Other interesting effects can be expected in heavy-lepton production, as for example apparent lepton-number nonconservation in $\mu^-\mu^-e^+$ events. Also the M^--μ^- mixing may bring in a neutral-current coupling allowing $M^-\to\mu^-\mu^-\mu^+$ single-step decay into three leptons at the few-percent level. Such trimuons would have a unique invariant mass m_{--+}, higher individual muon energies, and higher visible energy than the cascade-decay events; it is possible that event 119-017991 of Ref. 1 may be in this category.

We thank D. Cline and D. D. Reeder for discussions.

Note added.—Details of our Monte Carlo calculations of heavy-lepton production and cascade decay are described in a comprehensive paper.[9] In each amplitude the lepton helicities are correctly followed from the production through the final stage of the decay; we neglect interference between amplitudes with different intermediate configurations. We use standard scaling structure functions at the hadron vertex. An extensive analysis of branching fractions for the cascade-decay chain is given.[10] The inclusive trimuon branching fraction $B(M^-\to\mu^-\mu^-\mu^+X)=0.04$ is found for $m_-=7.0$ GeV, $m_0=2.5$ GeV, consistent with the branching fractions used in this Letter; of this 0.022 comes from the $X=\nu\bar{\nu}$ states calculated here. Finally, in Ref. 6 we point out that universality restrictions on the magnitude of ϵ can be circumvented in our SU(2)⊗U(1) model by a common mixing of leptons with heavy leptons and quarks with heavy quarks.

*Work supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the U. S. Energy Research and Development Administration under Contract No. E(11-1)-881, COO-595.

†On leave at the University of Wisconsin, supported by the Commission of Cultural Interchange between Spain and U. S. A.

[1]A. Benvenuti *et al.*, Phys. Rev. Lett. 38, 1110 (1977), and second preceding Letter [Phys. Rev. Lett. 38, 1183 (1977)].

[2]B. C. Barish *et al.*, Phys. Rev. Lett. 38, 577 (1977).

[3]V. Barger and D. V. Nanopoulos, University of Wisconsin Report No. COO-881-583, 1977 (to be published).

[4]M. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975), and Phys. Lett. 63B, 466 (1976).

[5]P. Baird *et al.*, Nature (London) 264, 528 (1976).

[6]V. Barger, D. V. Nanopoulos, and R. J. N. Phillips, University of Wisconsin Report No. COO-881-597, 1977 (to be published).

[7]A. Benvenuti *et al.*, Phys. Rev. Lett. 35, 1199 (1975).

[8]V. Barger, T. Gottschalk, and R. J. N. Phillips, University of Wisconsin Report No. COO-881-564, 1977 (to be published), and to be published.

[9]V. Barger, T. Gottschalk, D. V. Nanopoulos, J. Abad, and R. J. N. Phillips, University of Wisconsin Report No. COO-881-596, 1977 (to be published).

[10]V. Barger, T. Gottschalk, D. V. Nanopoulos, and R. J. N. Phillips, University of Wisconsin Report No. COO-881-598, 1977 (to be published).

Test of the Quark Parton Model with Data from Electroproduction of Pions*

J. F. Martin
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

L. S. Osborne
Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 3 December 1976)

We have measured the relative cross sections of π^+ and π^- electroproduction from protons and neutrons, and combined them in a way which permits a new, detailed comparison with the quark parton model of the nucleon. We confirm the expected behavior with $1/\omega$ of the quark charge ratio and the quark fragmentation function.

It has been pointed out by several authors[1,2] that a measurement of the relative production of $\pi^\pm$ and π^0 mesons from lepton-nucleon collisions can be related to the parameters of quark parton models. Given that quarks have not been seen, one may wonder whether their charge and their distribution functions (for example) are realities or merely parameters adjusted to fit data. We develop a method to separate these parameters and use our data to check the model's assumptions; the ratio of up- to down-quark charge is one of these since it should be an integer.

