have intensities of approximately 1% of the intensity of the main line and the fourth-order satellites appear to be slightly stronger, especially the S_{2-} transition which is almost 2% of the intensity of the allowed line. Finally, these fourth-order intensities disagree with the ones predicted by the perturbation theory if we use the field of 3 kV/cm in the calculation, revealing the inconsistencies of that model in the solution of this problem in turbulent plasmas.

We believe that our observations of Stark broadening in a turbulent plasma for the case of helium have shown the complicated nature of this problem and the failure of presently available theoretical calculations. Additional investigations along the lines given by Nee and Griem[6,7] would be of interest since further understanding of this problem is of great importance because of its applicability to the detection and classification of high-frequency fields in turbulent plasmas.

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Theory of the Glory*

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A new theory of the optical glory is given. The results are in good agreement with the exact Mie solution. The physical effects responsible for the glory are explained.

The optical glory, a meteorological effect first reported[1] in 1735, is a strong enhancement in the backscattering of light from clouds, often accompanied by colored rings. It arises from individual water droplets, with size parameters $\beta = ka$ (k is the wave number and a is the droplet radius) usually[2] in the range 10^2–10^3. Exact Mie partial-wave series summed by computer in this range show[3] that the backscattered intensity, as a function of β, undergoes very complicated, rapidly varying, quasiperiodic fluctuations, with a quasiperiod $\Delta\beta \approx 0.815$ (cf., Fig. 3), that have also been observed experimentally.[4] Since the number of partial waves that contribute is $\gtrsim \beta$, such results provide no answer to the question we address here: What physical effects are responsible for the glory?

We apply asymptotic techniques first developed for a scalar field[5] and later extended to electromagnetic waves.[6] For $\theta = \pi$, the scattering amplitudes $S_1(\beta,\theta)$ and $S_2(\beta,\theta)$ are given by[7]

$$S_1(\beta,\pi) = S^M(\beta) + S^E(\beta) = -S_2(\beta,\pi), \qquad (1)$$

where the contributions from magnetic multipoles (M) and electric multipoles (E) are of the same order (cross-polarization effect). We start from the Debye multiple-reflection expansion[5]

$$S_j(\beta,\theta)=\sum_{p=0}^{\infty}S_{j,p}(\beta,\theta),\quad (j=1,2) \tag{2}$$

where $S_{j,p}$ is associated with waves that undergo $p-1$ internal reflections (only external reflection for $p=0$). We must (a) determine which p values yield significant contributions at a given θ and (b) find suitable asymptotic expansions for these contributions. Both problems are solved by applying the modified Watson transformation[5] to each Debye term. The following types of contributions are found: (i) geometrical-optic contributions, associated with isolated real saddle points in the λ plane (λ is the complex angular momentum); (ii) surface waves, associated with complex Regge-Debye poles; (iii) rainbow terms, associated with confluent saddle points[5,8]; (iv) Fock-type contributions, interpolating smoothly between the first two types.[5]

For the refractive index $N\approx 1.33$ of water, geometrical-optic contributions are far too small to account for the glory.[3,5] Van de Hulst[2] conjectured that surface waves for $p=2$ are dominant.

