Quasi–Two-Body Scaling: A Study of High-Momentum Nucleons in Nuclear Matter

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I show that the shapes of the differential cross sections for the "backward" production of high-energy particles ($p,d,t,\ldots,B^{11}$), by projectiles of various types (p,d,α), from nuclei of any A (Be, ..., U), and for a large range of incident momenta and backward angles, can be related by quasi–two-body kinematics. This permits the direct determination of the shape of the momentum distribution of high-momentum nucleons in nuclear matter. This distribution is shown to be $F(k)=\exp(-k/k_0)/k$ over the range 0.4 to 1.5 GeV/c.

The *forward* scattering of protons or electrons[1,2] from the quasifree nucleons in a nucleus has long been employed to determine the low-momentum components of the nucleon momentum distribution in nuclei. This process, kinematically allowed in the free nucleon-nucleon interaction, is however dominated by the nature of the nucleon-nucleon interaction although the effect of the internal nucleon momentum, k, is to perturb the nucleon-nucleon scattering. This method is useful for studying low internal momenta, corresponding to kinetic energies of up to about 20–40 MeV, and thus probes the details of the average nuclear potential.

Backward production of nucleons is of course *forbidden* in free nucleon-nucleon collisions. On nuclei it occurs only because of the internal motion of the nucleons. Hence one might expect that the direct observation of the nucleon ejected by the incoming projectile, rather than the modified motion of the projectile itself, would be a more direct and sensitive measure of nucleon momenta, capable of studying momentum components corresponding to very high energies (~800 MeV) and therefore of studying the short-range nucleon-nucleon interaction in nuclear matter. (In this connection it is important to recognize that, since the nucleon-nucleon interaction is dominated by small momentum transfers, the backwardly observed particles are predominantly the target nucleons.)

I demonstrate in this Letter, from an analysis of available data on the low-momentum-transfer "backward" production of particles in the reaction anything + anything → anything + X, that *this is indeed the case.* The differential cross sections for these reactions can all be used to extract the form of the high-momentum components.

I start the discussion of "quasi-two-body scaling" with the momentum diagram and definitions shown in Fig. 1. Amado and Woloshyn[3] have made the crucial point that, because the internal momentum distribution falls so rapidly in nuclei, the cross sections should be dominated by the lowest internal momentum $\vec{k}$ that can produce the external momentum $\vec{q}$. This configuration is shown by the collinear arrangement in Fig. 1 for which we have the important transformation between k and q which I denote by "quisi-two-body scaling":

$$k_{\min}=|\vec{p}-\vec{q}|-|\vec{p}'|=(p^2+q^2-2pq\cos\theta)^{1/2}-[(E_p+E_k-E_q)^2-m_p^{\,2}]^{1/2}. \quad (1)$$

Here p and q (E_p and E_q) are the momenta (energies) of the incident projectile and detected particles, respectively; p' is the momentum of the scattered projectile; and k is the internal momentum of the struck nucleon. $E_k \equiv m_Q-(T+\epsilon+\Delta m)$ is the effective energy corresponding to the momentum k, with m_Q and Q the mass and mass

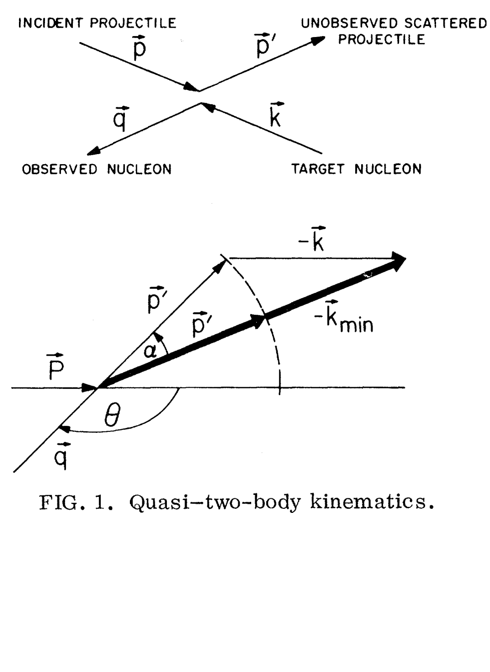

FIG. 1. Quasi–two-body kinematics.

