Localized Structures and Anomalous dc Resistivity*

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(Received 6 December 1976; revised manuscript received 28 April 1977)

A new mechanism is presented for anomalous dc resistivity when the electron drift velocity is close to the electron thermal velocity and $T_e \gg T_i$. The turbulent plasma can break up into homogeneous essentially field-free regions (of width $\gtrsim 60\lambda_{De}$) separated by ion density spikes ($\delta n_i \simeq n_i$) and potential jumps ($e\Delta\varphi/kT \geq 1$) (double layers). dc resistivity results from the thermalization via an electron-electron two-stream instability of the energy gained by electrons accelerated over a potential jump. No runaway electrons are produced in this process.

Ion acoustic waves are unstable in a plasma if the electron drift velocity V_d is above the sound velocity, $C_s = (kT_e/M)^{1/2}$ and the electron- to ion-temperature ratio is high enough. The plasma is unstable to an electron-ion two-stream instability[2] if $V_d > 1.3v_{th,e}$, where $v_{th,e} = (kT_e/m)^{1/2}$. Theory,[3,4] particle simulation calculations,[5] and experiments[6] indicate that the waves are associated with an anomalously high resistivity. The theoretical predictions for the anomalous resistivity vary over a wide range depending on the nonlinear mechansim assumed responsible. Detailed comparisons of theory and experiment have not been possible due to the lack of crucial data. However, there is one striking discrepancy between theory and experiment. Theory predicts and previous simulations show a large fraction of runaway (freely accelerated) electrons, which are not observed in experiments with fields well above the runaway field.[7]

Another characteristic of current-carrying plasmas is the creation of double layers.[8] When the current density exceeds a certain critical value, one or more double layers[9] are formed in the plasma. These double layers are sheaths where potential jumps exist and quasineutrality is violated. The double layers are usually stationary. Clearly this rather unusual plasma state should affect the plasma resistivity. Theory[10] and experiments[11] have shown that the threshold electron drift velocity for double-layer formation is roughly the electron thermal velocity, i.e., $V_d \gtrsim v_{th,e}$.

We have investigated anomalous dc resistivity for $V_d \approx v_{th,e}$ using one- and two-dimensional[12] cloud-in-cell particle simulation codes. We find that under certain conditions propagating, localized density spikes and potential jumps (double layers) are formed in the plasma. The conditions are (1) $T_e \gg T_i$ and (2) $V_d/v_{th,e}$ must remain $\gtrsim 1$ during the formation time. Ion turbulence which is also formed under these conditions can prevent the formation of the localized structures. This is because ion turbulence can itself result in strong anomalous resistivity which can reduce $V_d/v_{th,e}$ to < 1 before the structures are formed. However, if the plasma is created with large-amplitude density perturbations,[13] the structures grow at the locations of density minima (Fig. 1) before the ion turbulence is formed. Lutsenko[14] observed that propagating double layers were triggered in a current-carrying discharge by reducing the local plasma density in agreement with our results. The structures are formed in one-dimensional (1D) simulations (Fig. 2) of initially homogeneous plasma, where the resistivity induced by ion acoustic waves is very small.[3] They are not

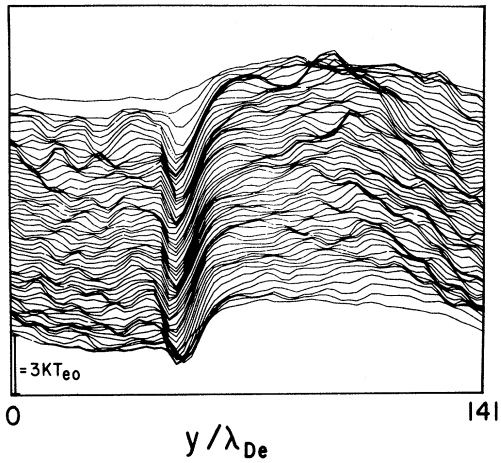

FIG. 1. Self-consistent potential surfaces (time averaged over one plasma period) at $t\omega_{pe}=144$. The plasma is intialized with a density depression (width $=18\lambda_{De}$ and depth, $\delta n/n_0=0.3$). A double layer has formed in the density depression. From a 2D simulation with parameters: $V_d/v_{th,e}=1.4$, $L_x=L_y=256$ cells $=141\lambda_{De}$, $M/m=100$, $T_e=T_i=20$, $N_e=N_i=157$ K, and $\Delta t\omega_{pe}=0.35$. The system is periodic in the drift (y) direction and has conducting particle–reflecting transverse boundaries.

