Inertial Induction in an Open Universe

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Integration of Einstein's inertial-mass induction to the particle horizon in Robertson-Walker space yields an equation for the scale factor of the universe. A generalized Milne-McCrea theorem is used, and G is allowed to vary slowly in order to preserve the weak equivalence principle. Among the results given is that at the present time $d(G/G_0)/dt = -0.22H_0$ for $q_0 = 0.03$. The age is somewhat less than for Friedman models. These results differ minimally from general relativity.

As early as 1922, Einstein[1] showed that, according to general relativity, the presence of ponderable mass near a test body increases the inertial mass of the body by the fractional amount

$$\delta = (G/c^2)\int \rho\, dV/r, \tag{1}$$

the volume integration being over the ponderable mass, whose density is ρ; G is the gravitational constant and c the speed of light. But general relativity assumes equivalence of inertial and gravitational mass; thus it is presumed that the gravitational mass of the body increases in the same proportion. If there is such an effect, the assumption of equality of the two masses is strongly supported by the recent analyses of the lunar laser-ranging experiments[2,3] following a suggestion by Nordtvedt.[4] These results eliminate many theories that allow inertial and gravitational mass to differ, either for small bodies or for ones of astronomical size.

The purpose of this paper is to discuss the application of Eq. (1) in an open universe on the assumptions that inertial and gravitational mass are identical and that all inertial mass is generated by an interaction like Eq. (1).[1] It is part of a study of minimal modifications of general relativity capable of coping with inertial induction in an open universe. The general relativistic case is treated first, and then a plausible model in which G is slowly varying in time is developed.

I begin by asserting that

$$m^* = Km\int dM/r, \tag{2}$$

where m^* is the inertial mass of a particle, m its gravitational mass, M the gravitational mass of other matter in the effective universe within the particle horizon, and r the distance between interacting masses. K is a proportionality factor whose dimensions are those of G/c^2. It is assumed that $m^* = m$ identically, for particles or for extended bodies.

First I consider the case of general relativity and discuss Eq. (2) as applied in a homogeneous isotropic universe with no cosmological term and negligible pressure. The Friedman equation[5] for the scale factor $a(t)$ is applicable:

$$(da/dt)^2 = 8\pi\rho_0 G/3a - k, \tag{3}$$

where ρ_0 is the present-day density and $k = -c^2/R_c^2$, R_c^{-2} being the magnitude of the present-day (negative) spatial curvature.

In Eq. (2) I use the element of proper volume and the coordinate distance $r = a(t)R_c\sigma$ (σ is the radial Robertson-Walker coordinate) to obtain

$$\frac{1}{K} = \int_0^{\sigma_1} \frac{4\pi R_c^3 a^3(t)\sigma(t)\sigma^2}{R_c a(t)\sigma(1+\sigma^2)^{1/2}}\, d\sigma, \tag{4}$$

where $\sigma_1(t)$ is the particle horizon. The functions of t can be removed from the integral sign and with the substitution[6]

$$\begin{aligned}\sigma_1(t) &= \sinh[(c/R_c)\textstyle\int_0^t dt/a] \\ &= \sinh[2\sinh^{-1}(Ca)^{1/2}],\end{aligned} \tag{5}$$

where $C = 3c^2/8\pi G\rho_0 R_c^2$, we obtain

$$\rho(t) = (G\rho_0/3Kc^2)a^{-3}(t), \tag{6}$$

showing that $m = \text{const}$ because the volume goes as $a^3(t)$. While this part of the result is in agreement with general relativity, to obtain $\rho = \rho_0$ at the present epoch, we must have $K = G/3c^2$, which disagrees numerically with Eq. (1) and also with Thirring's revised value[7] $K = 3G/c^2$.

This treatment is oversimplified in that radiation pressure has been omitted and such measures as the hypothesis of a special mass distribution[8] have not been considered. Moreover, there is the question as to whether Eq. (2) should apply as assumed; though two observers in comoving coordinates are each in a local inertial frame, they are not in the same inertial frame. But Einstein's hypothesis that all inertial mass should arise from this source and the internal consistency of a system so constructed are sufficient motivation to proceed with this fundamental approach. It may be useful to remark in this connection that Eq. (2) (or what amounts to it) was a motivating factor behind the Brans-Dicke[8] theory. Here I am attempting to give it an exact interpretation in a setting closely resembling general relativity.

To overcome the difficulty discussed above, the mass could be made time variable (though $m^* = m$), the order of the variation being $d(m/m_0)/dt = H_0$, the Hubble constant. But a time-variable mass seems undesirable on various grounds, including alteration of galactic red shifts and the timing of atomic clocks as compared to the cosmic time of the Robertson-Walker metric.

