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Simple Form of the Fixed-Scatterer Approximation: An Application to e^--H Scattering

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A tractable form of the fixed-scatterer approximation retaining up to double-scattering terms has been proposed. This formalism can be employed to investigate the elastic as well as inelastic processes at intermediate and high energies. The results for the differential cross section for the elastic scattering and for excitation of the $n=2$ states of H atom in e^--H scattering are found to be very encouraging, computational labor involved for s-s transitions being comparable to that of the first Born calculations.

The fixed-scatterer approximation (FSA) which assumes a stationary configuration of the target enjoys wide application in intermediate- and high-energy scattering. Chase[1] seems to have been the first to have suggested this model. The idea of a fixed-nuclei model (Golden *et al.*[1]) has its origin long before Chase's prescription. Stier[1] and Fisk[1] have applied this model, which is similar to the present one, to investigate the e^--molecule scattering. The Glauber multiple-scattering model[1] is the most commonly used method underlying this assumption. Apart from its application in other fields, Glauber multiple-diffraction theory has been extensively used in investigating atomic collision processes.[2] In addition to the frozen-nucleus assumption, Glauber theory assumes additivity of the phases and neglect of the longitudinal momentum transfer; moreover, it neglects the off-shell contributions. As a consequence of these approximations, the Glauber elastic amplitude gives a logarithmic singularity in the forward direction, and it fails to predict even the qualitative nature of the cross section for the large-angle region. The Glauber model suffers from some other practical difficulties. The results for e^--H scattering are identical to those for e^+-H in the framework of Glauber's method. The simplicity of the Glauber prescription is lost when one is interested in applying his formalism to complex atoms or incorporating exchange effects.

Considering all these points, in the present study I have used the FSA to investigate the scattering problem. I have assumed that triple and higher-order scattering terms are negligible. At high energies, this assumption is always valid. As in the present study, the Glauber approximation also neglects the triple and higher-order scattering terms. The present formalism can distinguish between the results obtained by using electrons and positrons as projectiles.

I will now give a brief outline of the present formalism. In the center-of-mass system, we can write the Schrödinger equation for e^--H scattering processes in the FSA as

$$(E-\epsilon_T-T_1)\psi_{\vec{k}_i}^{+}(\vec{r}_1,\vec{r}_2) = V(\vec{r}_1,\vec{r}_2)\psi_{\vec{k}_i}^{+}(\vec{r}_1,\vec{r}_2). \quad (1)$$

In the framework of FSA, the total wave function $\psi_{\vec{k}_i}^{+}(\vec{r}_1,\vec{r}_2)$ in the incident channel may be expressed as

$$\psi_{\vec{k}_i}^{+}(\vec{r}_1,\vec{r}_2)=\varphi_0(\vec{r}_2)F_{\vec{k}_i,\vec{r}_2}^{+}(\vec{r}_1), \tag{2}$$

where $\varphi_0(\vec{r}_2)$ is the wave function of the target ground states, $V(\vec{r}_1,\vec{r}_2)$ is the perturbed potential, and $\vec{k}_i$ is the incident momentum. Here the function $F_{\vec{k}_i,\vec{r}_2}^{+}$, which has parametric dependence on the bound electron at $\vec{r}_2$, satisfies the usual boundary condition, i.e., a plane wave plus an outgoing spherical waves. Equation (1) can be written using expression (2) as

$$(E-\epsilon_T-T_1)F_{\vec{k}_i,\vec{r}_2}^{+}(\vec{r}_1)=V(\vec{r}_1,\vec{r}_2)F_{\vec{k}_i,\vec{r}_2}^{+}(\vec{r}_1). \tag{3}$$