formed in initially homogeneous 2D simulations. However, they are formed if the electrons are cooled so that $V_d/v_{th,e}$ remains nearly constant (Figs. 3 and 4). The cooling algorithm involved replacement of every electron velocity as follows: $v \leftarrow (v-V_d)\alpha+V_d$, where α was usually 0.98. This algorithm was activated twice per plasma period if the spatially averaged electron temperature exceeded $1.2T_{e0}$, where T_{e0} is the initial electron

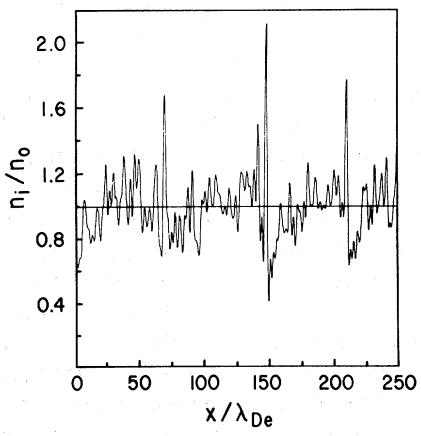

FIG. 2. Ion density at $t\omega_{pe}=236$. Three density spikes—double layers have formed. From a 1D simulation with parameters: $V_d/v_{th,e}=1.4$, $L=1024$ cells $=256\lambda_{De}$, $M/m=100$, $T_e/T_i=100$, $N_e/N_i=32$ K, and $\Delta t\omega_{pe}=0.2$.

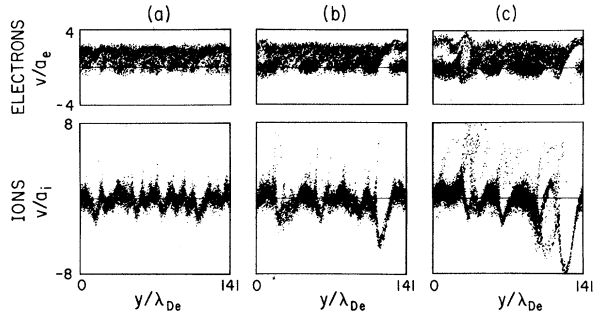

FIG. 3. Time history of electron and ion phase in the direction of the drift [y, v_y/a_α; where $a_\alpha=(2kT_{\alpha 0}/m_\alpha)^{1/2}$ with α standing for electrons or ions]. From a 2D simulation with parameters: $V_d/v_{th,e}=1.4$, $L_x=L_y=256$ cells $=141\lambda_{De}$, $M/m=1000$, $T_e/T_i=20$, $N_e=N_i=157$ K, and $\Delta t\omega_{pe}=0.5$. The x direction is divided into sixteen segments (each of width $9\lambda_{De}$), and the ninth segment is shown at (a) $t\omega_{pe}=540$, (b) $t\omega_{pe}=780$, and (c) $t\omega_{pe}=1260$. A double layer is forming in (a). The resistivity is maximum at a time close to (c).

temperature. These calculations model experiments[15] in which a steady state is obtained (however, a steady state is not obtained in all experiments[16]). This steady state results if the electron heating due to anomalous resistivity is balanced by electron heat loss (due to convection or electron-neutral collisions).

