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SU(3)⊗U(1) Gauge Theory of the Weak and Electromagnetic Interactions

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We describe an extension of the gauge theory of weak and electromagnetic interactions to SU(3)⊗U(1). The extended theory *naturally* insures universality, absence of right-handed currents in β and muon decay, flavor conservation in neutral currents, etc.; gives good quantitative agreement with observations of neutral currents; and accounts for recently observed trimuon events.

Within the modern unified theory[1] of weak and electromagnetic interactions, those models based on the SU(2)⊗ U(1) gauge group of the original example[2] stand out for the natural way in which they account for observed general features of the weak interactions. The existence of neutral currents makes it clear that any successful model must incorporate SU(2)⊗ U(1) gauge invariance, but it is possible that new observations, such as the recently reported high-energy trimuons,[3] might require SU(2)⊗ U(1) to be embedded in a larger gauge group. It would be premature to conclude from the limited trimuon data that SU(2) ⊗ U(1) must be expanded, but it is worth asking how, if this became necessary, it would be possible to enlarge the gauge group without losing the natural features of the simple SU(2)⊗ U(1) model. We offer an example of such a model.

The gauge group is SU(3)⊗ U(1); the charge is the sum of the SU(3) generator $\frac{1}{2}(\lambda_3+\lambda_8/\sqrt{3})$ plus the U(1) generator y. The quarks of each color[4] and each chirality form $y=0$ triplets with charges $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$, plus $y=\frac{2}{3}$ singlets with charges $\frac{2}{3}$. In contrast to SU(3) theories, the leptons of each chirality form $y=-\frac{2}{3}$ triplets with charges 0, -1, and -1, and there are additional left-handed[5] singlets with charge 0.

In general, each of the quark and lepton fields in such a theory would be mixtures of fields corresponding to known and undiscovered particles of definite mass. In consequence, the couplings of the various neutral intermediate vector bosons would generally not conserve strangeness; the charged vector bosons would induce right-handed transitions among known particles; and universality would be lost in a plethora of mixing angles.[6]

This can be avoided if we assume that the theory is invariant under a discrete symmetry R, which leaves gauge bosons and right-handed fermions invariant, and changes the sign of left-handed fermions. The R symmetry forbids bare fermion mass terms, so that the quark and lepton masses must arise from R noninvariant vacuum expectation values of scalar fields. Another consequence is that, for at least a finite range of parameters of the Lagrangian, these vacuum expectation values will naturally leave a symmetry

RU unbroken, where U is the SU(3) transformation represented (in some basis) by a diagonal 3 $\times 3$ matrix with elements -1, -1, and $+1$. (This has been checked by explicit calculation.) We assume that the vacuum expectation values are actually invariant under RU.

The RU symmetry imposes a distinction between particles that are even or odd under RU: Even (odd) quarks and leptons appear as the first and second members of left- (right-) handed triplets, as the third members of right- (left-) handed triplets, and as right- (left-) handed singlets. There is no mixing between fermions of different RU parity. The triplets then take the form

$$(u', d, b')_L, \quad (t', b', d)_R,$$
$$(c', s, h')_L, \quad (g', h', s)_R, \text{ etc.},$$
$$(\nu_e, e^-, E^{-\prime})_L, \quad (E^{0\prime}, E^{-\prime}, e^-)_R,$$
$$(\nu_\mu, \mu^-, M^{-\prime})_L, \quad (M^{0\prime}, M^{-\prime}, \mu^-)_R,$$
$$(\nu_\tau, \tau^-, T^{-\prime})_L, \quad (T^{0\prime}, T^{-\prime}, \tau^-)_R, \text{ etc.},$$

where primes indicate possible mixing between fermions of the same charge, color, and RU parity. We will identify the even fermions u, d, c, s, ν_e, e^-, ν_μ, μ^-, ν_τ, and τ^- with the "known" quarks and leptons, with τ^- responsible for the μe events at SPEAR.[7,8] The odd fermions b, h, $t, g, E^-, E^0, M^-, M^0, T^-, T^0$ are all so far undiscovered, except that M^- and M^0 may be responsible for the trimuon events[3] at Fermilab. The odd fermion of lowest mass must be absolutely stable, but this may be one of the neutral leptons E^0, M^0, or T^0.

