for $m(x)$, then it is found that the rate at which the detector makes transitions from an energy eigenstate corresponding to a frequency ν_1 to another eigenstate corresponding to another frequency $\nu_2=\nu_1+\nu$ is

$$R(\nu|\nu_1)=\epsilon^2|\langle\nu_1+\nu|m(0)|\nu_1\rangle|^2\int_{-\infty}^{\infty}d\tau\, e^{i\nu\tau}\langle 0|\varphi(\tau)\varphi(0)|0\rangle \tag{1}$$

$$=\epsilon^2|\langle\nu_1+\nu|m(0)|\nu_1\rangle|^2(2\pi\xi^2)^{-1}\nu/(e^{2\pi\nu}-1). \tag{2}$$

The last equality follows by explicitly evaluating the integral, and is, of course, just Unruh's result.[9] We wish to emphasize that (1) is a fluctuation-dissipation theorem since $R(\nu|\nu_1)$ determines not only the *equilibrium* internal state of the charge but also the *rate* at which equilibrium is approached, or equivalently the *dissipation rate of any correlations* that might initially be present. We note also that (1) shows that $R(\nu|\nu_1)$ is essentially determined by the Fourier transform of the autocorrelation function of the scalar field which, by the Wiener-Khinchin theorem, is just the power spectrum of the *noise* evaluated along the worldline of the particle.

The observation that (1) is a fluctuation-dissipation relation does not of itself explain the remarkable fact that the spectrum in (2) should be Planckian. This property appears to be intimately related to the explicitly stationary character of the Rindler manifold in combination with its causal and analytic structure.[11] We hope to return to this question elsewhere.

Let us now consider a particle detector that is constrained to remain near the horizon of a Kerr black hole and to corotate with it. That is, we take the detector to follow the path

$$x(\tau)=(t+\tau, r, \theta, \varphi+\Omega\tau),$$

where (t, r, θ, φ) are Boyer-Lindquist coordinates and Ω is the angular velocity of the horizon.

Since we are mainly concerned with gravitational disturbances, we shall consider the detector as weakly coupled to the fluctuations in the gravitational field rather than to those of a scalar field. The path of the detector follows the trajectory of a Killing vector so the regime is again a stationary one. We would expect that the detector would make transitions between its various energy levels at a rate dictated by the spectrum of the vacuum fluctuations of the gravitational field in much the same way as in our previous example.

As a measure of this spectrum we shall take the Fourier transform of the autocorrelation function of the gravitational shear[12] σ in the Hartle-Hawking tetrad. We choose σ since it determines all the nontrivial perturbations of the metric[13]; moreover, since it is gauge invariant, it is a suitable variable to quantize. The problem is, of course, not well posed until we choose a "vacuum state" for the gravitational field. Of the three vacua usually considered, namely those of Boulware,[14] Hawking and Hartle,[15] and Unruh,[9] only that of Unruh meets the requirements that the renormalized values of physical observables are well behaved on the future horizon and that at large radii it corresponds to an outgoing flux of black-body radiation.[16] In this sense then, the Unruh vacuum seems to approximate best the state that would obtain following the gravitational collapse of a star and that would give rise to the Hawking dissipation, which we are aiming to relate to a fluctuation spectrum. We shall therefore compute the expectation value of the autocorrelation function of σ in the Unruh vacuum. A direct calculation shows that asymptotically, as

$r \to r_+$,

$$\int_{-\infty}^{\infty} d\tau\, e^{i\omega\tau} \langle U|\,\sigma^*(\tau)\sigma(0)\,|U\rangle \sim \frac{4}{5}\,\frac{1}{r-r_+}\,\frac{\omega(\omega^2+4\kappa^2)}{e^{2\pi\omega/\kappa}-1},$$

where κ is the surface gravity of the hole, r_+ is the radius of the outer horizon, and by $\sigma(\tau)$ we mean $\sigma(x)$ evaluated at $x(\tau)$.

