Spin Waves in ^{3}He-B

D. D. Osheroff, W. van Roosbroeck, H. Smith,* and W. F. Brinkman
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 1 November 1976)

A set of harmonic-oscillator-like standing spin-wave modes have been observed in superfluid ^{3}He-B confined by a set of parallel plates. The frequencies and intensities of the modes are in close agreement with computer solutions of the theoretical equations. Their observation represents the first direct confirmation of the existence of spin-wave dispersion in either superfluid phase of ^{3}He.

In the past, spin dynamics of ^{3}He-B have been investigated extensively, and for uniform motion of the spin system the theoretical predictions[1] have been tested and found valid.[2] Similarly, many static spatial variations of the spin anisotropy axis, $\hat{n}$, as predicted by the theory of textures[3-5] in ^{3}He-B have been observed.[6] Prior to this work no experiments have been successfully performed, however, in which nonuniform spin precession was observed in a geometry designed to test the importance of spin-wave dispersion in the resulting spin dynamics. In the present work, we have been able both to excite a series of standing spin-wave modes inside a parallel-plate array, and to predict the energies and intensities of those modes with reasonable precision.

Spin-wave dispersion in superfluid ^{3}He arises from the stiffness of the anisotropic superfluid order parameter with respect to spatial variations induced by a nonuniform spin precession. To create a nonuniform spin precession with a uniform radio-frequency field we have taken advantage of the known textural behavior of ^{3}He-B. Consider a sample of ^{3}He-B contained between two parallel plates as shown in the inset in Fig. 1. Apply an intense static field $\vec{H}_0$ ($\gtrsim 1$ kOe in this work) in the $\hat{x}$ direction and orient the plates so that the surface normal vector is in the $\hat{z}$ direction. In the presence of the magnetic field, suppression of certain components of the order parameter by the surfaces orients $\hat{n}$ in the direction $\hat{n} = 5^{-1/2}(1, \sqrt{3}, 1)$ at those surfaces. In the interior between the plates, however, the orientation of $\hat{n}$ is determined by the competition between the bulk field anisotropy energy which tends to orient $\hat{n}$ parallel to $\vec{H}_0$, and the bulk bending energy terms which resist spatial variations of the order parameter. As a result, $\hat{n}$ can only change its orientation over the characteristic distance $R_H \propto 1/H_0$, the field texture bending length. When the plate spacing $l \lesssim R_H$, the variations of $\hat{n}$ are nearly quadratic in z as measured from the center of the ^{3}He-filled space. The interesting feature of ^{3}He-B which we will utilize is that this variation of $\hat{n}$ acts on an oscillating magnetization wave like a confining potential. Under appropriate circumstances the quadratic variation of $\hat{n}$ can thereby produce a set of harmonic modes of the oscillating magnetization.

In order to understand the above behavior, consider the equation of motion for the precessing transverse magnetization, S^+, in a field, H_0, when the Larmor frequency, γH_0, is large compared to the longitudinal resonant frequency,[5] Ω_L:

$$\{E - n_\perp^2(z)\}S^+ = -\{R_s^2 f(\hat{n})\partial^2/\partial z^2\}S^+. \quad (1)$$

Here $E = (\omega - \gamma H_0)/(\Omega_L^2/2\gamma H_0)$ is the frequency shift of the spin-wave mode measured relative to γH_0 and normalized to the transverse resonance shift measured when $\hat{n} \perp \vec{H}_0$, R_s ($\equiv 2c\gamma^2/\chi_B \Omega_L^2)^{1/2}$ is the dipolar bending length, $n_\perp$ is the component of $\hat{n}$ perpendicular to the field $\vec{H}_0$, and $f(\hat{n})$ is a scalar of order unity which varies by about 40% depending upon $\hat{n}$. The quantity c is a