The gauge bosons of this theory consist of an octet $V_a{}^\mu$ and a singlet $V_0{}^\mu$, which interact respectively with the currents $-\frac{1}{2}ig\bar{\psi}\gamma_\mu\lambda_a\psi$ and $-\frac{1}{2}ig'\bar{\psi}\gamma_\mu y\psi$. The RU symmetry then tells us that the gauge bosons of definite mass consist of an even charged boson $W_\pm{}^\mu = (V_1{}^\mu \mp iV_2{}^\mu)/\sqrt{2}$; an odd charged boson $U_\pm{}^\mu = (V_4{}^\mu \mp iV_5{}^\mu)/\sqrt{2}$; two even massive neutral vector bosons which are linear combinations of

$$Z^\mu = (g^2 + \tfrac{1}{3}g'^2)^{-1/2}\left[\tfrac{1}{2}g(\sqrt{3}V_3{}^\mu + V_8{}^\mu) - (g'/\sqrt{3})V_0{}^\mu\right],$$
$$Y^\mu = \tfrac{1}{2}(V_3{}^\mu - \sqrt{3}V_8{}^\mu);$$

plus two odd massive neutral vector bosons $X_1{}^\mu$ and $X_2{}^\mu$ which are linear combinations of $V_6{}^\mu$ and $V_7{}^\mu$; and the massless photon

$$A^\mu = (g^2 + \tfrac{1}{3}g'^2)^{-1/2}\left[gV_0{}^\mu + \tfrac{1}{2}g'(V_3{}^\mu + V_8{}^\mu/\sqrt{3})\right].$$

The neutral-current interactions that involve only "known" (even) fermions are mediated by Z and Y; these automatically conserve all quark and lepton flavors. The charged-current interactions that involve only "known" fermions are mediated by W; these are automatically of the $V-A$ type, and satisfy Cabibbo universality.

To go further, we must say what scalar fields are in the theory. The scalar fields which are needed to generate fermion masses are $y = -\frac{2}{3}$ triplets Ω_i and $y=0$ complex octets Φ_i, both with $R=-1$. Conservation of RU and charge requires their vacuum expectation values to take the form

$$\langle\Phi_i\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & a_i \\ 0 & b_i & 0 \end{pmatrix}, \quad \langle\Omega_i\rangle = \begin{pmatrix} v_i \\ 0 \\ 0 \end{pmatrix}. \tag{1}$$

There may be other scalars, whose Yukawa interaction with fermions is forbidden. For instance, a scalar $y = +\frac{1}{3}$ triplet with $R=1$ would have vacuum expectation values of the form $(0, 0, v')$. If v' were much larger than v, a, or b, the theory would effectively revert back to a pure SU(2)$\otimes$U(1); the U and X would be much heavier than the W, and just one linear combination of Z and Y would have a mass of order m_W. In this case, the M^- production in the reactions $\nu_\mu d \to M^- t$ would be suppressed by a factor $[m_W/m_U]^4$, but this suppression would be unrelated to considerations of universality.