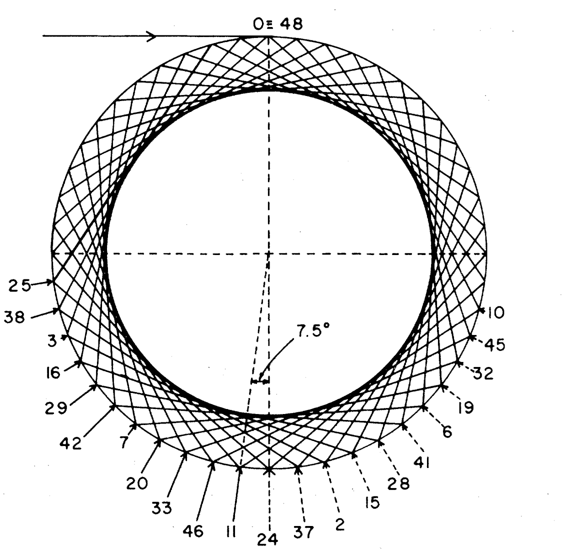

FIG. 1. Path of a tangentially incident ray for $N=[\cos(11\pi/48)]^{-1}\approx 1.330\,07$. The values of p for the dominant Debye terms are indicated next to the arrows (—, rainbow terms; ----, surface-wave terms). The directions of the arrows determine the corresponding values of ζ_p; e.g., $\zeta_{11}=-7.5°$. The ordering by increasing ζ_p (surface waves) or $|\zeta_p|/p$ (rainbow terms) is indicated by the lengths of the arrows.

It was shown, however,[5] that higher-order Debye terms must play an important role.

For $p\gg 1$, all relevant contributions arise from the edge domain $|\lambda-\beta|\lesssim\beta^{1/3}$, where the reflectivity is close to unity. Besides surface-wave contributions, those due to higher-order rainbows formed near $\theta=\pi$ are also important.[6] This is due to the usual rainbow enhancement[5,8] by $\beta^{1/6}$, which persists at considerable deviations from the rainbow angle $\theta_{R,p}$ because the width of the rainbow region increases with p. The backward direction falls within the dark side of the relevant rainbows, where the amplitude is exponentially damped, as it is also for the surface-wave contributions. The damping exponent is roughly proportional to $\beta(|\zeta_p|/p)^{3/2}$ for the large-p rainbow terms and to $\beta^{1/3}\zeta_p$ for the surface-wave terms, where $\zeta_p\equiv\pi-p\theta_t$ (mod2π), $-\pi\leqslant\zeta_p<\pi$, $\theta_t=2\times\cos^{-1}(1/N)$. Positive ζ_p values represent the angle described by a surface wave before emerging at $\theta=\pi$; negative ζ_p, for the rainbow terms, represent approximately the deviation $\epsilon_{R,p}=\pi-\theta_{R,p}$ from the rainbow angle. We expect, therefore, that the dominant Debye contributions arise from values of p satisfying $-\theta_t\leqslant\zeta_p<\theta_t$, and that the lowest ζ_p are dominant for surface waves, whereas the lowest $|\zeta_p|/p$ dominate for rainbow terms.

We have made numerical comparisons with the exact results[9] for $N=[\cos(11\pi/48)]^{-1}\approx 1.330\,07$. For this N, $\zeta_{24}=0$, so that a tangentially incident ray forms a closed path, a regular 48-sided star-shaped polygon[10] inscribed within the droplet.

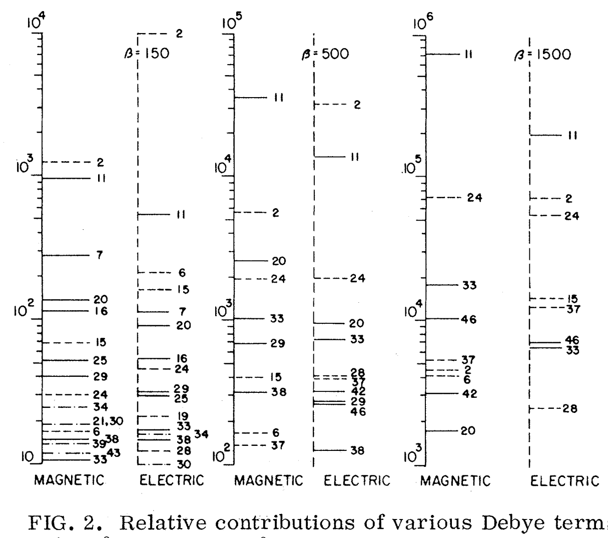

FIG. 2. Relative contributions of various Debye terms to $|S^M(\beta)|^2$ and to $|S^E(\beta)|^2$: —, rainbow terms; ----, surface-wave terms. For $\beta=150$, there appear some terms not present in Fig. 1 (shown by -·-·-).