The data are taken from a very-high-statistics experiment performed in a 20.5-GeV/c electron beam at Stanford Linear Accelerator Center. One previous Letter describes the experimental apparatus,[3] and another recently published Letter[4] describes the data and the basic analysis procedure which allows us to separate π's from other hadrons. This study differs from that of Ref. 4 in that here we have calculated the cross section for producing a π^+ or π^- rather than the yield. By cross section, we mean the production divided by incident electrons and by target nuclei; by yield, we mean the production divided by the number of scattered electrons from the target. Since we are comparing absolute cross sections, we monitored the density of both hydrogen and deuterium targets with a hydrogen bulb thermometer.

Concentrating on only π^+ and π^-, we write the following general expressions for cross sections, leaving out the factors common to neutron and proton:

$$\pi_p^+ = u_p q_u^2 D_u^+ + d_p q_d^2 D_d^+ + \beta, \tag{1a}$$

$$\pi_p^- = u_p q_u^2 D_u^- + d_p q_d^2 D_d^- + \beta, \tag{1b}$$

$$\pi_n^+ = u_n q_u^2 D_u^+ + d_n q_d^2 D_d^+ + \beta, \tag{1c}$$

$$\pi_n^- = u_n q_u^2 D_u^- + d_n q_d^2 D_d^- + \beta, \tag{1d}$$

where $\pi_i^\pm$ is the relative cross section of $\pi^\pm$ mesons from a struck hadron, with $i=p$ or n (i.e., proton or neutron[3]); u_i and d_i are, respectively, the relative probabilities that a u or d quark was present to strike[3]; $q_{u(d)}$ is the charge of the u (d) quark; $D_{u(d)}^\pm$ is the fragmentation function, i.e., the probability for struck quark u (d) giving rise to a $\pi^\pm$ meson; β is contribution to the cross section from the vacuum quarks. We assume, naturally, that this contribution is charge- and source-symmetric.

π, u, D, and β can be differential in all experimental variables, which are, (1) for the electron, the electron energy E_0, the four-momentum transfer $-Q^2$, the electron energy loss ν, $x=q^2/2m\nu = 1/\omega$, and $y=\nu/E_0$; (2) for the hadron, in addition to the above and in the hadron center-of-mass system with the collisions axis taken in the direction of the momentum of the virtual photon, $z=p_l/p_{l\,\max}$, the square of the transverse momentum $p_\perp^2$, and azimuthal angle φ ($\varphi=0$ points toward the electron).

Isotopic-spin reflection gives,

$$\begin{aligned} &D_u^+ = D_d^- \equiv D_1, \quad D_u^- = D_d^+ \equiv D_2, \\ &u_p = d_n \equiv u_1, \quad u_n = d_p \equiv u_2. \end{aligned} \tag{2}$$

We form the following sums and differences:

$$A' = (\pi_p^+ + \pi_p^-) + (\pi_n^+ + \pi_n^-), \tag{3a}$$

$$A = A' - 4\beta, \tag{3b}$$

$$B = (\pi_p^+ + \pi_p^-) - (\pi_n^+ + \pi_n^-), \tag{3c}$$

$$C = (\pi_p^+ - \pi_p^-) + (\pi_n^+ - \pi_n^-), \tag{3d}$$

$$D = (\pi_p^+ - \pi_n^-) - (\pi_n^+ - \pi_n^-). \tag{3e}$$

Note that the β term cancels out in the last three equations.

We also define a ratio,

$$R = A/A'. \tag{4}$$

This ratio gives the fractional contribution to the cross sections from valence quarks. It could, in principle, be measured from the y distributions in neutrino events,[5,6] but these data are not yet sufficiently accurate for use.

Expressions for A through D in Eqs. (3) are all products of sums and differences of the numbers q_i^2, u_i, and D_i; for example,

$$B = (q_u^2 - q_d^2)(u_1 - u_2)(D_1 + D_2). \tag{5}$$

By taking appropriate ratios, we can solve for three expressions relating measurable cross sections to functions of the fractions q_u^2/q_d^2, u_1/u_2, and D_1/D_2 alone. These functions, however, include R. For the q's,

$$\begin{aligned} B\cdot C/A\cdot D &= B\cdot C/R\cdot A'\cdot D \\ &= (q_u^2 - q_d^2)^2/(q_u^2 + q_d^2)^2. \end{aligned} \tag{6}$$

Similar ratios hold for the u's and the D's but they have not been used in the analysis.