In what follows, we will explore the opposite case, in which Φ_i and Ω_i are the only scalar fields.[9] (We put no limit on their number.) In the usual way,[1,2] we find then that the vector fields of definite mass are $W^\pm$, $U^\pm$, Z^0, Y^0, A^0, and $x_1{}^\mu = V_6{}^\mu \cos\frac{1}{2}\epsilon + V_7{}^\mu \sin\frac{1}{2}\epsilon$; $X_2{}^\mu = -V_6{}^\mu \sin\frac{1}{2}\epsilon + V_7{}^\mu \times \cos\frac{1}{2}\epsilon$, where ϵ is the phase of $\sum_i a_i{}^* b_i$. Their masses are now such that $m_U{}^2 = m_W{}^2$, $m_Y{}^2 = 4l(1+l)^{-1}m_W{}^2$, $m_Z{}^2 = \frac{4}{3}(1-w)^{-1}(1+l)^{-1}m_W{}^2$, and $m_{X_1}{}^2 = m_{X_2}{}^2 = 2l(1+l)^{-1}(1 \mp \delta)m_W{}^2$, where $2l = \sum_i(|a_i|^2 + |b_i|^2)/\sum_i|v_i|^2$, $\delta = 2|\sum_i a_i{}^* b_i|/\sum_i(|a_i|^2 + |b_i|^2)$, and $w = g'^2/(g'^2 + 3g^2)$. The charge of the electron here is $-\frac{1}{2}g\sqrt{3w}$, and the Fermi constant $G_F/\sqrt{2}$ is $g^2(8m_W{}^2)^{-1}$, so that

$$m_W = \left[\frac{4\pi\alpha}{3\sqrt{2}G_F w}\right]^{1/2} \simeq \frac{43.2}{\sqrt{w}} \text{ GeV}.$$

It is striking that the odd fermions are much heavier than the corresponding even fermions. This observation can be incorporated into our theory by supposing that the Lagrangian has an additional *approximate* discrete symmetry $\sqrt{R}$, which leaves gauge bosons and right-handed fermions invariant, and multiplies the left-handed fermion fields and scalar fields with factors of $-i$ and $+i$, respectively. If $\sqrt{R}$ were exact, it would be natural for the vacuum expectation values to preserve invariance under $\sqrt{R}\sqrt{U}$, where $\sqrt{U}$ is the SU(3) transformation represented by a diagonal 3×3 matrix with elements $+i$, $-i$, and $+1$. The vacuum expectation values would then be of the form (1), with $b=0$. Also, the $\sqrt{R}$ symmetry would allow Yukawa couplings which can give masses to odd fermions, but would forbid couplings which could give masses to even fermions. To the extent that $\sqrt{R}$ is a good symmetry of the Lagrangian, we expect that $|b_i| \ll |a_i|$; then even fermions will be much lighter than odd fermions, and $\delta \ll 1$, so that X_1 and X_2 should be almost degenerate.

We now turn to the physical consequences of this theory.[10] In assessing these, we assume that the Φ couplings are subject to an additional discrete symmetry in such a way as to forbid mixing among b and h, or E^-, M^-, and T^-.

(1) The $K_L{}^0$-$K_S{}^0$ mass difference contains a $\overline{d}_L s_L \to \overline{s}_L d_L$ term proportional to $G_F{}^2(m_c{}^2 - m_u{}^2) \times \sin^2\theta \cos^2\theta$, and a $\overline{d}_R s_R \to \overline{s}_R d_R$ term proportional to $G_F{}^2(m_t{}^2 - m_g{}^2)\sin^2\theta' \cos^2\theta'$, where θ is the Cabibbo angle, and θ' is its analog for t and g. The second term must be at most of the same order of magnitude as the first.

(2) We have compared the predictions of this theory with experimental data on deep inelastic neutral-current reactions, elastic $\nu_\mu p$ and $\overline{\nu}_\mu p$ scattering, and $\nu_\mu e$, $\overline{\nu}_\mu e$, and $\overline{\nu}_e e$ scattering, and find a good fit to all data for l and w both in the neighborhood of 0.2. If, for illustration, we take $l = w = 0.2$ and $\delta = 0$, the intermediate-vector-boson masses are $m_W = m_U \simeq 100$ GeV, $m_Z \simeq 115$ GeV, $m_Y \simeq 80$ GeV, and $m_{X_{1,2}} \simeq 55$ GeV. The electron interacts with nuclei through both Z and Y exchange. Y exchange is parity conserving, and the $\overline{e}eZ$ coupling is pure vector, so the parity nonconservation is not enhanced in heavy atoms.

