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Binding Energies of *sd*-Shell Nuclei with a Realistic Effective Hamiltonian

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The nuclear shell model with a second-order effective Hamiltonian derived within Brueckner theory from the free nucleon-nucleon interaction is shown to yield accurate binding energies of nuclei with $16<A<40$. This agreement is obtained by choosing the spectrum of low-lying unoccupied orbitals in a justified manner and, when necessary, by employing a statistical method to approximate the lowest eigenvalue of very large shell-model diagonalizations.

There is much current interest in whether microscopic nuclear theory based on Brueckner theory[1] of the effective interaction with no adjustments will succeed in describing properties of nuclei. The total binding energy and one-body density of infinite nuclear matter[2] and of finite nuclei[3,4] have been the traditional foci of critical tests. For finite nuclei these tests may be carried out within a Brueckner-Hartree-Fock[3] (BHF) or shell-model[4] (SM) framework.

Residual discrepancies between BHF results and experiment[5] have been a recurring cause for uncertainty about the eventual success of Brueckner theory. More recently, Jastrow-type variational calculations of infinite nuclear matter[6] have yielded significant differences from lowest-order Brueckner theory (LOBT), thus raising new questions. One may speculate that nuclear theory based on the Brueckner reaction matrix G is correct and that these difficulties are due to significant intermediate-range correlations such as strong three- (or higher) body cluster effects. These many-body correlations are absent in BHF and LOBT which accommodate strong two-body correlations within a mean-field approximation (single Slater determinant). Therefore, we argue that one should examine the Brueckner-SM approach since it allows a linear combination of Slater determinants and thus accomodates essential many-body cluster effects.

The chief limitations to a conclusive demonstration in the SM framework are the following: (1) Convergence of the effective-interaction expansion based on the G matrix has not been established; (2) exact SM diagonalizations with a significant number of nucleons are not possible; and (3) the effective radius operator should also be evaluated in the same perturbation theory as the effective Hamiltonian. Overcoming these limitations will require a substantial theoretical effort. In the search for justification for such a major attack, we make the following speculation regarding these limitations: If one (1) uses current guidelines for calculating the effective Hamiltonian through second order in G, (2) uses a reasonable procedure to estimate the results of a large SM diagonalization, (3) employs a basis representation with a reasonable radius, and thereby demonstrates agreement with experimental binding energies (BE), then one has obtained substantial motivation for microscopic nuclear theory based on Brueckner theory. Here we summarize the methods and present results of such a test.

In the realistic nuclear SM based on Brueckner theory, one diagonalizes an effective Hamiltonian, $H_{\rm eff}=H_0+V_{\rm eff}$ in a defined model space of valence single-particle states.[7] We choose an ^{16}O core and valence orbitals in the $1s$-$0d$ shell of the harmonic oscillator (HO) with intershell spacing $\hbar\Omega = 14$ MeV. H_0 possesses an eigenspectrum of occupied orbitals with HO eigenvalues, valence orbitals with experimentally determined eigenvalues, and unoccupied orbitals with HO spacings but an additional spacing C between valence and unoccupied orbitals.[8]

It is evident that $V_{\rm eff}$ depends on H_0 (which depends on the parameters $\hbar\Omega$ and C) whereas the true Hamiltonian $H=T+V$ with kinetic energy op-

erator T depends only on the free nucleon-nucleon interaction V. Ideally, then, H_{eff} should be independent of H_0 (and hence independent of $\hbar\Omega$ and C) but when evaluated to fixed order of perturbation theory some residual dependence is inevitable. A popular philosophy is to minimize this dependence by a physical choice for H_0 (that which yields a single-particle spectrum and wave functions in reasonable accord with experiment). Some justification is derived from calculations showing that this "physical" choice minimizes classes of higher-order corrections[9,10] and thus optimizes the convergence of V_{eff}.

From BE calculations which include higher-order effects[9,10] one can conclude that $C \cong 20-25$ MeV is the optimum choice for sd-shell nuclei since it locates the low-lying unoccupied HO states near their average kinetic energies.[8] Results are presented here for this "justified" H_0.

The interaction V_{eff} is obtained by solving[7] the linked diagrams of

$$V_{\text{eff}} = G + GQ'(E_0 - H_0)^{-1}V_{\text{eff}} \quad (1)$$

through second order in G, where Q' projects onto states outside the model space but not included in G. We consider only the two-particle component of V_{eff} and select E_0 to be their unperturbed energy. The renormalized interaction G is the solution of

$$G = V + VQ(E_0 - H_0)^{-1}G, \quad (2)$$

where Q projects onto unoccupied two-particle states. The Reid soft-core interaction is chosen for V and the solution is obtained as in Ref. 8.