If the charge ratio q_u^2/q_d^2 is known from the quark model and corroborated by (6) then the u's and D's can be obtained using similarly derived expressions

$$\frac{D}{C} = \frac{(u_1 - u_2)}{(u_1 + u_2)} \frac{(q_u^2 + q_d^2)}{(q_u^2 - q_d^2)}, \tag{7a}$$

$$\frac{D}{B} = \frac{(q_u^2 + q_d^2)}{(q_u^2 - q_d^2)} \frac{(D_1 - D_2)}{(D_1 + D_2)}. \tag{7b}$$

Since these expressions do *not* contain R they can

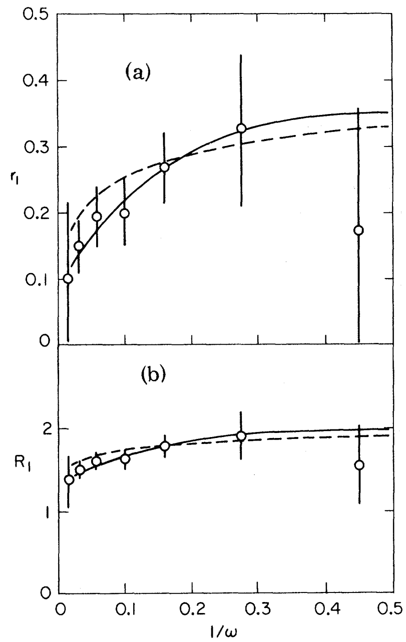

FIG. 1. (a) The measured value of r_1, defined in the text, vs $1/\omega$. From r_1, we have also calculated the ratio, R_1, which is shown in (b). The solid line is the expectation from Ref. 8, the dashed line from Ref. 7.

be used reliably even in the absence of information on the vacuum quarks.

Equation (7a) is a recasting of Eq. (4.11) of Gronau, Ravndal, and Zarmi.[2] A simple quark model would require $B \cdot C/A \cdot D$ to be independent of all variables, u_1/u_2 (and hence D/C) to be dependent on x and independent of the emitted-hadron variables, and D_1/D_2 (and hence D/B) to be independent of x. Equivalent expressions can be derived for neutrino interactions.

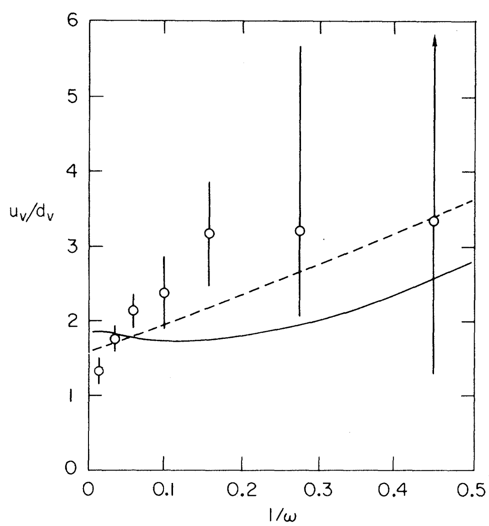

FIG. 2. A plot of $u_v/d_v(u_1/u_2)$ extracted from Eq. (7a) under the assumption that $q_u^2/q_d^2=4$. The solid (dashed) line is the expected behavior from Ref. 8 (7).

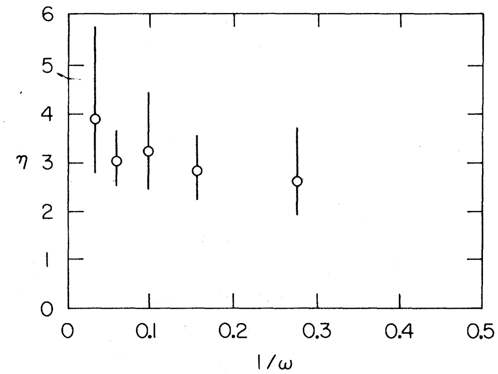

FIG. 3. A plot of η extracted from Eq. (7b) with the assumption $q_u^2/q_d^2=4$. η is the ratio of $\overline{D}_1$ to $\overline{D}_2$, i.e., D_1 and D_2 averaged over z from $z=0.3$ to 0.85 and over φ and $p_\perp^2$.