The function $F_{\vec{k}_i,\vec{r}_2}^{+}(\vec{r}_1)$ must satisfy the Lippmann-Schwinger equation

$$F_{\vec{k}_i,\vec{r}_2}^{+}(\vec{r}_1)=|\vec{k}_i\rangle+\frac{1}{k_i^2/2\mu_i+i\epsilon-T_1}V(\vec{r}_1,\vec{r}_2)F_{\vec{k}_i,\vec{r}_2}^{+}(\vec{r}_1)$$
$$=|\vec{k}_i\rangle+2\mu_i\int d^3k''\frac{|\vec{k}''\rangle\langle\vec{k}''|V(\vec{r}_1,\vec{r}_2)|F_{\vec{k}_i,\vec{r}_2}^{+}(\vec{r}_1)\rangle}{k_i^2-k''^2+i\epsilon}. \tag{4}$$

Now one can obtain the integral equation of the fixed scattering T matrix as (Chaudhuri, Ghosh, and Sil[3])

$$\langle\vec{k}_f|V|F_{\vec{k}_i,\vec{r}_2}^{+}\rangle=\langle\vec{k}_f|V|\vec{k}_i\rangle+2\mu_i\int d^3k''\frac{\langle\vec{k}_f|V|\vec{k}''\rangle\langle\vec{k}''|V|F_{\vec{k}_i,\vec{r}_2}\rangle}{k_i^2-k''^2+i\epsilon}, \tag{5}$$

where k_f is the scattered momentum. The above equation shows that it has a parametric dependence on the position vector $\vec{r}_2$ of the target electron. As a result, one would find it very difficult to solve the integral equation, even for one-electron targets, by making a partial-wave analysis. For many-electron targets, solution is impracticable. So I have retained up to double-scattering terms in Eq. (4) and the resulting T matrix is

$$\hat{T}_{\vec{k}_i,\vec{k}_f}(\vec{r}_2)=T_{\vec{k}_i,\vec{k}_f}^{S}(\vec{r}_2)+2\mu_i\int d^3k''\frac{\hat{T}_{\vec{k}'',\vec{k}_f}^{S}(\vec{r}_2)\hat{T}_{\vec{k}_i,\vec{k}''}^{S}(\vec{r}_2)}{k_i^2-k''^2+i\epsilon}, \tag{6}$$

where

$$\hat{T}_{\vec{k}_\alpha,\vec{k}_\beta}^{S}=\langle\vec{k}_\beta|V|\vec{k}_\alpha\rangle \text{ and } \hat{T}_{\vec{k}_\alpha,\vec{k}_\beta}=\langle\vec{k}_\beta|V|F_{\vec{k}_\alpha,\vec{r}_2}^{+}\rangle. \tag{7}$$

The first term corresponds to single scattering and the second term, to double scattering. The transition matrix element from the initial state $|i\rangle$ to the final state $\langle f|$ is given by

$$\langle\varphi_f|\hat{T}_{\vec{k}_i,\vec{k}_f}|\varphi_i\rangle=\langle\varphi_f|\hat{T}_{\vec{k}_i,\vec{k}_f}^{S}|\varphi_i\rangle+\langle\varphi_f|\hat{T}_{\vec{k}_i,\vec{k}_f}^{D}|\varphi_i\rangle, \tag{8}$$

where

$$\hat{T}_{\vec{k}_i,\vec{k}_f}^{D}=2\mu_i\int d^3k''T_{\vec{k}'',\vec{k}_f}^{S}T_{\vec{k}_i,\vec{k}''}^{S}/(k_i^2-k''^2+i\epsilon). \tag{9}$$

It may be mentioned that the present theory may be extended to study nuclear and particle scattering processes without much trouble.

The present formalism is similar to second Born approximation. Under certain assumption, one can obtain a double-scattering term similar to the present one from the corresponding second Born term by using the closure relation.[4] In that case, however, one has to replace the energy difference by an average excitation energy. In the present case the energy difference is zero. As a consequence, the position of the pole of the Green's function will be different from that of the present one.