In addition to these physical conditions for the formation of the structures, there are numerical conditions on the simulation calculations. The

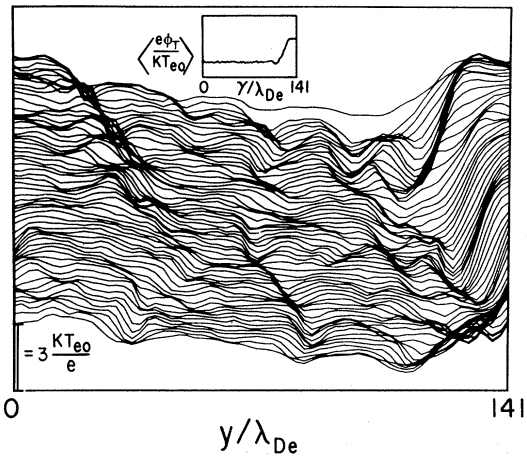

FIG. 4. Self-consistent potential surfaces (time averaged over one plasma period) at $t\omega_{pe}=1225$ from the 2D simulations shown in Fig. 3. The x-averaged total potential (sum of self-consistent plus external) is shown schematically in the inset.

cell size, λ_c, must be small enough to resolve the structures ($\lambda_{De}/\lambda_c > 2$). In addition, the length of the simulation plasma must be longer than the structure spacing ($\simeq 60\lambda_{De}$ for $V_d/v_{th,e}=1$ and $M/m=100$).

Once formed, the double layers induce a large anomalous resistivity. The external potential drop which drives the current is almost shielded out of the plasma and appears almost entirely in the potential jumps. Electrons on the low-potential side are accelerated over the jump. These electrons form a beam and interact with electrons trapped on the high-potential side via the e-e two-stream instability. Since the beam density is comparable with the trapped electron density, the two-stream instability thermalizes the beam energy in a very short distance ($\lesssim 60\lambda_{De}$).

The resulting anomalous dc resistivity can be understood with use of a simple model. The electrons accelerated over the jump gain a directed energy of $e\Delta\varphi$, where $\Delta\varphi$ is the change in potential across the jump. This energy is thermalized by the two-stream instability. Consider a plasma of length L which has N double layers, where $N=L/\lambda_s$ and λ_s is the double-layer spacing. The energy thermalized in a distance L is just $e\Delta\varphi N$. We can obtain an effective collision frequency by noting that, if energy was absorbed uniformly in space, the energy gained per particle in a length L would be $\approx mV_d^2\nu_{eff}L/V_d$, where ν_{eff} is the effective collision frequency. Equating this expression to $e\Delta\varphi N$ results in

$$\frac{\nu_{eff}}{\omega_{pe}}=\frac{e\Delta\varphi}{kT_e}\frac{v_{th,e}}{v_d}\frac{\lambda_{De}}{\lambda_s}. \quad (1)$$

In general, this relation is confirmed by the simulations. Double layers are not formed at low drift velocities ($V_d/v_{th,e}\lesssim 0.8$), and the resistivity is low (see Fig. 5, $T_e/T_i=20$). As the drift velocity is increased from $V_d/v_{th,e}=1$ to $V_d/v_{th,e}=1.4$, the double-layer spacing decreases and the potential-jump amplitude increases so that the effective collision frequency increases rapidly [Eq. (1)]. The effective collision frequency also *increased with* mass ratio ($\nu_{eff}/\omega_{pe}=0.022$ for $M/m=100$ to 0.035 for $M/m=1000$, i.e., the case shown in Figs. 3 and 4). 1D simulations give much lower resistivity values. If $T_e=T_i$, double layers are not formed and the 1D and 2D simulations agree quite well (Fig. 5).

In the 2D calculations the double layers are formed randomly in the direction perpendicular to the drift, but they finally line up to make a structure somewhat like a cliff (Fig. 4). The resistivity also maximizes when the cliff is formed. In the 2D simulations with an infinite magnetic field parallel to the drift, they are formed randomly as before, but they do not line up. The resistivity was 20% less. No runaway electrons were observed in the simulations.