number of the detected particle,[4] T the kinetic energy of the recoiling nucleus of mass m_{A-Q}, ϵ the mean excitation energy of the final state, and $\Delta m = (m_{A-Q}+m_Q)-m_A$. Values of ϵ can be obtained from Moniz *et al.*,[2] but it is important to realize that the effect of ϵ on $k_{\min}$ is to produce an almost A- and $k_{\min}$-independent *shift* in $k_{\min}$. This kinematic relation applies as long as no other high-speed nucleons are ejected from the nucleus and, because T is usually small, applies over a wide range of the angle α near $\alpha = 0$ (see Fig. 1). For this reason, Eq. (1) describes the majority of the observed events.

It is apparent from Eq. (1) that p', the momentum of the *forward*-going projectile, is a very insensitive function of k, since it appears only in the small term T. However, for $q \ll m_q$ the *backward* target nucleon (180°) has a momentum $q \cong k$ and therefore closely mirrors the internal momentum. [At 90°, the relationship is markedly different and, for $q \ll p$, $k \cong (q^2/2m_Q)J$, with $J \cong 1 + m_Q/p$].

The simplest assumption that one can make for the cross section, $d\sigma/d^3q$, is that it is just the average of the inclusive cross section for the scattering of a particle from momentum $\vec{k}$ to momentum $\vec{q}$ over the internal momentum distribution, $F(k)$. Rather than carry out such (Monte Carlo) averaging, thereby obscuring the physics, I have made use of the fact that the inclusive cross sections vary much less rapidly with k than the internal momentum distribution so that the integral should be factorizable into a part, C, that is proportional to the elementary interaction with a nucleon of momentum $k_{\min}$, and a part depending on the probability of finding a nucleon in the nucleus of momentum $k_{\min}$. At low energies where elastic scattering dominates, one can simplify the formulation of Amado and Woloshyn to obtain a form useful for direct scaling of the experimental data,

$$d\sigma/d^3q = CG(k_{\min})/|\vec{p}-\vec{q}|, \tag{2}$$

with $G(k_{\min}) = \int F(k)k\,dk$ evaluated at $k = k_{\min}$.[5]

For *illustrative purposes*, in this Letter I shall take C as unity and show plots of $|\vec{p}-\vec{q}|d\sigma/d^3q$ vs $k_{\min}$ to determine the shape of G.

Before embarking on the verification of (1) and (2) from examination of available data, I assert, without justification in this Letter, that (a) the same expression for $k_{\min}$ applies for incident and ejected particles *other* than nucleons ($p, d, \alpha, \ldots$), and (b) a corresponding factorization appears for inclusive scattering as for elastic scattering.

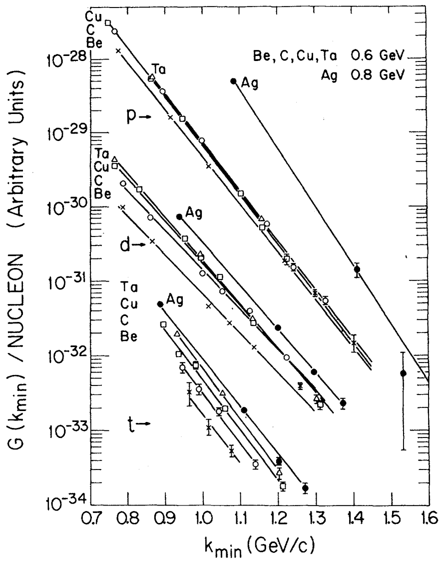

FIG. 2. $G(k_{\min})$ vs $k_{\min}$: $p + A \to (p,d,t) + X$ (180°, Ref. 6).

