goes down by 30%. If we compare the uncorrected experimental number 5×10^{-4} for $R=\sigma(\mu^-\mu^-\mu^+)/\sigma(\mu^-)$, with $E_\nu > 100$ GeV, we note that $\sigma(\nu_\mu + N \to M^- + X)/\sigma(\nu_\mu + N \to \mu^- + X)$ is 14% for our model with the same energy cut. Model-dependent but reasonable estimates[12] for the two branching ratios are in the range 7 to 15%, yielding $R=(7-30)\times 10^{-4}$. The agreement between these values sugfests that there is little room for a mixing angle at the production vertex. In other words, the leptonic vertex couples the ν_μ-M^- left-handed with a new charged gauge boson (not the regular W^+). Finally we point out that while the heavy-lepton-cascade chain will lead to some excess of events at large y in the reaction $\bar{\nu}_\mu + N \to \mu^+ + X$, it is clear that the magnitude of this contribution is much too small to explain the whole effect.

We are very grateful to A. K. Mann and D. Cline for stimulating discussions on the heavy-lepton interpretation of the FHPRW data. We would also like to acknowledge useful conversations with P. Langacker, B. W. Lee, H.-T. Nieh, H. Primakoff, G. Segre, and R. E. Shrock.

*Work supported in part by the National Science Foundation, under Grant Nos. PHY-75-05467A01 and PHY-76-15328.

†Permanent address.

[1]A. Benvenuti *et al.*, Phys. Rev. Lett. 38, 1110 (1977).
[2]B. C. Barish *et al.*, Phys. Rev. Lett. 38, 577 (1977).
[3]A. Benvenuti *et al.*, preceding Letter [Phys. Rev. Lett. 38, 1183 (1977)].
[4]A. Benvenuti *et al.*, Phys. Rev. Lett. 35, 1199, 1203, 1249 (1975); B. C. Barish *et al.*, Phys. Rev. Lett. 36, 939 (1976).
[5]L. M. Sehgal and P. M. Zerwas, Nucl. Phys. B108, 483 (1976); B. Varger, R. J. N. Philips, and T. Weiler, Phys. Lett. 62B, 227 (1976).
[6]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).
[7]M. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975), and Phys. Lett. 63B, 466 (1976).
[8]C. H. Albright, Phys. Rev. D 13, 2508 (1976).
[9]A. Soni, Phys. Rev. D 9, 2092 (1974), and 11, 624 (1975); C. H. Albright and C. Jarlskog, Nucl. Phys. B84, 467 (1975); C. H. Albright, C. Jarlskog, and L. Wolfenstein, Nucl. Phys. B84, 493 (1975).
[10]L. N. Chang, E. Derman, and J. N. Ng, Phys. Rev. Lett. 35, 6 (1975), and Phys. Rev. D 12, 3539 (1975); C. H. Albright, Phys. Rev. D 12, 1319 (1975); A. Pais and S. B. Treiman, Phys. Rev. Lett. 35, 1206 (1975).
[11]Conversely the experimental efficiency for the detection of trimuons is 50%.
[12]Y.-S. Tsai, Phys. Rev. D 4, 2821 (1971), and 13, 771(E) (1976); J. D. Bjorken and C. H. Llewellyn Smith, Phys. Rev. D 7, 887 (1973); C. H. Albright, C. Jarlskog, and M. O. Tijia, Nucl. Phys. B86, 535 (1975).

Lepton-Cascade-Decay Interpretation of Neutrino-Produced Trimuons*

V. Barger, T. Gottschalk, and D. V. Nanopoulos
Physics Department, University of Wisconsin, Madison, Wisconsin 53706

and

J. Abad†
Physics Department, University of Zaragoza, Spain

and

R. J. N. Phillips
Rutherford Laboratory, Chilton, Didcot, Oxon, England
(Received 29 March 1977)

We explain neutrino-produced trimuon events with a gauge model incorporating heavy leptons M^- and M^0 with the cascade-decay chain $M^- \to M^0\mu^-\bar{\nu}$ and $M^0 \to \mu^-\mu^+\nu$. Comparisons with trimuon data are made for M^- and M^0 masses 7 and 3.5 GeV, respectively. This mechanism predicts same-sign dimuons at five times the trimuon rate.

