A final remark is in order. My solutions possess finite action and finite $F_{\mu\nu}{}^a$, but in the gauge in which I am working, the four-dimensional gauge field $A_\mu{}^a$ is actually singular at $r=0$. This is obvious from (1), where I see that the field is nonsingular only if $\varphi_2=1$ and $\varphi_1=0$ at $r=0$. It is necessary to perform a gauge transformation on the solutions to satisfy these conditions; such a transformation always exists because I have $\varphi^2=1$ at $r=0$. In the language of (6), (8), and (9), a suitable gauge function is

$$h=-i\prod_{i=1}^{k}(a_i{}^*+z)^2.$$

Thus,

$$\psi=\ln\frac{2r}{(1-g^*g)(h^*h)^{1/2}}$$

and

$$\varphi_1-i\varphi_2=h\frac{dg}{dz}e^\psi$$

give nonsingular four-dimensional gauge fields that satisfy the equations of motion.

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Where is the Dip Structure in pp Elastic Scattering?

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The structureless $\exp(1.8t)$ behavior of recent pp data for $|t|\gtrsim 2$ GeV^2, $\sqrt{s}=53$ GeV, is shown to be in sharp disagreement with dip-structure predictions from popular models of diffraction. Flip amplitudes, real parts, and large-angle effects are quantitatively insufficient to resolve the discrepancy. Modifications of some familiar ideas on diffraction (like eikonalization) seem necessary.

Recently, the CERN-Hamburg-Orsay-Vienna (CHOV) collaboration published accurate results on pp elastic scattering at $\sqrt{s}=53$ GeV extending out to $|t|\simeq 9$ GeV^2.[1] This is a substantial extension of previous results, which were limited to $|t|\lesssim 3$ GeV^2.[2] The purpose of this Letter is to show how the t dependence of the new data necessitates modification of current ideas on diffraction scattering.

The CHOV data have two noteworthy features (see Fig. 1): (i) $d\sigma/dt$ has the well-known dip at $|t|\simeq 1.3$ GeV^2, but there is *no additional second dip* below $|t|\simeq 7$ GeV^2. (ii) The data are essentially *a structureless exponential* beyond the first maximum (at $|t|\simeq 2$ GeV^2) with a slope $B_2=1.8$ GeV^{-2}—considerably smaller than a typical slope $B_1\simeq 12$ GeV^{-2} in the forward peak.

The above features are in sharp conflict with the expectations of currently popular models[3] of high-energy diffraction scattering (which I shall also refer to as the Pomeron). I now demonstrate this disagreement by looking at various models which have some physical basis and more or less agree with previous ($|t|\lesssim 3$ GeV^2) data.

The pp elastic amplitude is customarily given by[3]

$$A(s,t)=P(s,t)+C(s,t). \quad (1)$$

The Pomeron contribution $P(s,t)$ is approximately pure imaginary, dominates at small angles, and contains dip structure. The large-angle contribution $C(s,t)$ is smoothly behaved (no dips) and approximately real for pp scattering. The real phase is established by use of dispersion relations[4] or derivative analyticity relations.[5] It can be understood in the duality framework, since the pp channel is exotic. Since P and C are approximately out of phase, no significant interference occurs and $d\sigma/dt\propto P^2+C^2$.

Most current Pomeron models have a single, pure imaginary amplitude and can be roughly classified into two categories.

(A) Models with an s-channel viewpoint.—Such models are usually described by the amplitude