TABLE I. Exact $S_{11}^{M}(\beta,\pi)$ compared with rainbow term.

β	Exact	Asymptotic
1500.1	$-333-794i$	$-304-823i$
1500.2	$860-43i$	$878-8i$
1500.3	$-249+824i$	$-289+829i$
1500.4	$-690-514i$	$-681-553i$
1500.5	$719-474i$	$752-454i$

The lowest values of p satisfying the above criteria are indicated in Fig. 1; further values are obtained by adding multiples of the period $\Delta p=48$. Figure 2 shows the contributions from Debye terms that contribute up to ~0.1% to $|S^M(\beta)|^2$ and $|S^E(\beta)|^2$ for $\beta=150$, 500, and 1500. Both the dominant p values and their relative ordering generally agree with our expectations. Discrepancies are due to factors not included in the above discussion: (a) The damping at each internal reflection, with damping exponent proportional to $p\beta^{1/3}$, favors lower p values, especially at lower β. (b) For low β, the transition angular regions become wide, and damping is less effective at low p. (c) The difference between $\epsilon_{R,p}$ and $|\zeta_p|$ is significant at low p; e.g., $|\zeta_{11}|=7.5°$, but $\epsilon_{R,11}\approx 3°$, making $p=11$ the leading rainbow term.

The validity of our physical interpretation of the dominant terms follows from their asymptotic representation. We illustrate this for S^M near $\beta=1500$. The leading rainbow term $p=11$ is treated by the Chester-Friedman-Ursell method, previously described[8] for $p=2$. The results are compared with the exact partial-wave sum in Table I. The leading surface-wave term for this β is $p=24$. Since $\zeta_{24}=0$, we get an asymptotic representation in terms of Fock-type functions.[6] The results are compared with the exact ones in Table II. In both cases, our physical interpretation is confirmed.

TABLE II. Exact $S_{24}^{M}(\beta,\pi)$ compared with surface-wave term.[a]

β	Exact	Asymptotic
1500.1	$183-231i$	$214-193i$
1500.2	$-250-102i$	$-245-111i$
1500.3	$52+244i$	$38+255i$
1500.4	$205-187i$	$229-178i$
1500.5	$-254-117i$	$-236-131i$

[a] Both exact and asymptotic results include summation over $\Delta p=48$.

The dominant Debye terms undergo a phase change of $\approx\pi$ for $\Delta\beta=5\pi/[22(N^2-1)^{1/2}]\approx 0.814$,