Energetic trimuon events have recently been observed in neutrino experiments.[1,2] The rate and kinematics of these events seem incompatible with dilepton production at the hadron vertex, e.g., from associated charm production. In this Letter we show that all characteristics of the trimuon events can be understood in a gauge-theory model which includes M^- and M^0 heavy leptons of masses 7 and 3.5 GeV, respectively. The essential feature of this interpretation is neutrinoproduction of M^-, which then decays via

$$M^- \to M^0\mu_1^-\bar{\nu}, \quad M^0 \to \mu_2^-\mu_3^+\nu. \qquad (1)$$

The general properties expected for three muons originating in this way from the neutrino vertex are evident in the data. A heavy-lepton cascade-decay interpretation of neutrino-produced trimuons was first suggested on the basis of a theoretical gauge model.[3]

For concrete illustration we consider an SU(2) ⊗ U(1) gauge model with the following new lepton doublets,

$$\begin{pmatrix} E^0 \\ E^- \end{pmatrix}_L, \begin{pmatrix} M^0 \\ M^- \end{pmatrix}_L, \begin{pmatrix} E^0 \\ e^- \end{pmatrix}_R, \begin{pmatrix} M^0 \\ \mu^- \end{pmatrix}_R. \qquad (2)$$

Here E^- is the SPEAR heavy lepton.[4] The $(E^0, e^-)_R$ doublet explains the apparent absence of parity nonconservation in atomic physics.[5] Then e-μ symmetry requires the $(M^0, M^-)_L$ and $(M^0, \mu^-)_R$ doublets; these doublets provide the cascade decay of interest for trimuons, provided M^- is heavier than M^0. In this specific model the M^- coupling to the incident ν_μ beam is achieved by M^--μ^- mixing. The experimental test of μ-e universality in π_{l2} decay limits the ν_μ-M^- coupling to $\epsilon G_F/\sqrt{2}$ with $\epsilon \leq 0.2$, which is of the same order as the Cabibbo angle; this assumes that the electron doublet is unmixed. However, in a larger gauge group or with a correlated mixing scheme,[6] this restriction on ϵ could be avoided entirely.

If we define $\langle\sigma_G(M^-)\rangle$ as the spectrum-averaged M^- production cross section for full Fermi coupling strength, the ratio of trimuon to single-muon rates is

$$\frac{N(\mu^-\mu^-\mu^+)}{N(\mu^-)} \simeq \eta^3\epsilon^2 B_\mu(M^-)B_\mu(M^0)\frac{\langle\sigma_G(M^-)\rangle}{\langle\sigma(\mu^-)\rangle}. \qquad (3)$$

Here η represents the mean detection efficiency per muon in trimuon events, and the B_μ denote muonic branching ratios. For energies $E_\nu > 100$ GeV and the Harvard-Pensylvania-Wisconsin-Fermilab (HPWF) quadrupole-triplet spectrum[1] we find $\langle\sigma_G(M^-)\rangle/\langle\sigma(\mu^-)\rangle \simeq 0.15$. From Monte Carlo calculations with detection energy cuts $E_\mu > 4$ GeV we obtain $\eta \simeq 0.88$. Roughly estimating B_μ as ~ 0.2 from quark-lepton counting and taking $\epsilon^2 = 0.04$ we find

$$\frac{N(\mu^-\mu^-\mu^+)}{N(\mu^-)} \simeq 2\times10^{-4}. \qquad (4)$$

The reported experimental rates are 5×10^{-4} for Ref. 1 and 3×10^{-4} for Ref. 2, based on six events and two events, respectively. If higher-statistics data indicate a substantial discrepancy with Eq. (4), a larger gauge group or a correlated mixing scheme[6] will be needed. However, the essential dynamical features of our model calculations will still apply.