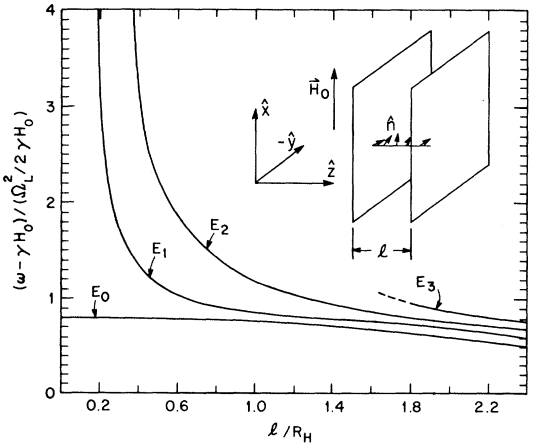

FIG. 1. The normalized frequency shifts (see text) of the four lowest symmetric spin-wave modes of a sample of ^{3}He-B contained in the parallel-plate geometry shown, as a function of l/R_H. These are computer simulations valid for $T/T_c = 0.44$ and $H_0 = 1$ kOe.

coefficient which multiplies the bending free-energy expression, and is proportional to the spin superfluid density,[3] and χ_B is the B-phase spin susceptibility. This equation is just a Schrödinger equation with $n_\perp^2(z)$ acting as a potential. The question is whether one can actually confine spin waves by this potential without large effects due to the walls and whether such waves are observable.

To predict accurately the characteristics of the spin-wave modes, we have determined the variation of $\hat{n}(z)$ between the parallel plates by simultaneously minimizing the bending energy and the surface and field anisotropy energies on a computer. With this result, we then use the computer to solve (1) with the boundary condition that at the surfaces $\partial S^+/\partial z = 0$. This insures that no spin currents flow into the walls. Figure 1 shows results of computer solutions for the eigenvalues of (1) in which it is assumed that $R_H/R_s = 20$. This is characteristic of ^{3}He-B at melting pressures for $T/T_c \simeq 0.55$ and $H_0 \simeq 1$ kOe. Notice the gradual transition from a quadratic spin-wave spectrum for $l \ll R_H$ to a set of uniformly spaced modes for $l \gtrsim 2R_H$.

To understand the computer results, consider the cases $R_H \gg l$ and $R_H \sim l$. In the limit $R_H \gg l$, the even solutions of (1) are $S^+ = \cos(mkz)$, where $k = 2\pi/l$, and m is an integer. The corresponding eigenvalues are $E_m = n_\perp^2 + R_s^2 f(\hat{n}) m^2 k^2$. The modes with $m > 0$ are simply standing plane waves in a constant potential, and the lowest mode is the solution to the uniform Leggett equations. In this limit, the modes with $m > 0$ have no net magnetization, and consequently do not couple into a uniform radio-frequency field such as is normally used in NMR experiments. For the case $R_H \sim l$, it has been shown[7] that $n_\perp^2 = \text{const} + \frac{13}{60} z^2/R_H^2$. By incorporating this variation of $\hat{n}$ into (1) we are left with an expression which is identical to the Schrödinger equation for a harmonic oscillator provided we set $f(\hat{n})$ equal to a constant. When the quadratic term in $n_\perp^2$ is sufficient to localize the solutions inside the ^{3}He-filled channels, this expression predicts spin-wave modes whose energies are evenly spaced. The net magnetization of these states is nonzero, and they can be excited by a uniform rf field. Their intensities drop off approximately as $1/m$, where m is the mode number.

Using these results as a guide, we constructed an accurately spaced array of twenty fused quartz plates approximately 0.7 cm × 1.1 cm × 0.013 cm each, spaced by much smaller plates at the four