It seems preferable to allow G to vary with time, as has been proposed before, as for example in the Brans-Dicke theory.[8] Here, however, I employ a new approach and use the Milne-McCrea theorem[5] to obtain

$$d^2a/dt^2 = -(4\pi\rho_0 G_0/3)a^{-2}(t)f(t), \tag{7}$$

where $f(t) = G(t)/G_0$. Let $f(t) = a^{-n}$ near $t = t_0$, an appropriate form justifiable on pragmatic grounds; n can be determined by numerical means. Equation (7) can now be integrated to obtain

$$(da/dt)^2 = (8\pi\rho_0 G_0/3)a^{-(1+n)}/(1+n) - k, \tag{8}$$

$-k$ being the constant of integration. As in the application of the Milne-McCrea theorem to the general-relativistic case, k is identified as $-c^2/R_c^2$. I use Eq. (8) only at $t = t_0$ to obtain

$$h^2 = 1 - 2q_0/(1+n), \tag{9}$$

where $h = c/R_c H_0$ and q_0 is the deceleration parameter. I now assume that Robertson-Walker space is appropriate[6] and in Eq. (4) set $K = 3G(t)/c^2 = 3f(t)G_0/c^2$; solving for $f(t)$ and substituting it in Eq. (7) yield

$$(d^2a/dt^2)a\left[\cosh\left(hH_0\int_0^t dt/a\right) - 1\right] = -h^2 H_0^2/9. \tag{10}$$

This nonlinear equation for $a(t)$ has been reduced by numerical methods to obtain the results in Table I. Though not given here in detail, $a(t)$ is virtually indistinguishable from that of the Friedman models for red shift $z \lesssim 1$; then it drops rapidly to give smaller ages. The density of luminous matter corresponds roughly to $q_0 = 0.01$.[5] Gott *et al.*[9] favor $q_0 \simeq 0.03$ on the basis of several different arguments. Using this value as an example, we see that the variation of G is not excessive, viz., $G(t) = a^{-0.22}$ at present, or

$$d(G/G_0)/dt = -0.22H_0 = -1.1\times 10^{-11}\ \mathrm{yr}^{-1} \tag{11}$$

if $H_0 = 50$ km/sec Mpc (1 pc = 1 parsec).[10] At $z = 0.4$ this would result in a galactic-luminosity increase of only 0.3 magnitude if it goes as G^4.[11,12] Geophysical effects are much smaller than in the Hoyle-Narlikar theory.[13]

TABLE I. Properties of inertial inductive model.

q_0	n	Age H_0t_0	Friedman H_0t_0
0.07	0.472	0.763	0.875
0.05	0.352	0.808	0.898
0.03	0.221	0.864	0.927
0.02	0.151	0.899	0.944
0.01	0.078	0.941	0.966

This rate of change of G is smaller in magnitude than that reported by Van Flandern,[14] viz., $(-8\pm 5)\times 10^{-11}\ \mathrm{yr}^{-1}$, but the error quoted is large and the rate could even be positive at the 2σ level. It is conjectured that when pressure is added, the inertial induction model will predict even less variation.[15] As it is, the rate is so small that it would be very difficult to measure since it is comparable to the stability of atomic clocks.[16]

The consequences of variable G in this model are thus quite moderate. The departure from general relativity is slight, involving only the slowly varying gravitational constant and somewhat altered cosmological results. The main difference may be in nucleosynthesis in the early universe because of the change in time scale at that epoch.

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SU(3)⊗U(1) Gauge Theory of the Weak and Electromagnetic Interactions

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We describe an extension of the gauge theory of weak and electromagnetic interactions to SU(3)⊗U(1). The extended theory *naturally* insures universality, absence of right-handed currents in β and muon decay, flavor conservation in neutral currents, etc.; gives good quantitative agreement with observations of neutral currents; and accounts for recently observed trimuon events.

Within the modern unified theory[1] of weak and electromagnetic interactions, those models based on the SU(2)⊗U(1) gauge group of the original example[2] stand out for the natural way in which they account for observed general features of the weak interactions. The existence of neutral currents makes it clear that any successful model must incorporate SU(2)⊗U(1) gauge invariance, but it is possible that new observations, such as the recently reported high-energy trimuons,[3] might require SU(2)⊗U(1) to be embedded in a larger gauge group. It would be premature to conclude from the limited trimuon data that SU(2)⊗U(1) must be expanded, but it is worth asking how, if this became necessary, it would be possible to enlarge the gauge group without losing the natural features of the simple SU(2)⊗U(1) model. We offer an example of such a model.

The gauge group is SU(3)⊗U(1); the charge is the sum of the SU(3) generator $\frac{1}{2}(\lambda_3+\lambda_8/\sqrt{3})$ plus the U(1) generator y. The quarks of each color[4] and each chirality form $y=0$ triplets with charges $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$, plus $y=\frac{2}{3}$ singlets with charges $\frac{2}{3}$. In contrast to SU(3) theories, the leptons of each chirality form $y=-\frac{2}{3}$ triplets with charges 0, -1, and -1, and there are additional left-handed[5] singlets with charge 0.

In general, each of the quark and lepton fields in such a theory would be mixtures of fields corresponding to known and undiscovered particles of definite mass. In consequence, the couplings of the various neutral intermediate vector bosons would generally not conserve strangeness; the charged vector bosons would induce right-handed transitions among known particles; and universality would be lost in a plethora of mixing angles.[6]

This can be avoided if we assume that the theory is invariant under a discrete symmetry R, which leaves gauge bosons and right-handed fermions invariant, and changes the sign of left-handed fermions. The R symmetry forbids bare fermion mass terms, so that the quark and lepton masses must arise from R noninvariant vacuum expectation values of scalar fields. Another consequence is that, for at least a finite range of parameters of the Lagrangian, these vacuum expectation values will naturally leave a symmetry