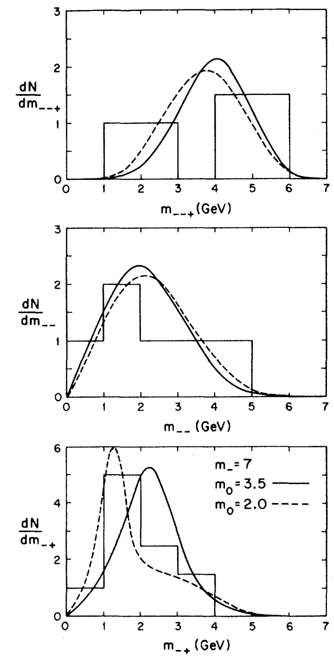

FIG. 1. Invariant-mass distributions for trimuon events in our heavy-lepton cascade-decay model, compared with data from Ref. 1.

The cascade-decay mechanism also predicts same-sign $\mu^-\mu^-$ dimuon events, when the μ^+ is undetected, or when the $\mu^+\nu$ vertex is replaced by an $e^+\nu$ or hadron vertex. The ratio of same-sign dimuon to trimuon events is

$$N(\mu^-\mu^-)/N(\mu^-\mu^-\mu^+) \simeq (1-\eta)/\eta + (1-B_\mu)/(B_\mu\eta) \simeq 5. \qquad (5)$$

Same-sign dimuons have been observed,[7] but the rate relative to trimuons is not yet established.

We have calculated the trimuon experimental distributions shown in Ref. 1 and have found broad overall agreement between the data and the model described above. Our calculations average over the neutrino spectrum of Ref. 1 and incorporate detection energy cuts $E_\mu > 4$ GeV. We present some of the more interesting comparisons here. Figure 1 shows the distributions of the observable invariant-mass combinations $m_{--+} = m(\mu^-\mu^-\mu^+)$, $m_{--} = m(\mu^-\mu^-)$, and $m_{-+} = m(\mu^-\mu^+)$ summing over the two $\mu^-\mu^+$ alternatives in the latter case. The kinematical endpoint of all three distributions is the mass m_- of the M^- parent. The m_{--+} distribution reflects this scale most clearly, with $\langle m_{--+}\rangle \simeq m_-/2$. The lower peaking of m_{-+} is partly due

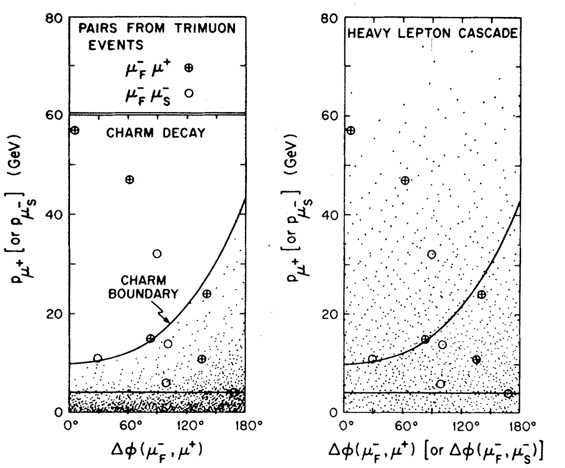

FIG. 2. Momentum-azimuth correlation test of hadronic vs heavy-lepton origin of trimuon events. (i) Momentum of the slow charm decay muon (μ^+ or μ_S^-) vs its azimuthal separation from the fast muon (μ_F^-). (ii) Momentum of slow cascade decay products of heavy leptons (μ^+ or μ_S^-) vs the azimuthal separation from the faster negative muon (μ_F^-). The trimuon data of Ref. 1 are compared with these predictions. The Monte Carlo theoretical distributions are normalized to 1000 events in each case.