The experimental determination of these ratios requires good statistics and small systematic uncertainties. The errors on these ratios are a sum in quadrature of the statistical error, and the systematic errors which are dominated by our ± 2% uncertainty in the value (target density times integrated incident beam) for hydrogen versus deuterium. Other systematic corrections (as described in Ref. 4) have small effects and very much smaller uncertainties. The errors for $1/\omega$ below 0.1 are largely systematic, and above 0.1 are largely statistical. The data is summed over $p_\perp$, $0.3<z<0.85$, and φ excluding a small region near the beam pipe where background contamination might be possible. Note that we are actually comparing the average D_i $(\equiv \overline{D}_i)$ for the range $0.3<z<0.85$. In Fig. 1 we plot $B \cdot C/A' \cdot D$ and the value of $R_1 = |q_u/q_d|$ (effective) derived from it, assuming $R=1$. Our result is clearly compatible with $q_u=\frac{2}{3}e$ and $q_d=-\frac{1}{3}e$ for large x. Shown also is the deviation from 2 expected from vacuum quark contributions with use of two published parametrizations.[7,8]

From the above we feel justified in accepting the charge ratio as -2 and using this number in Eqs. (7a) and (7b) to find u_1/u_2 (i.e., u_v/d_v) and $\eta = \overline{D}_1/\overline{D}_2$ $(0.3<z<0.85)$. These are plotted in Figs. 2 and 3, respectively. We see that u_v/d_v is *not* a constant with respect to x. It clearly falls below the expected average value of 2 at low x and is consistent with a number greater than 2 at large x.[9] We also note that η is independent of

x even in the small x region where the vacuum quarks contribute appreciably to the cross section.

In summary, we have derived expressions which allow us to calculate the parameters of a general quark model directly from experimental data with no fitting. We find that the ratio of up- to down-quark charge is consistent with -2. We further find the ratio of the average fragmentation functions, $\bar{D}_i$, to be independent of x. Finally, the majority- to minority-quark ratio is not a constant with x, falling well below 2 at small x.

The authors wish to thank their collaborators on the Stanford Linear Accelerator Center experiment E-97 (Ref. 4) for making the data possible. They particularly thank D. Roth for his contribution to the data analysis.

---

*Work supported in part by the U. S. Energy Research and Development Administration.

[1]J. T. Dakin and G. J. Feldman, Phys. Rev. D 8, 2862 (1973).

[2]M. Gronau, F. Ravndal, and Y. Zarmi, Nucl. Phys. B51, 611 (1973).

[3]J. F. Martin *et al.*, Phys. Rev. Lett. 34, 288 (1973).

[4]J. F. Martin *et al.*, Phys. Lett. 65B, 483 (1976).

[5]C. H. Llewellyn Smith, Phys. Rep. 3C, No. 5 (1972).

[6]*Proceedings of the Seventeenth International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford High Energy Laboratory, Didcot, Berkshire, England, 1975).

[7]T. Kuti and V. F. Weisskopf, Phys. Rev. D 4, 3418 (1971), and R. McElhaney and S. F. Tuan, Phys. Rev. D 8, 2267 (1973).

[8]Listed by R. Blankenbecler *et al.*, Stanford Linear Accelerator Center Report No. SLAC-PUB-1531, 1975 (to be published), and attributed to G. Farrar, G. Chu, and J. Gunion. See also J. F. Gunion, Phys. Rev. D 10, 242 (1974); and Glenys R. Farrar, Nucl. Phys. B77, 429 (1974).

[9]A. Bodek *et al.*, Phys. Rev. D 7, 1362 (1973). The "disappearance" of the minority quark at large x ($d_v \to 0$) is the usual explanation for the νW_2 of neutron to proton going to a ratio smaller than $\frac{2}{3}$.