We recognize this relation as a fluctuation-dissipation theorem, with $\omega^2+4\kappa^2$ playing the double role of impedance function and density-of-states factor. This factor deviates from the more familiar ω^2 form in a manner reminiscent of the effect on radiation damping of enclosing a charge in a finite box.[17] In the black-hole situation, the deviation is important for wavelengths $1/\omega$ which are not small compared to the length scale $1/\kappa$ of the gravitational field. In fact, for a field of spin s the density-of-states factor is proportional to $\omega^2+s^2\kappa^2$.[18]

Let us now consider the rate at which a black hole dissipates a gravitational perturbation. If the disturbance is purely gravitational and $\sigma^*\sigma$ is slowly varying, then to lowest order the area of the horizon increases at a rate given by[12]

$$dA/dv = 2\kappa^{-1}\int \sigma^*\sigma\, dA, \tag{4}$$

where v is a suitably defined time coordinate on the horizon. Since the balck-hole entropy is proportional to the area, this formula determines the dissipation rate in a macroscopic process such as the slowing down of a rotating black hole by a moon.

If we now interpret σ as a quantum operator, take a vacuum expectation value and renormalize, then we may employ (4) to describe also the rate at which the mass of the black hole is dissipated via Hawking radiation. We would then have

$$\frac{dA}{dv} = \frac{2}{\kappa}\int \langle U|\sigma^*\sigma|U\rangle_{\text{renorm}}\, dA \tag{5}$$

$$= \frac{-4}{\kappa}\sum_{l,m}\int \frac{dp\,(p-m\Omega)T_{lm}(p,a)}{e^{2\pi(p-m\Omega)/\kappa}-1}, \tag{6}$$

where $T_{lm}(p,a)$ is a transmission factor for waves of frequency p and angular eigenvalues l and m for a black hole of specific angular momentum a.

The second equality follows either by considering the luminosity at infinity, or by observing that, due to the invariance of the Hawking vacuum under time reversal, $\langle H|\sigma^*\sigma|H\rangle_{\text{renorm}}=0$ and hence, as $r\to r_+$,

$$\langle U|\sigma^*\sigma|U\rangle_{\text{renorm}} \sim \langle U|\sigma^*\sigma|U\rangle_{\text{renorm}} - \langle H|\sigma^*\sigma|H\rangle_{\text{renorm}} = \langle U|\sigma^*\sigma|U\rangle - \langle H|\sigma^*\sigma|H\rangle,$$

which may be calculated directly. Equations (4) and (5) show that the dissipation rate is quadratic in the perturbation, so that the black hole is a linear system in the sense of Onsager.[6]

In virtue of this, we can fit into our picture the emission by a black hole of Hawking gravitational radiation. We know from macroscopic theory that a nonstationary black hole with a nonvanishing shear on its horizon would radiate gravitational waves to infinity, and in consequence would reduce the shear of the horizon and approach a stationary state. Now, according to Onsager's[6] point of view, a linear system (that is, one with no memory) behaves on average in the same way in a given configuration whether it reached that configuration by a spontaneous fluctuation or by an externally induced perturbation. Accordingly we would expect that the quantum fluctuations of the shear would also lead to the emission of gravitational radiation, and since the shear fluctuations have the stochastic properties of black-body radiation at a temperature $\kappa/2\pi$, we would expect the gravitational radiation to have the same properties. This is precisely Hawking's result (6). A more graphic expression of this point of view is that the black hole may be thought of as possessing internal coherence by virtue of the localization of its mass; when the vacuum fluctuations of the gravitational field are allowed to act on it, this coherence is corrupted and the mass is sapped away.

There is an important proviso, however. There will be a net emission of gravitational radiation only provided that phase relations have been chosen which do not exactly suppress the flux. If the black hole is the result of stellar collapse, then one would not expect the collapse to respect correlations that might initially be present. This is essentially Hawking's point of view. By contrast, if we are dealing with an eternal Kruskal black hole, it is not clear what are the "correct" initial conditions to take. The problem is an academic one and is usually phrased in terms of choosing a particular "vacuum state." If one were to choose either the Boulware or the Hawking vacuum, there would be no net flux at infinity since in these cases correlations would have been chosen which would exactly suppress the radiation.

The resulting situation is then essentially the same as for an atom in its ground state which has been coupled for a long time to the electromagnetic vacuum. There is then on average no exchange of energy between the atom and the electromagnetic field because the zero-point fluctuations of the field drive those of the atomic moments and produce complete interference.[19,20] Correspondingly we can regard this final state as one of equilibrium between two systems of zero-point fluctuations.[20] From this point of view, Hawking's great discovery is that a black hole formed by gravitational collapse would not achieve such an equilibrium state (unless it were contained in a sufficiently small box[21]).

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