(3) The transitions $\mu \leftrightarrow M + X$ make a contribution to the gyromagnetic ratio of the muon of order $\sqrt{2}G_F m_\mu m_M (m_W/m_X)^2 \delta/\pi^2$. In order to keep this contribution within experimental limits, we must have $\delta < 10^{-1}$ (*mutatis mutandis* for the electron). This is in accord with our expectations based on $\sqrt{R}$ symmetry—we would anticipate that δ should be of order m_μ/m_{M^-} or m_s/m_h, or roughly 10^{-2}.

(4) We have not assumed muon conservation. If E^0, M^0, and T^0 mix, then the process $\mu \to e\gamma$ will occur in much the same manner as described by Cheng and Li,[11] with no "left-right" terms.[12] For $m_W \simeq 100$ GeV, and the heaviest of E^0, M^0, and T^0 with a mass of about 4 GeV, the expected branching ratio for $\mu \to e\gamma$ would be $\lesssim 1.4\times 10^{-10}$.

(5) We also have not assumed CP conservation. If CP is not conserved in the Lagrangian or is spontaneously broken, we would expect nonvanishing values for phases in fermion mixtures and for the mixing angle ϵ. There would still be no CP nonconservation in first-order W, U, Z, or Y exchange, but X exchange may produce a first-order CP nonconservation in processes like $E^- \to e^- \mu^- M^+$. The imaginary part of the $K_L{}^0$-$K_S{}^0$ mass differences arises in order $G_F{}^2$ from the process $\overline{s}d \to U^+U^- \to \overline{d}s$; for $m_t{}^2 - m_g{}^2$ of order $m_c{}^2 - m_u{}^2$ and θ' of order θ, we find that CP-nonconservation phases must be of order 10^{-3}. This theory does not explain why these phases are so small. The transition $d \to b + X$ gives the neutron an electric dipole moment of order $eG_F m_b \varphi \delta\pi^{-2}$, where φ is a typical CP-nonconservation phase. With $m_b \simeq 5$ GeV, $\delta \approx 10^{-2}$, and φ of order 10^{-3}, we expect an electric dipole moment or order 10^{-24} $e\cdot$cm.

(6) This model does not contain a $\overline{b}_R u_R$ current which could explain the reported high-y anomaly[13] in $\overline{\nu}N$ reactions. However, the production of heavy leptons and quarks by ν and $\overline{\nu}$ expected in this model would greatly complicate the interpretation of these experiments.

(7) The M^- can be produced in $\nu_\mu N$ collisions[14] by the process $\nu_\mu d \to M^- t$; the $\mu^-\mu^-\mu^+$ trimuons can then be produced by the decay chains $M^- \to \nu_\mu \overline{M}_0 \mu^-$, $\overline{M}^0 \to \mu^+ \tau^- \overline{T}^0$, $\tau^- \to \nu_\tau \mu^- \overline{\nu}_\mu$; or $M^- \to \overline{\nu}_\mu M^0 \mu^-$, $M^0 \to \mu^- \mu^+ E^0$, etc. Counting M^- decay channels, we estimate that the total probability of $\mu^-\mu^-\mu^+$ production in M^- decay ranges up to 4%.[15] Also, M^- production is suppressed by phase space and the right-handed d-t coupling, by a factor of about 0.03,[16] so that the overall probability of high-energy $\mu^-\mu^-\mu^+$ production in $\nu_\mu N$ collisions at Fermilab could be as large as 10^{-3}. It is a specific prediction of this theory that the trimuon events are accompanied by production of the t quark. Muons from the decay of the t quark are quite soft, but if observable, they would contribute to the rate for both "right"- and "wrong"-sign trimuons, and even tetramuons.

The combined gauge group $SU(3)\otimes U(1)\otimes SU(3)$ of weak, electromagnetic, and strong interactions allows a promising unification in the simple group SU(6), with fermions in representations $\underline{1}$ and $\underline{15}$. This will be dealt with in a future communication.