The main advantage of this V_{eff} is accuracy since the infinite sum over intermediate states in the second-order term was performed until convergence was achieved near 300-MeV particle-hole (p-h) excitation energy.[8] For comparison we also consider the second-order V_{eff} of Kuo[11] calculated in the same theoretical framework [Eqs. (1) and (2)] but restricted to intermediate p-h states through only ~30 MeV excitation. This major difference in the effective interactions[12] is responsible for the large differences in the calculated binding energies.

Beyond five valence nucleons, full SM diagonalizations are unfeasible so that we employ a statistical method to approximate the BE relative to ^{16}O. We employ the SM level densities

$$\rho(E, J) = N_J(2\pi\sigma_J)^{-1/2}\exp[-(E-\epsilon_J)^2/2\sigma_J{}^2], \quad (3)$$

where N_J is the number of levels with spin J.[13] When the centroids (ϵ_J) and widths (σ_J) are obtained from the same SM H_{eff}, this two-moment distribution gives a remarkably accurate approximation to the SM level distributions, especially for large configuration spaces.[14,15] A systematic error is known for this two-moment distribution for even-even nuclei where the pairing character of the force produces a skewness of the actual level density. This pairing-force contribution of V_{eff} can be more completely included[16] by adding higher-moment terms to Eq. (3) but would not affect the main conclusions of this work. The moments were calculated using Ginocchio's reduction formula.[17,18]

We estimate the BE via a continuous-to-discrete transcription[15] for the known ground-state spin J.

$$\int_{-\infty}^{E_0}\rho(E, J)\,dE = \tfrac{1}{2}. \quad (4)$$

The inherent fluctuation error in Eq. (4) is less than the local intralevel spacing.[15,16,19]

To estimate net uncertainties, we compare, where feasible, with diagonalization. For ^{19}F, ^{21}Ne, ^{20}F, ^{19}O, ^{21}F, and ^{20}O, the differences between diagonalization and predictor methods using V_{eff} with $C = 25$ MeV are 4.5, 1.5, 1.1, 0.2, 0.1, and 2.3 MeV, respectively. Since the statistical method becomes more accurate with increasing matrix dimensions the uncertainties for the predicted BE in the middle of the sd shell should be less than those above.[20]

We compare with experimental BE corrected for Coulomb contributions using an empirical formula.[18] Differences between calculated and experimental BE for $T = 0$, $\frac{1}{2}$, 1, $\frac{3}{2}$, and 2 sd-shell nuclei are shown in Fig. 1 for three choices of V_{eff}. The $C = 20$ and $C = 25$ V_{eff} of Ref. 8 yield BE results labeled $V20$ and $V25$, respectively. Results with use of the V_{eff} of Kuo[11] are also shown. Exact diagonalizations were performed for masses 19, 20, and 21.

For each T value one has 10–20 MeV underbinding on the upper end of the sd shell and 10–15 MeV overbinding in the middle of the sd shell. For even-even nuclei the systematic shift towards less binding in the calculations relative to experiment yields a sawtooth pattern and is primarily due to lack of a skewness term[16] in $\rho(E, J)$ as discussed above. It is remarkable that these two realistic interactions result in reasonable agreement with experimental BE for all T values considered, especially since only second-order contributions are included in V_{eff}.

In comparing the $V25$ results with and without the second-order term for $T = \frac{1}{2}$, $A = 35$ nuclei, it

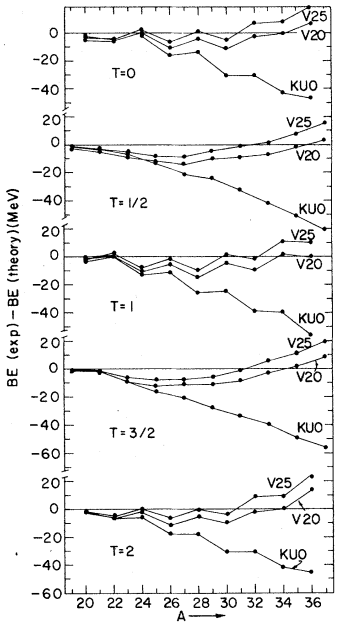

FIG. 1. Difference between experimental (Coulomb corrected) and theoretical BE of *sd*-shell nuclei with respect to ^{16}O as a function of the atomic number A for nuclei of differing isospin T. The effective interactions are described in the text.

is found that second-order contributions are essential and change the ground state from 90 MeV overbinding to 8 MeV underbinding. In general, BE errors due to deficiencies of interactions are amplified as the number of valence particles is increased.[18,21] The BE for the V_{eff} of Kuo[11] in Fig. 1 exhibit this behavior; the BE for the V_{eff} of Vary and Yang[8] do not. This difference is traced primarily to the above-mentioned different treatments of intermediate-state sums[12] in V_{eff}.