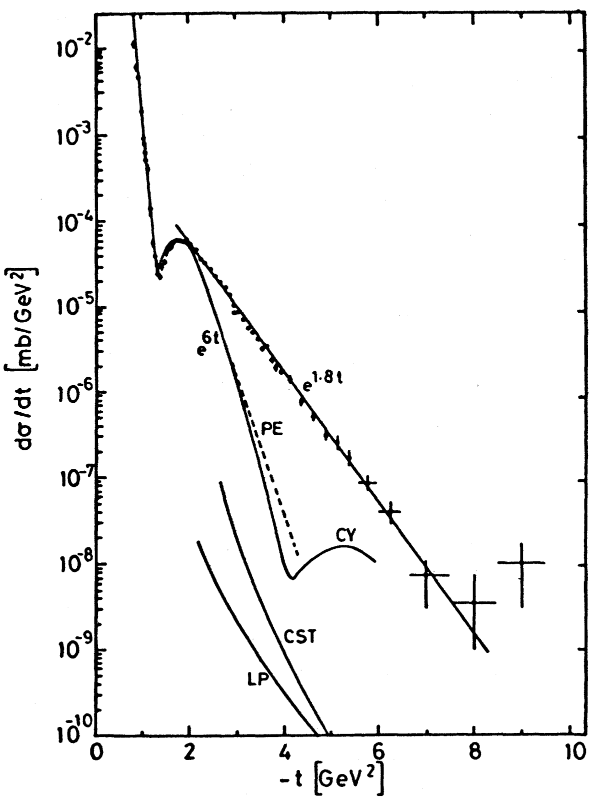

FIG. 1. Comparison of theoretical models and pp data from the CHOV collaboration. The expectations of two popular models of diffraction are labeled CY (Chou-Yang) and PE (Pomeron exchange). Curves corresponding to two successful large-angle models are labeled LP (Landshoff-Polkinghorne) and CST (Coon-Sukhatme-Tran).

$g(s, b)$ in impact-parameter space (since unitarity becomes simple). They primarily give information about the t dependence of $d\sigma/dt$ and have flexible or *ad hoc* energy dependence.

(i) Optical or geometrical models: These typically predict a diffraction pattern corresponding to zeros of $J_1(R\sqrt{-t})/R\sqrt{-t}$. $R=0.66$ fm gives the observed first dip at $|t|\simeq1.3$ GeV^2. Additional dips are also predicted, in particular a second dip at $|t|\simeq4.4$ GeV^2.

(ii) Eikonal models: The basic theoretical input is the eikonal $\chi(s, b)$. The amplitude is obtained by "eikonalization":

$$g(s, b)=1-\exp(-\chi)=\chi-\tfrac{1}{2}\chi^2+\tfrac{1}{6}\chi^3-\dots. \quad (2)$$

This prescription can be derived by summing the generalized ladder diagrams of field theory. In most physically motivated models (e.g., Cheng-Wu,[6] Chou-Yang,[7,8] Regge eikonal[9]), the eikonal by itself describes the forward peak of $d\sigma/dt$ well, but contains no additional dip structure. I call such models "conventional." The alternating signs in Eq. (2) generate dip structure. The slope B_2 beyond the first maximum comes mainly from the $\frac{1}{2}\chi^2$ term. To describe the forward exponential peak, χ must be approximately Gaussian,

$$\chi(b)\propto\exp(-b^2/2B_1), \quad B_1\simeq12\ \mathrm{GeV}^{-2}. \quad (3)$$

Therefore, $\frac{1}{2}\chi^2\propto\exp(-b^2/B_1)$ which corresponds to a slope $B_2=\frac{1}{2}B_1\simeq6$ GeV^{-2}—much steeper than the CHOV data (see Fig. 1). The second dip arises roughly by the cancellation of contributions from the $\frac{1}{2}\chi^2$ and $\frac{1}{6}\chi^3$ terms in Eq. (2). For example, in the Chou-Yang model [with eikonal $\chi\propto b^3K_3(\mu b)$ obtained from the dipole form factor] the second dip is at $|t|\simeq4.1$ GeV^2 (see CY curve in Fig. 1). Note the marked disagreement between "conventional" eikonal models and the CHOV data.

(iii) Inelastic overlap-function models: Here, models for inelastic processes give the overlap function $O(s, b)$.[10] Elastic scattering then comes from s-channel unitarity, ignoring Re$g(s, b)$. Attempts using multiperipheral models[11] and a multiparticle extension of the Chou-Yang model[12] have not yet proved successful. Of course, good parametrizations of the overlap function exist[13] which reproduce the CHOV data, but they are physically unmotivated and lack predictive power.