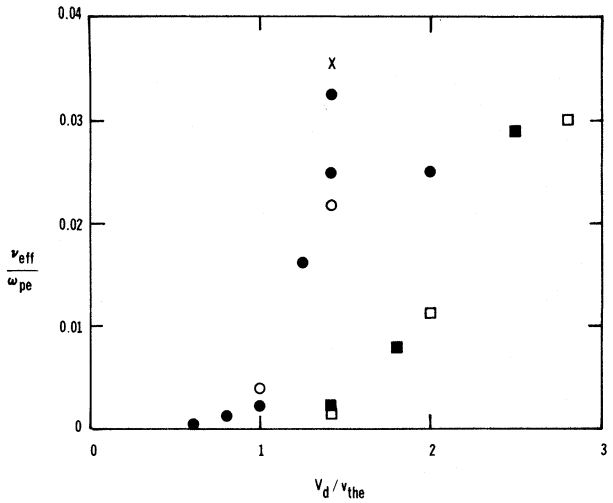

FIG. 5. Simulation results for the maximum effective collision frequency. The effective collision frequency is given by $\nu_{eff}\equiv -eE_e/mV_d$, where E_e is the external electric field required to maintain constant current. Points are from 2D simulations with $T_e/T_i=20$ and $L=50\lambda_{De}$ (●), $L=141\lambda_{De}$ (○ and ×). ■ and □ are for 1D and 2D simulations with $T_e/T_i=1$, respectively. $M/m=100$ for all points except × where $M/m=1000$. The maximum resistivity depends on the double layers lining up. The alignment was complete for the larges ν_{eff} shown (×). The spread in values at $V_d/v_{th,e}=1.4$ (○, ●) is due to incomplete alignment. The collision frequency due to the numerical model is less than any shown.

In summary, localized density and potential structures can form in a current-carrying plasma. If $V_d > v_{th,e}$ and $T_e \gg T_i$ these structures are formed from large-amplitude density perturbations. They are also formed, if $V_d > V_{th,e}$, $T_e \gg T_i$, and the electron temperature is evolving on a slow enough time scale. Electrons accelerated over a potential jump interact with the plasma electrons on the high-potential side of the jump and are thermalized via two-stream instabilities. No runaway electrons are produced in this process.

*Work supported by Lawrence Livermore Laboratories under Intramural Orders No. 9037305 and No.

3233503 and U. S. Energy Research and Development Administration Contract No. E(04-3)-326.

†Also with Lawrence Livermore Laboratory.

‡Hertz predoctoral fellow.

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Orientational Epitaxy—the Orientational Ordering of Incommensurate Structures

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(Received 28 March 1977)

A new class of structures is predicted to exist for monolayer films on solid surfaces. These structures involve two incommensurate lattices—the monolayer lattice and the surface lattice— which have a preferred relative orientation. The precise orientation depends upon the lattice constant and symmetry of each lattice. We believe this orientationally ordered incommensurate phase to be present in many physisorbed films. In particular, the existence of this phase appears to explain recent low-energy electron diffraction data for rare gases adsorbed on homogeneous substrates.

The growth of crystals on dissimilar substrates plays an important role in microelectronic device technology, and is also of considerable scientific interest. We discuss here aspects of the interactions of a monolayer film on a crystalline substrate and demonstrate the existence of a new class of epitaxy involving the relative orientations of substrate and adsorbate crystal axes for substances whose lattice parameters are mutually incommensurate. It is shown that, even for incommensurate systems, the ground state of the composite system is characterized by a definite relative orientation which is, in general, not a symmetry angle.

In many cases involving two crystals of differing bulk lattice parameters, distortions at the interface cause the crystals to be in registry.[1] That is, they have locally commensurate lattice parameters and relative orientations determined by symmetry. These results appear to obtain when adsorbate-substrate and adsorbate-adsorbate interactions are of comparable strength. There are, however, examples with weak adsorbate-substrate interactions in which the lattice parameters are incommensurate and the orientation is unrelated to symmetry. A well-stud-