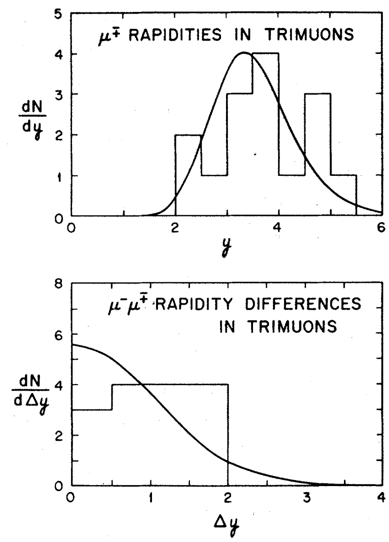

FIG. 3. Rapidity distributions of muons from heavy-lepton-cascade decays. The $y^{\pm}$ distributions for $\mu^{\pm}$ have similar forms and are shown combined. The rapidity differences Δy^{-+} and Δy^{--} also resemble one another and are shown combined. Trimuon data are from Ref. 1.

to the M^0 component, which has $\langle m_{-+}\rangle \simeq m_0/2$. The invariant-mass distributions are somewhat insensitive to m_0; to illustrate this, Fig. 1 also shows the case $m_0 = 2$ GeV (with $m_- = 7$ GeV as before). Reducing m_0 increases the mean energy of the fast μ^-, and reduces the mean energy of the slow μ^- and μ^+.

Consider $\nu N \to \mu^- H$ events, with H a hadron jet: The $\mu^- H$ azimuthal angular difference about the neutrino axis is $\Delta\varphi = 180°$. As the energy of a particle in the jet increases, its momentum becomes more closely correlated with the jet axis, and its azimuthal distribution relative to the μ^- becomes increasingly peaked at $\Delta\varphi = 180°$. This is a crucial and distinctive property of hadron-jet constituents, shared also by their decay products, e.g., muons from charm decay.[8] Figure 2 illustrates this effect for dimuons from charm decay ($\nu N \to \mu^- DX$, $D \to K^* \mu^+ \nu$); the correlation between the momentum of the decay μ^+ and $\Delta\varphi(\mu^-, \mu^+)$ is evident. If the trimuons were of hadronic origin, i.e., the μ^+ and slow μ^- come from decaying hadrons, similar correlations between $p(\mu^+)$ and $\Delta\varphi(\mu_F^-, \mu^+)$ and between $p(\mu_S^-)$ and $\Delta\varphi(\mu_F^-, \mu_S^-)$ should occur (here subscripts F and S denote fast and slow). When the trimuon data[1] are plotted in this fashion, it is clear that a hadron-vertex interpretation is excluded. Figure 2 also shows the corresponding predictions from our heavy-lepton production and decay sequence, which agree with the data.

Predicted distributions in rapidity $y = \frac{1}{2}\ln[(E + p_z)/(E - p_z)]$ are compared with results from trimuon events in Fig. 3. Individual muon rapidities are large but rapidity differences are small, in complete accord with the heavy-lepton-decay picture.

The two undetected neutrinos, carrying substantial missing energy and missing transverse momentum are an essential feature of our lepton cascade interpretation. With the HPWF neutrino spectrum,[1] we calculate the following average energies in GeV units: 175 (incident ν_μ), 126 (M^-), 76 (M^0), 30 (μ_1^-), 25 (μ_2^-), 39 (μ_F^-), 16 (μ_S^-), 27 (μ_3^+), 24 (ν), 20 ($\bar{\nu}$), for the case $m_- = 7$ and $m_0 = 3.5$. The mean missing energy carried by the neutrinos is 54% of the 3μ energy. Hence the incident neutrino energy is appreciably higher than the visible energy, a prediction that can be tested with narrow-band beams. The predited mean missing momentum transverse to the beam axis is 1.7 GeV. The average energies from the six trimuon events of Ref. 1 are 59 (μ_F^-), 13 (μ_S^-), 31 (μ^+), in general agreement with the predicted values above, given the present experimental and theoretical uncertaintites.