Figure 2 shows $|\vec{p}-\vec{q}|d\sigma/d^3q$ vs $k_{\min}$ from data at 600 and 800 MeV[6] in the reaction $p + A \to (p, d, t) + X$. Note first that, whereas the experimental differential cross sections fall off with q less rapidly as A increases and the cross sections per nucleon rise faster than A,[6] $G(k)$ *per nucleon is A independent* in the primary reaction $p + A \to p + X$. This result is important for two reasons: (i) The A independence provides support for the single-scattering hypothesis, and (ii) the high-momentum components depending on short-range interactions should be A independent. Note, second, the surprising result that although the *shapes* of $d\sigma/d^3q$ vs q for d and t production are quite different and highly A dependent, the shapes of the $G(k)$ are not, and the values of k_0 in $G(k) = \exp(-k/k_0)$ are remarkably similar[7] for p, d, and t.

Figure 3 shows plots of $G(k_{\min})$ vs $k_{\min}$ for data[8] at 137° for $p + \mathrm{C} \to (p, d) + X$ up to 5.7 GeV. For comparison, the differential cross section versus q^2 are also shown. Here again $G(k_{\min})$ has the same shape for p and d.

Figure 4 shows data[9] at 93° for $p + \mathrm{Pt} \to (p, d, t) + X$. The scaling appears to work even at 93° where $k \sim q^2$ and not $\sim q$.

Figure 5 shows data[10] at 90° for $p + \mathrm{U} \to$ (He, Li, Be, B) $+ X$ showing that, even at the low range of k covered by those data, fragments heavier than tritons also show evidence of quasi-two-body *scaling*.

Since we are considering small-momentum-

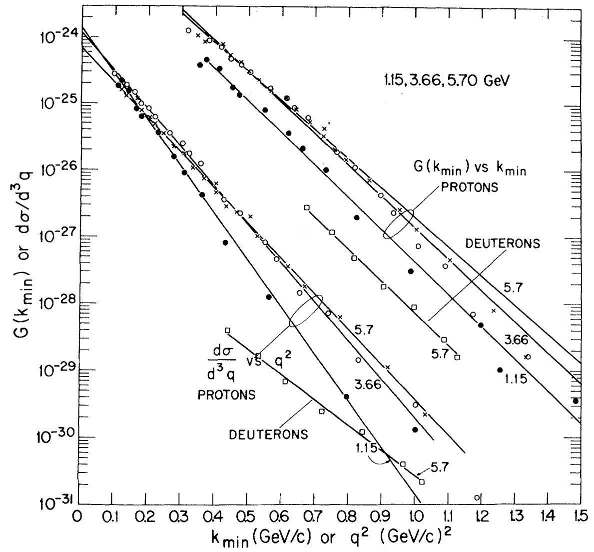

FIG. 3. $G(k_{\min})$ vs $k_{\min}$: $p+\mathrm{C}\to(p,d)+X$ (137°, Ref. 8). The deuteron ordinates are arbitrary.

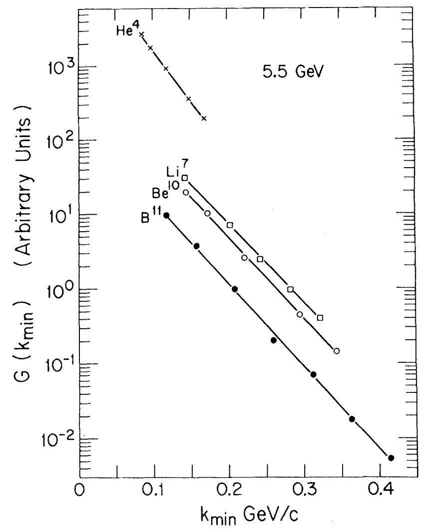

FIG. 5. $G(k_{\min})$ vs $k_{\min}$: $p+\mathrm{U}\to(\mathrm{He,Li,Be,B})+X$ (90°, Ref. 10). Note that the upper range of $k_{\min}$ is much lower for these data than for the data in the previous figures.