To test the validity of this formalism, I have investigated e^--H scattering. Taking account of the proper normalization, we can write the expression for scattering amplitude [Eq. (8)] as

$$f_{fi}(\hat{k}_f\cdot\hat{k}_i)=f_{fi}^{S}(\hat{k}_f\cdot\hat{k}_i)+\frac{1}{4\pi^2\mu_i}\int dk''f_{fi}^{D}(\hat{k}_f,k'',\hat{k}_i),$$

where μ_i is the reduced mass of the system in the initial channel. The elastic scattering amplitude

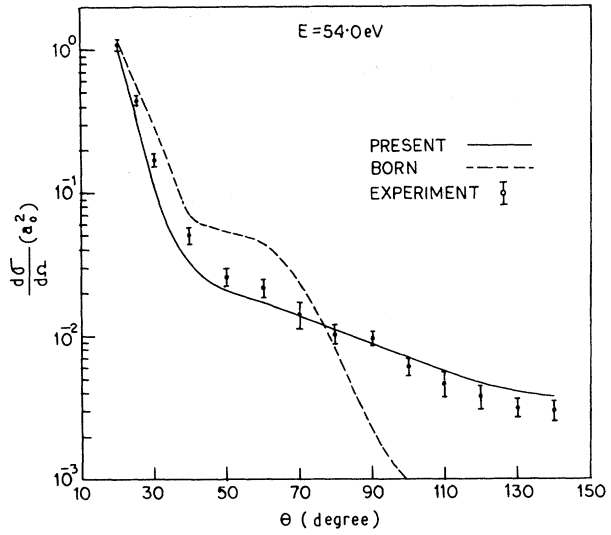

FIG. 1. Differential cross section for elastic e^--H scattering.

for e^--H scattering is given by

$$f_{1s,1s}(\hat{k}_f\cdot\hat{k}_i) = \left\{\frac{2}{K^2+4}+\frac{8}{(K^2+4)^2}\right\}+\left\{\frac{A}{K^2+4}+\frac{B}{(K^2+4)^2}\right\},$$

where

$$\vec{K}=\vec{k}_i-\vec{k}_f$$

and

$$A=\frac{2}{k_i}\tan^{-1}\left(\frac{1}{k_i}\right)-\frac{4}{4k_i^2+1},$$

$$B=\frac{8}{k_i}\tan^{-1}\left(\frac{1}{k_i}\right).$$

I have also calculated the $1s$-$2s$ and $1s$-$2p$ excitation amplitudes. I have not given the final expressions for these inelastic amplitudes as they are lengthy. It may be mentioned that the s-s transition amplitudes can be evaluated analytically whereas one-dimensional integration is to be performed numerically for the s-p transition amplitudes.

The computational labor involved to calculate the present amplitude is comparable to that of the first Born approximation. The results for the differential cross section at three projectile energies are plotted in Fig. 1. At all three energies, the present results are superior to Glauber[5] or first Born (single-scattering) results. My results underestimate the cross sections near the forward direction, although the value of the cross