Other interesting effects can be expected in heavy-lepton production, as for example apparent lepton-number nonconservation in $\mu^- \mu^- e^+$ events. Also the M^--μ^- mixing may bring in a neutral-current coupling allowing $M^- \to \mu^- \mu^- \mu^+$ single-step decay into three leptons at the few-percent level. Such trimuons would have a unique invariant mass m_{--+}, higher individual muon energies, and higher visible energy than the cascade-decay events; it is possible that event 119-017991 of Ref. 1 may be in this category.

We thank D. Cline and D. D. Reeder for discussions.

Note added.—Details of our Monte Carlo calculations of heavy-lepton production and cascade decay are described in a comprehensive paper.[9] In each amplitude the lepton helicities are correctly followed from the production through the final stage of the decay; we neglect interference between amplitudes with different intermediate configurations. We use standard scaling structure functions at the hadron vertex. An extensive analysis of branching fractions for the cascade-decay chain is given.[10] The inclusive trimuon branching fraction $B(M^- \to \mu^- \mu^- \mu^+ X) = 0.04$ is found for $m_- = 7.0$ GeV, $m_0 = 2.5$ GeV, consistent with the branching fractions used in this Letter; of this 0.022 comes from the $X = \nu\bar{\nu}$ states calculated here. Finally, in Ref. 6 we point out that universality restrictions on the magnitude of ϵ can be circumvented in our $SU(2) \otimes U(1)$ model by a common mixing of leptons with heavy leptons and quarks with heavy quarks.

*Work supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the U. S. Energy Research and Development Administration under Contract No. E(11-1)-881, COO-595.

†On leave at the University of Wisconsin, supported by the Commission of Cultural Interchange between Spain and U. S. A.

[1]A. Benvenuti *et al.*, Phys. Rev. Lett. 38, 1110 (1977), and second preceding Letter [Phys. Rev. Lett. 38, 1183 (1977)].

[2]B. C. Barish *et al.*, Phys. Rev. Lett. 38, 577 (1977).

[3]V. Barger and D. V. Nanopoulos, University of Wisconsin Report No. COO-881-583, 1977 (to be published).

[4]M. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975), and Phys. Lett. 63B, 466 (1976).

[5]P. Baird *et al.*, Nature (London) 264, 528 (1976).

[6]V. Barger, D. V. Nanopoulos, and R. J. N. Phillips, University of Wisconsin Report No. COO-881-597, 1977 (to be published).

[7]A. Benvenuti *et al.*, Phys. Rev. Lett. 35, 1199 (1975).

[8]V. Barger, T. Gottschalk, and R. J. N. Phillips, University of Wisconsin Report No. COO-881-564, 1977 (to be published), and to be published.

[9]V. Barger, T. Gottschalk, D. V. Nanopoulos, J. Abad, and R. J. N. Phillips, University of Wisconsin Report No. COO-881-596, 1977 (to be published).

[10]V. Barger, T. Gottschalk, D. V. Nanopoulos, and R. J. N. Phillips, University of Wisconsin Report No. COO-881-598, 1977 (to be published).

Test of the Quark Parton Model with Data from Electroproduction of Pions*

J. F. Martin
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

L. S. Osborne
Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 3 December 1976)

We have measured the relative cross sections of π^+ and π^- electroproduction from protons and neutrons, and combined them in a way which permits a new, detailed comparison with the quark parton model of the nucleon. We confirm the expected behavior with $1/\omega$ of the quark charge ratio and the quark fragmentation function.

It has been pointed out by several authors[1,2] that a measurement of the relative production of $\pi^\pm$ and π^0 mesons from lepton-nucleon collisions can be related to the parameters of quark parton models. Given that quarks have not been seen, one may wonder whether their charge and their distribution functions (for example) are realities or merely parameters adjusted to fit data. We develop a method to separate these parameters and use our data to check the model's assump-