transfer reactions by the single-scattering mechanism, we might expect that projectiles other than protons (deuterons and α particles) would be just as capable of ejecting quasifree nucleons. A recent collaboration[11] provides experimental evidence for this view in the study of the reaction $(p, d, \alpha)+A\to p+X$ at 180°. We note that in the production of particles by (p, d, α) collisions at 90° Eq. (1) predicts that high-momentum projectiles of the same momentum will give identically shaped cross sections for any fragment. This is borne out beautifully by the data of Zebelman *et al.*[12] who showed that, for incident protons and

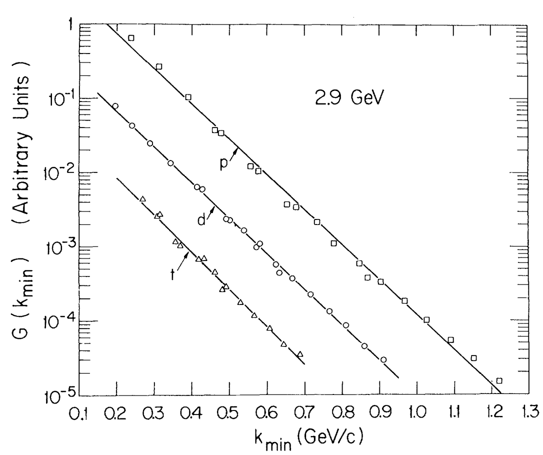

FIG. 4. $G(k_{\min})$ vs $k_{\min}$: $p+\mathrm{Pt}\to(p,d,t)+X$ (93°, Ref. 9). The ordinate scale is arbitrary.

deuterons of the same momentum, the ratios of $d\sigma/d^3q$ for all values of q and for a large variety of fragments are astonishingly q independent. They also point out that this is not the case for their α-particle data. Since their α particles had twice the momentum of the p or d, the cross-section ratios should not be q independent. The independence on projectile type is restored if one compares $G(k)$ for all three projectiles!

We now turn to $F(k)$. $G(k)$ appears to follow the form $G(k)=\exp(-k/k_0)$ so that $F(k)=\exp(-k/k_0)/k$, which conforms with the suggestion of Amado and Woloshyn.[13] More precise data and calculations would be needed to establish the precise power of k appearing in the denominator. The high-momentum data of Figs. 2–4 are consistent with $k_0\cong 80$ MeV/c. (The fragment data of Fig. 5 are in the "Fermi-gas" region and should fall off much more steeply.)

We next remark that once it is understood that the high-momentum, internal momentum distribution is exponential in k rather than Gaussian (as obtained either from a Fermi-gas model at finite temperature, or from a naive interpretation of the observed differential cross sections, confusing q with k), one can understand why k_0 is essentially independent of fragment mass. Whether one treats the interaction as the interaction of a projectile with a "quasifragment" inside the nucleus or treat (R. Amado, private communication) the process as the pickup of nucleons by the target nucleon to form fragments, the *exponential* nature of $F(k)$ produces the observed independence of k_0 on the mass of the ejected fragment.

Finally, we note that in the subthreshold production of antiprotons in p-nucleus collisions, Piroue and Smith[9] were led unalterably, a decade ago, to the need for an exponential form for $F(k)$. W. Frati and this author,[14] by refitting the antiproton data with $F(k)=\exp(k/k_0)/k$ and using the proper effective energy of the struck nucleon, find the value $k_0 \cong 95$ MeV/c. This provides another direct confirmation of the single-scattering hypothesis.