(B) Models emphasizing t-channel exchanges.—These models primarily give energy dependences, and there is considerable arbitrariness in the choice of t-dependent trajectory and residue functions. Let us, for simplicity, choose a linear Pomeron trajectory $\alpha(t)=\alpha_0+\alpha't$ where $\alpha_0\simeq1$ and $\alpha'\simeq0.25$ GeV^{-2}.[14] The Pomeron pole and cut contributions are familiar:

$$A(s, t)=g^2(t)s^{\alpha't}-N^2(t)s^{\alpha't/2}/\ln s + N'^2(t)s^{\alpha't/3}/\ln^2 s-\dots, \quad (4)$$

where g, N, and N' are the vertices coupling protons to one, two, and three Pomerons, respectively. (The following discussion is essentially unaffected even if enhanced Pomeron diagrams are included[15]). The observed exponential forward peak motivates an exponential parametrization of the residue functions:

$$g(t)=g(0)e^{K^2t}, \quad N(t)=N(0)e^{L^2t}, \quad N'(t)=N'(0)e^{L'^2t}. \quad (5)$$

The slopes of the forward peak and beyond the first and second maxima are then approximately given by

$$B_1 = 4R^2 + 2\alpha' \ln s; \quad B_2 = 4L^2 + \tfrac{1}{2}(2\alpha' \ln s);$$
$$B_3 = 4L'^2 + \tfrac{1}{3}(2\alpha' \ln s); \qquad (6)$$

and the dip locations roughly correspond to cancellation of successive terms in Eq. (4).

In principle, the quantities $g(0)$, $N(0)$, $N'(0)$, R^2, L^2, and L'^2 are free parameters, and numerical predictions cannot be made. However, a simple pole approximation for intermediate states in Pomeron vertices is sometimes used[15,16] which gives

$$g(0) = aN(0) = a^2N'(0), \qquad (7a)$$

$$R^2 = 2L^2 = 3L'^2. \qquad (7b)$$

Numerically, $R^2 = 2\ \mathrm{GeV}^{-2}$ gives the observed slope $B_1 \simeq 12\ \mathrm{GeV}^{-2}$ at $\sqrt{s} = 53$ GeV. Then B_2 and B_3 are predicted to be $B_2 = \frac{1}{2}B_1 \simeq 6\ \mathrm{GeV}^{-2}$, $B_3 = \frac{1}{3}B_1 \simeq 4\ \mathrm{GeV}^{-2}$ (see PE curve in Fig. 1). Note that the predicted slope B_2 is again much larger than experiment, presumably indicating that the pole approximation leading to Eq. (7) is naive.

Another approach is the strong-coupling solution of Gribov's Reggeon calculus. Recent calculations in the ϵ expansion are much below the CHOV data,[17] but the applicability of such calculations at $\sqrt{s} \sim 50$ GeV is somewhat doubtful.

To summarize, we see that whenever models of diffraction have predictive power beyond the first maximum, they predict a steep slope (typically $B_2 \simeq \frac{1}{2}B_1 \simeq 6\ \mathrm{GeV}^{-2}$) and a second dip in $d\sigma/dt$ near $|t| \simeq 4\ \mathrm{GeV}^2$—and both these features are simply not present in the CHOV data! Therefore, either the models considered above are wrong and need modifications or, alternatively, the models may be correct, but there exists some extra unconsidered reason responsible for filling in the second dip. In fact, three qualitatively plausible reasons come to mind.

(a) Several scattering amplitudes: Should high-energy pp scattering be described by a single (spin-averaged, non-flip) amplitude? After all, there are five independent amplitudes. In general, they would have different zero locations, and if several of them are of comparable size, dips in $d\sigma/dt$ would be washed out. This could be true, but several arguments suggest that a single amplitude suffices. Firstly, the dip at $|t| \simeq 1.3\ \mathrm{GeV}^2$ *is* seen, so a single-amplitude description seems adequate there. Secondly, large flip amplitudes usually give large polarization effects,[18] whose discovery at $\sqrt{s} \sim 50$ GeV would be a major surprise. Finally, if several amplitudes of comparable size were present, their zeros should produce some oscillatory structure beyond the first maximum, but the data do not show any.