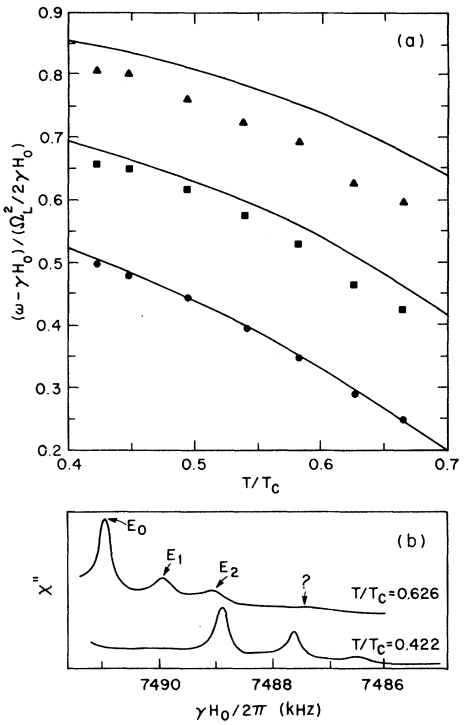

FIG. 2. (a) A comparison of computer-calculated frequency shifts of the three lowest symmetric spin-wave modes (solid lines) and experimentally determined shifts for a parallel-plate geometry with $l = 210$ μm and $H_0 = 2.31$ kOe. The circles represent data for the $m = 0$ mode, the squares for the $m = 1$ mode, and the triangles for the $m = 2$ mode. (b) Typical NMR absorption spectra of spin-wave modes whose frequency shifts are shown in (a).

corners of the large plates so that the resulting ^{3}He-filled channels are 210 ± 4 μm wide. This array was placed in the NMR tail piece of a compressional cooling apparatus similar to those described previously. The experimental techniques utilized in this work are also described in previous literature and will not be reviewed here.[8]

NMR absorption spectra were then obtained at resonant frequencies of 2.5, 3.5, 5, and 7.5 MHz by sweeping either H_0 or ω. In Fig. 2(b) are shown actual absorption spectra at 7.5 MHz obtained by sweeping H_0. The spin-wave modes are clearly sharpest at low temperatures. The rising baseline to the left is the result of NMR absorption in the portion of the sample outside the array. Because this unwanted contribution was temperature dependent, it could not be subtracted from the total signal. The linewidths of the spin-wave resonances were considerably larger than the Fermi-liquid linewidth. Much of this difference may be caused by the variations in plate

spacings since the spin-wave resonant frequencies depend very strongly on the ratio l/R_H.

To compare the positions of these spin-wave resonances with the computer solutions we first used the position of the lowest-frequency spin-wave mode from each spectrum to determine $R_H H_0$ accurately at each temperature at which measurements were made. This was possible because its position depends only very weakly upon R_s, but very strongly upon R_H. We could therefore use preliminary computer results for the purpose. In making this fit, values of $R_H H_0$ have been determined with an overall uncertainty of about ± 3%, and results at all fields agreed with one another to within a characteristic scatter of less than ± 3%. To specify (1) totally we then determined the appropriate value of R_s utilizing the definition (in Ref. 3 where this expression was first presented, $R_H H_0$ is referred to as $R_c H_B$) that

$$R_H H_0 \equiv (c/a)^{1/2}, \tag{2}$$

and that the coefficient multiplying the bulk field anisotropy energy, a, is given by[9]

$$a = \tfrac{5}{4}(\Delta g/g)\chi_B. \tag{3}$$

We obtain the B-phase susceptibility from the ratios χ_B/χ_N of Corruccini and Osheroff[10] and the value of the normal-state susceptibility, χ_N, given by Wheatley.[2] The quantity $\Delta g/g$ is the effective g-value shift to the transverse NMR frequency in ^{3}He-B when $n_\perp = 0$.[11] This quantity has recently been measured to an accuracy of about ± 2% by Osheroff and Brinkman from $T/T_c = 0.4$ to $T/T_c = 0.75$.[12]

Once $R_H H_0$ and R_s were determined, separate computer runs were made to find the eigenvalues of (1) and the relative intensities in each mode at a series of six temperatures for fields at which absorption spectra were measured. In Fig. 2(a) we show a comparison of the computer output (solid lines) and the frequency shifts which we measured at 7.5 MHz for the first three observed spin-wave resonances. The spacings observed are about 10% smaller than the computer predictions, but overall the agreement is reasonably good. Similar agreement is found at 3.5 and 5 MHz. The excellent agreement between the experimental and theoretical shifts for the lowest mode is insured by the fit which determined $R_H H_0$. The actual values of $R_H H_0$ were obtained by a best fit to data at four separate frequencies.