For V_{eff} with $C = 20$ to 35 MeV, the SM spectra for $A = 18$ are in reasonable agreement with experiment.[8] To test V_{eff} further, Fig. 2 presents experimental and SM spectra of ^{21}Ne. The four lowest SM states are properly ordered, but the Kuo results are somewhat compressed and the Vary-Yang[8] results are somewhat spread with respect to the data. Overall, the spectra are reasonable, considering that only second-order terms of H_{eff} are included.

Substantial encouragement for a Brueckner-based SM theory of nuclei has been obtained by showing that second-order calculations of H_{eff} yield BE and spectra of *sd*-shell nuclei in reasonable accord with data.

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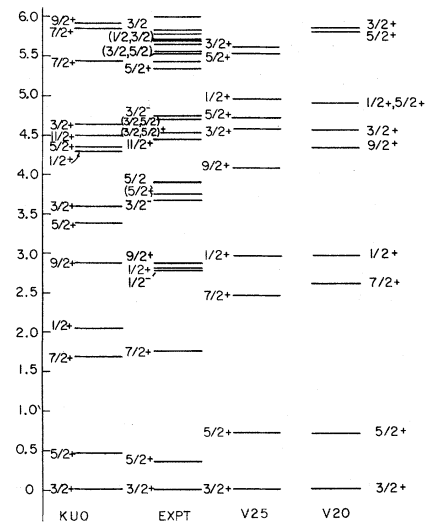

FIG. 2. Experimental and theoretical spectra for ^{21}Ne aligned according to the lowest state.

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Observation of Resonances in the Radiation Pressure on Dielectric Spheres

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We report an experimental check of the Mie-Debye theory for the variation of radiation pressure on dielectric spheres with wavelength and size using optical-levitation techniques. Sharp resonances are observed which are shown to be related to dielectric surface waves. They permit particle-size measurement to a precision of 1 part in 10^5 to 10^6.

We report the first observation of the variation of the radiation-pressure force on transparent dielectric spheres with wavelength and size. We use a technique based on optical levitation which we call force spectroscopy. The measured force shows a regular series of sharp optical resonances which are in excellent qualitative agreement with the limited force data presently available from Mie-Debye theory. This resonant behavior can provide the most precise check on Mie scattering theory and also a way of measuring sizes of spheres to an accuracy exceeding that of present far-field scattering techniques by at least two orders of magnitude. These resonances are thought to be due to dielectric surface waves. This view is strongly supported by the observation of the scattered-light distribution in the near field. Both the resonant coupling of light striking the sphere edge and its subsequent isotropic tangential scattering are seen. These measurements should stimulate more precise calculation of radiation pressure and of the scattered-light distributions in the near field. Resonant effects are also of interest for the interpretation of particle scattering processes using optical models.

The Mie theory[1] for scattering of light by a sphere large compared to a wavelength is the best understood and most carefully checked example of scattering of waves by a particle. It is the basis of a vast theoretical and experimental literature and is widely used for particle-size measurement.[2] Using Mie theory, Debye[3] calculated expressions for the radiation-pressure force on a sphere as a function of the size parameter $X = 2\pi a/\lambda$, where a is the radius and λ the wavelength. Recently Irvine[4] made the first computer evaluation of the radiation pressure and extended computations of the total scattering cross section to large values of X using relatively high resolution (up to $\Delta X/X = 10^{-4}$). At low X, calculations of the total scattering cross section for low-loss spheres show the well-known "ripple structure"[5] as X is varied. Ripples are experimentally observed in measurements of the total scattering cross section for small x,[6] in far-field radar backscatter,[7] and in 90° scattering.[8] They are attributed to dielectric surface waves as originally proposed by Van de Hulst.[5] Irvine[4] shows that the ripple structure is larger on the radiation pressure than on the total scattering cross section and that at large X and high index n it sharpens dramatically and eventually becomes an unresolved sequence of resonances. Unfortunately he did not increase the resolution of his calculation further, since recent advances in the study of radiation pressure have now made possible the observation of Mie-Debye resonances with a resolution exceeding these existing calculations.

Indeed, with focused laser beams of modest powers one can use radiation-pressure forces to