We conclude that "quasi-two-body scaling" gives a simple representation of a wide body of experimental data. The analyses are consistent with the hypothesis that the interactions that produce backward particles can be represented by the single scattering from an effective high-momentum distribution $F(k)=\exp(-k/k_0)/k$.

I wish to take this opportunity to thank Dr. William Frati for his splendid collaboration in our experimental work and for his encouragement and discussions during the course of this attempt to understand high-momentum distributions in nuclei. I thank D. Yang for his enthusiastic help with the computations, and R. Woloshyn and R. Amado for spirited discussions.

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[1]See, e.g., J. D. Walecka, in *High Energy Physics and Nuclear Structure* (Plenum, New York, 1970).

[2]E. J. Moniz *et al.*, Phys. Rev. Lett. 26, 445 (1971).

[3]R. D. Amado and R. M. Woloshyn, Phys. Rev. Lett. 36, 1435 (1976).

[4]m_p is the projectile mass and m_Q and Q are the mass and mass number, respectively, of the detected nucleus in the generalization of Eq. (1) to nuclear projectiles and fragments.

[5]R. M. Woloshyn obtains, for elastic scattering,

$$C=\frac{s(s-4m^2)}{32\pi^2 pmE_q}\frac{d\sigma(k_{\min}\to q)}{dt}.$$

We assume that neutron- and proton-momentum distributions in nuclear matter are identical; the normalization constant $N_k=\int_0^\infty F(k)\,d^3k$ depends mainly on the low-momentum behavior of $F(k)$ and hence on A.

[6]S. Frankel *et al.*, Phys. Rev. Lett. 36, 642 (1976).

[7]The absolute rates per nucleon are not A independent since d and t production do not involve single interactions.

[8]Yu D. Bayukov *et al.*, Yad. Fiz. 19, 1266 (1977) [Sov. J. Nucl. Phys. 19, 648 (1974)].

[9]P. A. Piroue and A. J. Smith, Phys. Rev. 148, 1315 (1966).

[10]A. M. Poskanzer, G. W. Butler, and E. K. Hyde, Phys. Rev. C 3, 882 (1971).

[11]H. Brody, S. Frankel, W. Frati, D. Yang, C. Perdrisat, J. Comiso, and K. Ziock, "180° Production of Protons in High Energy p, d, and α Reactions in Nuclei" (unpublished).

[12]A. M. Zebelman *et al.*, Phys. Rev. C 11, 1280 (1975).

[13]R. D. Amado and R. M. Woloshyn, Phys. Lett. 62B, 253 (1976).

[14]S. Frankel and W. Frati, to be published.

Sub-Coulomb Resonances in the ^{12}C-^{12}C System through the Reaction ^{12}C(^{12}C, ^{16}O)^{8}Be

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The excitation functions of the reaction ^{12}C(^{12}C, ^{16}O(g.s.)) ^{8}Be(g.s.) were measured at 35° lab (~90° c.m.) and at 20° lab (~50° c.m.) near and above the Coulomb barrier ($5.75 \leq E_{\rm c.m.} \leq 8.9$ MeV). Resonant structures were found at 5.94, 6.35, 6.69, 8.0, and 8.4 MeV. These resonant structures were correlated with those in the elastic channel. The angular distributions at 6.4, 6.65, and 8.0 MeV were measured, and the spin-parity assignments are 2^+, 2^+, and 4^+, respectively. The oxygen partial width at 5.94 MeV was determined to be 14 keV, which is as large as the carbon partial width.

Since the resonant structures in the ^{12}C-^{12}C system were first observed near and below the Coulomb barrier,[1] several experiments have been performed[2] and new information on this system has been accumulated. Theoretical work also has been forwarded to understand these structures.

In the simplest picture these resonances are represented as shape resonances in a potential well. Recent attempts to apply this picture to the ^{12}C-^{12}C and ^{12}C-^{16}O systems were reported by Park, Scheid, and Greiner[3] and by Nagorcka and Newton.[4] In other pictures, doorway-state mod-