We do not measure the spin-wave velocity, s, directly in these experiments. We can calculate

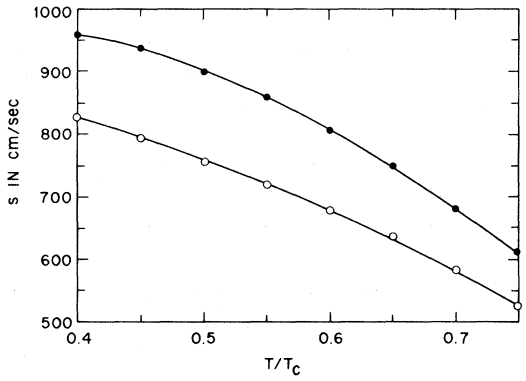

FIG. 3. Spin-wave velocities in ^{3}He-B at melting pressure. The open circles are obtained from the experimentally determined quantities $R_H H_0$ and $\Delta g/g$. The closed circles are obtained from the theoretical expression (see text). The solid lines through the two sets of points are for convenience only.

s, however, from the experimental values of $R_H H_0$ and $\Delta g/g$ using the relationship $s^2 = \frac{32}{65}\gamma^2 c/\chi_B$ and Eqs. (2) and (3). We then obtain

$$s^2 = \tfrac{8}{13}(\Delta g/g)\gamma^2 (R_H H_0)^2. \tag{4}$$

These values are shown in Fig. 3 as open circles.

Alternatively, we can substitute the theoretical expression for c into the above expression for s^2 to obtain[13]

$$s^2 = \rho_s{}^0 \gamma^2 \hbar^2 / 10 m \chi_B. \tag{5}$$

Here $\rho_s{}^0$ is the spin superfluid density, $\rho_s{}^0 = (m^*/m)^{-1}(1-Y)N/V$, where (m^*/m) is the effective mass enhancement, Y is the Yoshida function, N/V is the number density of atoms, and m is the mass of the ^{3}He atom. Values of Y are determined by inverting the Leggett expression[14] for the Fermi-liquid enhanced B-phase susceptibility. We use the Fermi-liquid parameters $F_1{}^s$ and $F_0{}^a$ obtained by Halperin *et al.*[15] in our analysis.

Despite the apparently small uncertainties in determining s from (5), the resulting values, which are shown as solid circles in Fig. 3 are consistently about 18% higher than the values determined using the experimental quantities. This disagreement, which lies well outside the uncertainties in both sets of calculated velocities, may be due to a relatively large and negative value of the Landau parameter $F_1{}^a$. This value ($F_1{}^a = -1.2$) is consistent with the experimental results of Corruccini *et al.*[16] provided one uses the Halperin value of $F_0{}^a = -0.77$ in the analysis of those results. A similar value of $F_1{}^a$ is obtained by Pethick, Smith, and Bhattacharyya[17] from a comparison of their calculated normal-

state viscosities with data obtained near melting pressures.

In summary, we observe that spin waves do exist in ^{3}He-B and that they are by no means overdamped modes. Further, we have shown that within the context of current theory, spin-wave effects can be calculated with reasonable precision. We believe that the (20%) discrepancy between theory and experiment is attributable to the need for a nonzero value of F_1^a.

We wish to thank W. O. Sprenger and S. B. Darack for their technical assistance in these experiments, D. D. Warner for his advice in programming the computer, S. Engelsberg who has long encouraged the experimental effort to observe standing spin waves in a parallel array, and M. C. Cross who provided theoretical assistance and advice on numerous occasions throughout the course of this work.

*Permanent address: H. C. Ørsted Institute, Copenhagen, Denmark.