(b) Real part of the Pomeron: Can the second dip be filled in by a substantial real part in the Pomeron amplitude? At high energies, a quick way of including real parts is derivative analyticity relations (DAR)[5]:

$$\mathrm{Re}P \simeq \frac{\pi}{2} \frac{d}{d(\ln s)} \mathrm{Im}P. \qquad (8)$$

This formula correctly gives the depth of the dip at $|t| \simeq 1.3$ GeV.[2] When applied at larger $|t|$, one finds that ReP is *not* sufficiently large to hide a zero in Im P at $|t| \simeq 4\ \mathrm{GeV}^2$. For example, the CY curve in Fig. 1 includes the real part.[19] Conversely, when accurate $d\sigma/dt$ measurements at several energies become available, ReA can be directly extracted using logarithmic DAR.[20] Available (insufficiently accurate) data[21] yield the estimate $\mathrm{Re}A/\mathrm{Im}A = 0.1$ to 0.5 at $|t| = 3\ \mathrm{GeV}^2$. This is also one indication that the large-angle regime has not been reached, since $\mathrm{Re}A = \mathrm{Im}A$ is a good criterion for the boundary between diffraction and large-angle pp scattering.

(c) Large-angle regime: Is there no second dip in the data because $|t| \simeq 4\ \mathrm{GeV}^2$ is in the large-angle regime and the amplitude C swamps the Pomeron? To answer this, I look at two models for C which agree very well with large-angle experiments at $\sqrt{s} \lesssim 8$ GeV, and perform large extrapolations in energy (up to $\sqrt{s} = 53$ GeV) and angle (down to $\theta_{\mathrm{c.m.}} \simeq 5°$ or $-t \simeq 5\ \mathrm{GeV}^2$). A typical model (roughly) in the dimensional counting mold is[22]

$$(d\sigma/dt)_{\mathrm{LP}} = (1.9 \times 10^8)s^{-9.7}(\sin\theta_{\mathrm{c.m.}})^{-14}\ \mathrm{mb/GeV}^2. \qquad (9)$$

This works well for $4 \lesssim \sqrt{s} \lesssim 8$ GeV and $|t| \gtrsim 2.5\ \mathrm{GeV}^2$. At $\sqrt{s} = 53$ GeV, it gives the LP curve in Fig. 1. Other such models[23] yield similar curves. A second type of model is a tachyon-free dual model with logarithmic trajectories,[24]

$$C(t,u) = \frac{c\, q^{\alpha_t \alpha_u} G(1-\alpha_t-\alpha_u)}{G(1-\alpha_t)G(1-\alpha_u)}, \qquad (10)$$

where $\alpha_t \equiv \ln(b-at)/\ln q$. $G(\alpha)$ is the analog of $\Gamma^{-1}(\alpha)$. This generalization of the Veneziano formula provides a remarkably good description of large-angle pp data. At $\sqrt{s} = 53$ GeV, it gives the CST curve in Fig. 1. It is clear that both large-angle models lie 1000 to 4000 times below experiment, indicating that the CHOV data are not in the large-angle regime. Of course, this conclusion is not possible if one questions the validity of extrapolating in energy and angle.

So, to the best of our current knowledge, the conflict between the CHOV data and the predictions of Pomeron models cannot be quantitatively resolved by large-angle effects, flip amplitudes, or real parts. Therefore, of necessity, the CHOV data require modification of current Pomeron models such that $B_2 \simeq 1.8$ GeV^{-2} and there is no second zero at least below $|t| \simeq 7$ GeV^2. Let us see how these requirements affect various types of models.

The discussion surrounding Eq. (3) demonstrates that "conventional" eikonal models, in which eikonalization is the sole source of the dip structure, cannot work. If eikonalization is to be retained, some dip structure must also come from the leading eikonal term itself.

Pomeron-exchange models give $B_2 \simeq 1.8$ GeV^{-2} if the vertex $N(t)$ has almost no t dependence, i.e., $L^2 \simeq 0$. Similarly, taking $L'^2 = 0$, one expects $d\sigma/dt$ to change slope from $B_2 = \alpha' \ln s$ to $B_3 = \frac{2}{3}\alpha' \times \ln s$ as $|t|$ increases. The second dip should appear between these two regions, but since real parts are substantial, only a break in slope may be seen. Its location is model dependent, but taking $N'(0)$ from Eq. (7a) as a rough guide the break lies between $|t| = 7$ and 11 GeV^2, and should be observable in forthcoming experiments at the CERN intersecting storage rings and at Fermilab.

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