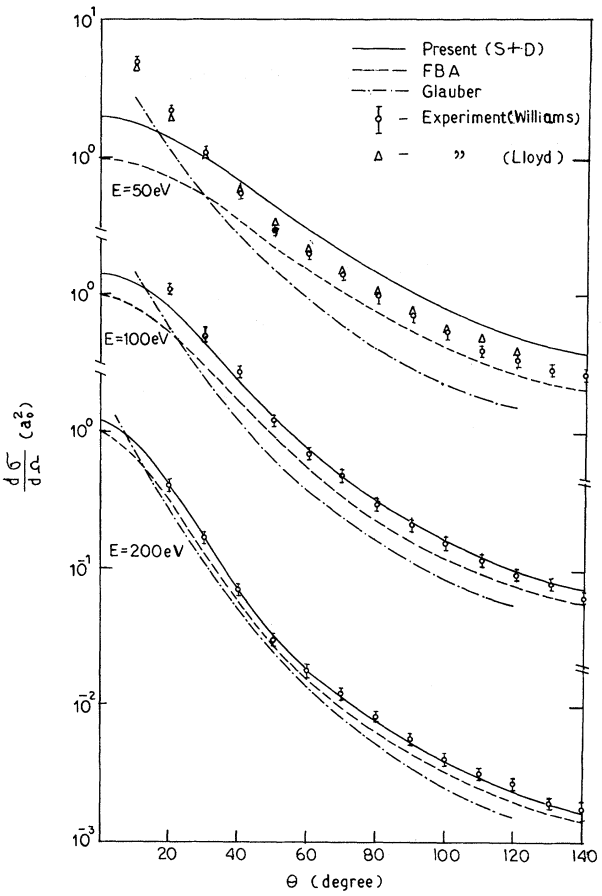

FIG. 2. Differential cross section for e +H(1s) → e +H(2s +2p) at 54 eV.

section at 50 eV in the forward direction is nearly double that of the corresponding value in the first Born approximation. The agreement between the present results and available measured values[6] is excellent at the incident energies 100 and 200 eV. The effect of the double scattering can be seen from the present and the corresponding single-scattering (first Born) curve. The contribution of the double scattering decreases with the increase of incident energy. This contribution is negligible at the incident energies $E \gtrsim 500$ eV. The present and the first Born results are indistinguishable in the high-energy range.

The disagreement between the present results and the measured values may be due to the following reasons. The neglect of third-order terms is equivalent to ignoring the contributions of the order of k_i^{-2} in the differential cross section in the high energies. It may also be mentioned that the exchange amplitude is also of order k_i^{-2} for large k_i. The discrepancies near the forward elastic

scattering angles at all energies are due to the fact that we have neglected the long-range forces. The inclusion of above-mentioned effects is expected to improve the results substantially.

Figure 2 represents my curve and the measured values.[7] The agreement between the present results and the experimental values is very satisfactory. The effect of the double scattering can be judged by comparing the Born (single scattering) and the present curves. This effect is appreciable for large scattering angle. It has been seen that with the increase of energy, the agreement between the present results and observed values are also improving.

The author is thankful to Mrs. Sunanda Guha for checking calculations.

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Doubly Excited Autoionization Resonances in the Absorption Spectrum of Li^+ Formed in a Laser-Produced Plasma*

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In a technique using a continuum generated by focusing a laser beam on a target of high atomic number, the spatially resolved absorption spectrum of Li^+ formed in a second laser-produced plasma was studied from 200 to 50 Å. The doubly excited resonances, $1s^2\,{}^1S$-$2snp\,{}^1P$, as well as the principal series and its adjoining photoionization continuum were observed. The Fano parameters, q and Γ, of the $2s2p\,{}^1P$ were derived from the absorption profile. Comparison is made with theory.

Controlled experiments on the absorption spectra of atomic ions are of importance because they can provide information which cannot be obtained from emission spectra, particularly on inner-shell and double electron excitation and also on photoionization continua. Little work however has been done in this field, although some recent experiments have shown promising results.[1-3] In this Letter we report some work on what promises to be a relatively simple procedure for obtaining the absorption spectra of a wide range of ionic species.

In our experiments, the absorption spectra of laser-produced plasmas were systematically studied. The main effort was concentrated on lithium because in its singly ionized state it constitutes the second member of the heliumlike isoelectronic sequence and consequently its double electron transitions, if observed and studied, would be of considerable theoretical interest. A single Q-switched ruby laser (pulse length, 40–50 nsec; pulse energy, 1 J) was used to produce both the ionized species and the background continuum. To generate the continuum, part of the laser beam was sharply focused on a tungsten target (Fig. 1). Following the work of Ehler and Weissler[4] we found, in a separate series of experiments,[5] that plasmas generated on tungsten, uranium and other targets of high atomic number gave continua of various degrees of uniformity