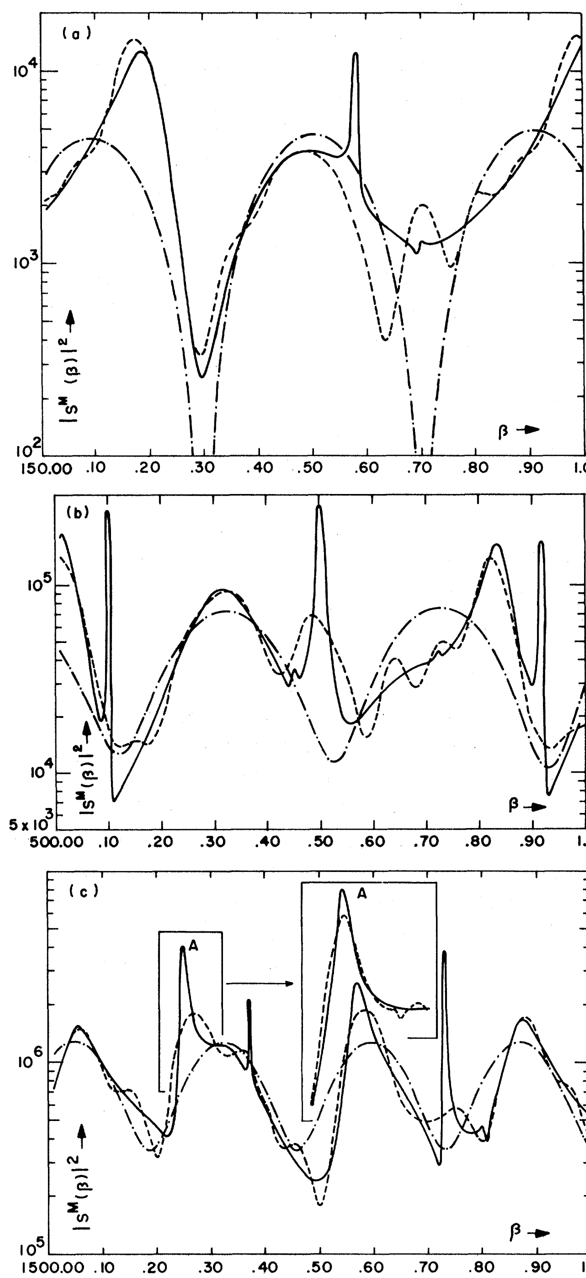

FIG. 3. Behavior of $|S^M(\beta)|^2$ within a quasiperiod: —, exact; -·-·-, contribution from the two leading Debye terms in Fig. 2; ----, contributions from all Debye terms in Fig. 1 (without summation over $\Delta p=48$): (a) near $\beta=150$; (b) near $\beta=500$; (c) near $\beta=1500$. The inset shows the effect of summation over $\Delta p=48$ on the approach to the spike marked A: —, exact; ----, contribution from all Debye terms in Fig. 1 summed over $\Delta p=48$.

in excellent agreement with the observed quasiperiod. The behavior of $|S^M(\beta)|^2$ within a quasiperiod near $\beta = 150$, 500, and 1500 is shown in Fig. 3. The main humps are seen to result from interference between the leading rainbow term and the leading surface-wave term. Inclusion of all terms indicated in Fig. 1 improves the agreement with the exact results, but still does not reproduce the sharp superimposed spikes. However, when one carries out the summation over $\Delta p = 48$, they are recovered [Fig. 3(c), inset]. Thus, the spikes represent "geometrical resonances" associated with the quasiperiodic orbits with $\Delta p = 48$. They occur at different β for S^M and S^E; the total intensity $|S_1(\beta, \pi)|^2$ shows further structure due to interference between S^M and S^E.

For $u = \beta(\pi - \theta)$ not $\gg 1$ (first few glory rings), we have

$$S_1(\beta, \theta) \approx 2S^M(\beta) J_1'(u) + 2S^E(\beta) J_1(u)/u, \qquad (3)$$

and $-S_2(\beta, \theta)$ is obtained by interchanging M and E. The angular distribution and polarization of the glory rings[2,11] can be understood in terms of these results. Detailed discussions will be given elsewhere.

We conclude that the glory arises from the following physical effects conjointly: (i) incidence in the edge domain (near the top of the centrifugal barrier); (ii) the consequent almost total internal reflection, leading to many Debye contributions (an effect related with "orbiting"); (iii) enhancement by axial focusing[2,5]; (iv) the cross-polarization effect; (v) surface-wave contributions from Regge-Debye poles; (vi) complex rays in the shadow of higher-order rainbows formed near $\theta = \pi$; (vii) geometrical resonances associated with closed or nearly closed quasiperiodic orbits; and (viii) the competition among various kinds of damping—radiation damping of surface waves, the damping of complex rays in the shadow of a caustic (rainbow), and damping by internal reflection. The glory represents a new domain in optics, where complex orbits give rise to dominant effects, suggesting that complex extremals of Feynman path integrals may play an important role under more general conditions.

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