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Note added.—After completion of this paper, we became aware of the work of Segrè and Weyers.[17] (We thank F. Wilczek for calling this paper to our attention.) There are several similarities between their work and ours, e.g., the use of a discrete symmetry to make the symmetry-breaking pattern natural. However, their model is different from ours in a number of important respects. Their model contains only fermion triplets and is strictly vectorlike; their discrete symmetry corresponds to our $\sqrt{R}$; and they assume the limit $|a| \ll |v'|$ with $b = v = 0$. In consequence, the phenomenological implications of the two models are quite different.

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[1]See, for example, E. S. Abers and B. W. Lee, Phys. Rep. 9, 1 (1973); S. Weinberg, Rev. Mod. Phys. 46, 255 (1974).

[2]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Physics*, edited by N. Svartholm (Almquist and Wiksells, Stockholm, 1968), p. 367.

[3]A. Benvenuti *et al.*, Phys. Rev. Lett. 38, 1110, 1183 (1977). See also B. C. Barish *et al.*, Phys. Rev. Lett. 38, 577 (1977).

[4]We tacitly assume the color gauge theory of strong interactions. Color indices are dropped everywhere, and SU(3) refers throughout to the flavor gauge group.

[5]We do not exclude the possiblity of there also being right-handed lepton singlets, in which case neutrinos might have finite masses.

[6]For an exhaustive bibliography on previous SU(3) or SU(3)⊗U(1) models up to 1974, see C. H. Albright, C. Jarlskog, and M. Tjia, Nucl. Phys. B86, 535 (1975). See also P. Fayet, Nucl. Phys. B78, 14 (1974); H. Fritzsch and P. Minkowski, Phys. Lett. 63B, 99 (1976); F. Gürsey and P. Sikivie, Phys. Rev. Lett. 36, 775 (1976); P. Ramond, Nucl. Phys. B110, 214 (1976); M. Yoshimura, Prog. Theor. Phys. 57, 237 (1977). The main difference between all these models and ours is that, in our theory, universality, suppression of right-handed currents, flavor conservation in neutral currents, etc., are consequences of the pattern of spontaneous symmetry breaking that emerges naturally in the theory.

[7]M. Perl *et al.*, Phys. Rev. Lett. 35, 195 (1975), and Phys. Lett. 63B, 466 (1976).

[8]We have considered the possiblity of only having two lepton triplets, with E^- identified with τ^-. However, this scheme does not seem to agree with observed features of τ^- decay, and also leads to a small trimuon probability.

[9]This is the counterpart of the assumption in Ref. 2 that all scalars belong to doublets.

[10]We gratefully acknowledge the help given us by R. E. Shrock in assessing phenomenological implications of this theory. This and other matters will be dealt with in detail in longer papers by B. W. Lee, R. E. Shrock, and S. Weinberg, to be published.

[11]T. P. Cheng and L.-F. Li, Phys. Rev. Lett. 38, 381 (1977).

[12]J. D. Bjorken, K. Lane, and S. Weinberg, to be published; S. B. Treiman, F. Wilczek, and A. Zee, to be published; B. Lee and R. Shrock, to be published.

[13]A. Benvenuti *et al.*, Phys. Rev. Lett. 37, 1478 (1976). There is a variant of our theory with only one quark singlet of each chirality, in which u_R replaces t_R. This leads to parity-conserving neutral currents.

[14]The possibility of trimuon production through M^- decay in a pure SU(2)×U(1) model has been considered by P. Langacker and G. Segrè, to be published; V. Barger, T. Gottschalk, D. V. Nanopoulos, J. Abad, and R. J. N. Phillips, Phys. Rev. Lett. 38, 1190 (1977).

[15]We have considered a number of cases. The trimuon yield in M^- decay depends sensitively on the ordering of masses and on possible mixing among E^0, M^0, and T^0.

[16]C. H. Albright, private communication. This estimate is based on the Fermilab ν energy distribution and on the assumed masses $m_{M^-} \simeq 7 \sim 8$ GeV and $m_t \simeq 4$ GeV. See also C. H. Albright, J. Smith, and J. A. M. Vermaseren, Phys. Rev. Lett. 38, 1187 (1977).

[17]G. Segrè and J. Weyers, Phys. Lett. 65B, 243 (1976).