[1]A. J. Leggett, Rev. Mod. Phys. 47, 331 (1975).
[2]J. C. Wheatley, Rev. Mod. Phys. 47, 415 (1975).
[3]W. F. Brinkman, H. Smith, D. D. Osheroff, and E. I. Blount, Phys. Rev. Lett. 33, 624 (1974).
[4]I. Fomin and M. Vuorio, J. Low. Temp. Phys. 21, 271 (1975).
[5]H. Smith, W. F. Brinkman, and S. Engelsberg, Phys. Rev. B 15, 199 (1976).
[6]See, for example, D. D. Osheroff, S. Engelsberg, W. F. Brinkman, and L. R. Corruccini, Phys. Rev. Lett. 34, 190 (1975); D. D. Osheroff, Phys. Rev. Lett. 33, 1009 (1974); and most recently, A. I. Ahonen, M. Krusius, and M. A. Paalanen, to be published.
[7]See Osheroff, Engelsberg, Brinkman, and Corruccini, Ref. 6.
[8]See, for example, D. D. Osheroff and W. F. Brinkman, Phys. Rev. Lett. 32, 584 (1974); D. D. Osheroff and P. W. Anderson, Phys. Rev. Lett. 33, 686 (1974).
[9]The expression for a was first presented incorrectly in Ref. 7. It is correctly derived in Ref. 5.
[10]L. R. Corruccini and D. D. Osheroff, Phys. Rev. Lett. 34, 695 (1975).
[11]See Osheroff, Ref. 6.
[12]D. D. Osheroff and W. F. Brinkman, to be published.
[13]W. F. Brinkman and H. Smith, Phys. Rev. A 10, 2325 (1974).
[14]A. J. Leggett, Phys. Rev. 140, 1869 (1965).
[15]W. P. Halperin, C. N. Archie, F. B. Rasmussen, T. A. Alvesalo, and R. C. Richardson, Phys. Rev. B 13, 2124 (1976).
[16]L. R. Corruccini, D. D. Osheroff, D. M. Lee, and R. C. Richardson, J. Low Temp. Phys. 8, 229 (1972).
[17]C. J. Pethick, H. Smith, and P. Bhattacharyya, to be published.

Dielectric Anomaly and Improper Antiferroelectricity at the Jahn-Teller Transitions in Rare-Earth Vanadates

H. Unoki and T. Sakudo
Electrotechnical Laboratory, Tanashi, Tokyo, Japan 188
(Received 11 October 1976)

Dielectric measurements have been carried out on the single crystals of $DyVO_4$, $TbVO_4$, and $TmVO_4$ through the Jahn-Teller transition temperatures. An anomalous increase of the dielectric susceptibility with temperature along the c axis was observed for $DyVO_4$, but not for $TbVO_4$ and $TmVO_4$. This behavior is explained as due to the antiferroelectricity driven by the soft B_{1g} strain mode through the "sublattice piezoelectricity."

Some of the zircon-type rare-earth vanadates and arsenides have attracted much interest because of their cooperative Jahn-Teller phase transitions.[1-5] In $DyVO_4$ and $DyAsO_4$, the ($x_1 - x_2$)-type (B_{1g} symmetry) spontaneous strains occur below the transition temperature (T_D), while in $TbVO_4$, $TmVO_4$, and $TmAsO_4$, x_6-type (B_{2g} symmetry) spontaneous strains occur; thereby softening of transverse acoustic phonons [or the elastic stiffness component $\frac{1}{2}(c_{11} - c_{12})$ for the former group and the component c_{66} for the latter group] takes place.[3-5] The transitions have been regarded as a uniform tetragonal-orthorhombic distortion comprising no internal strain because, according to Raman-scattering experiments,[2] there was no evidence of a soft optical mode either above and below T_D. We report here the first observation of the dielectric behavior near the transition temperature in $DyVO_4$, $TbVO_4$, and $TmVO_4$ single crystals and propose a new model —that for $DyVO_4$ at the low-temperature phase there should be an antiferroelectric ordering due to the interaction of some optical modes with the strain mode.

Temperature variations of the dielectric constants were measured for the samples of single