Comparison of Macroscopic and Microscopic Calculations of High-Energy $^{20}Ne + ^{238}U$ Collisions*

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For the reaction $^{20}Ne + ^{238}U$ at a laboratory bombarding energy per nucleon of 250 MeV we calculate the cross section $d^2\sigma/dE\,d\Omega$ for outgoing protons using four different approaches. These are relativistic fluid dynamics, classical many-body calculations with a hard-sphere nucleon-nucleon potential, and two versions of relativistic intranuclear-cascade calculations. These calculations reproduce some features of the experimental data, but some major discrepancies remain.

Many groups are now studying what happens when two heavy nuclei collide at high energies. Interest in this field stems from the possibility that during collisions at high energy, heavy nuclei may become compressed to more than their normal density. To describe high-energy heavy-ion collisions, a variety of theoretical approaches are currently being pursued. These include fluid dynamics with a given nuclear equation of state,[1-4] classical many-body calculations with a given nucleon-nucleon potential,[5-9] and intranuclear-cascade calculations with a given nucleon-nucleon cross section,[8-13] as well as simple geometric-thermodynamic models.[14] These approaches all use classical mechanics, but otherwise rely upon different approximations concerning the dynamics of the reaction. For the description of low-energy heavy-ion collisions, the time-dependent Hartree-Fock approximation is also a useful starting point.[15-17] However, because of the mean-field approximation that is involved, this method is unsuitable at the high energies considered here.

Here we compare some results calculated from relativistic fluid dynamics,[3,4] nonrelativistic classical many-body calculations with a hard-sphere nucleon-nucleon potential,[8,9] and two versions of relativistic intranuclear-cascade calculations.[11,12] In particular, we use each approach to calculate the double-differential cross section $d^2\sigma/dE\,d\Omega$ for outgoing protons from the reaction $^{20}Ne + ^{238}U$ at a laboratory bombarding energy per nucleon of 250 MeV, for which recent experimental data[14] are available.

In the first approach, we solve numerically in three spatial dimensions the classical relativistic equations of fluid dynamics.[3,4,18] These equations express the conservation of nucleon number, momentum, and energy, for a specified nuclear equation of state, which is currently taken from theory.[3,4,19] We neglect nuclear viscosity, surface energy, Coulomb energy, and single-particle effects, as well as the production of additional particles. From the particle density and velocity vectors at some large time we construct the energy and angular distributions for the expanding matter. Upon integrating these results over impact parameter and assuming that the proton density is a constant fraction $\frac{102}{258}$ of the matter density, we obtain $d^2\sigma/dE\,d\Omega$ for outgoing protons.

In the second approach, we solve numerically the classical nonrelativistic many-body equations of motion for a collection of twenty hard-sphere nucleons impinging on a collection of 238 hard-sphere nucleons.[8,9] The hard-sphere diameter is taken as 0.9 fm. The associated nucleon-nucleon potential is not realistic, but it leads to a many-body problem that is extremely simple to solve and that is roughly realistic in its implied nucleon-nucleon cross section of 25.4 mb, all elastic and isotropic. In choosing the microscopic initial conditions within the target and projectile, we neglect the Fermi motion. The cross section $d^2\sigma/dE\,d\Omega$ is calculated by counting each scattered nucleon as $\frac{10}{20}$ of a proton if it came originally from ^{20}Ne and $\frac{92}{238}$ of a proton if it came originally from ^{238}U.

In the third and fourth approaches, we calculated by Monte Carlo techniques the intranuclear cascade following a heavy-ion collision.[11,12] In both of these cascade calculations, individual hadrons interact with each other in accordance with experimental hadron-hadron cross sections.[20,21] Pion production and absorption are taken into account approximately in terms of nuclear resonances.[12,22,23] The primary difference between the two cascade calculations is that in the first version the cascades initiated by different projectile nucleons are treated as independent of one another,[11] whereas in the second version all of the nucleons in both the target and projectile evolved simultaneously in time.[12]

To elaborate, the first version corresponds essentially to a superposition of single-proton and single-neutron collisions with a ^{238}U target, except that the weighting over impact parameter is appropriate to nucleons inside a ^{20}Ne projectile. The Fermi motion and nuclear potential are taken into account for target nucleons but are ignored for projectile nucleons. We neglect the evaporation of protons from the target and the projectile. Their inclusion would increase the calculated spectra at energies below about 40 MeV and at energies near 250 MeV. The number of pions emerging from the nucleus was calculated to be less than 1% of the number of protons emerging. A calculation was also made with the pion production turned off, and the spectra remained the same to within statistical errors. However, because the Fermi motion is neglected in the projectile and because pion production increases rapidly with increasing energy at these nucleon energies, our first-version cascade calculation underestimates pion production.

In the second version of the intranuclear-cascade calculations, the projectile and target nucleons are treated on an equal footing. Baryon conservation is maintained by depleting the continuous nuclear density distribution by one nucleon for every cascade particle produced. The Fermi motion of the nucleons is taken into account in

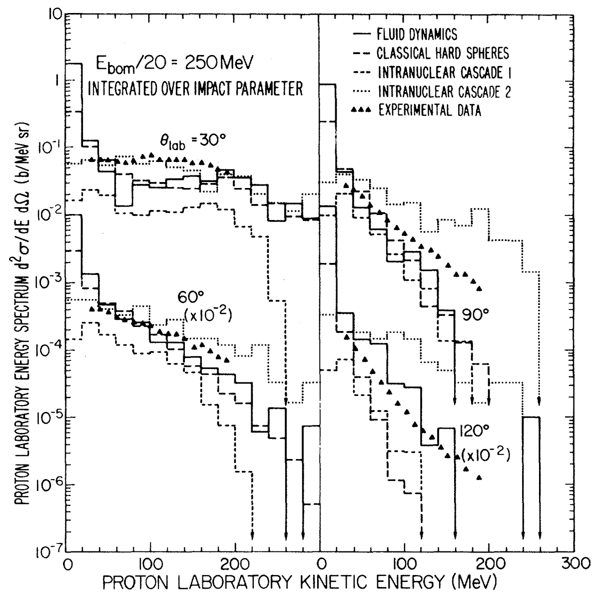

FIG. 1. Calculated and experimental (Ref. 14) proton energy spectra. Angular bins of 10° width are used for the calculated curves.

both the target and projectile, whose surfaces are taken to be diffuse. Single-particle binding-energy and evaporation effects are incorporated by placing all particles off their mass shell.[12]

Some results of these four calculations are compared in Fig. 1 with one another and with experimental data.[14] The figure shows proton energy spectra, $d^2\sigma/dE\,\Omega$ vs E, for four laboratory angles ranging from 30° to 120°. The theoretical results in the two lowest-energy bins (laboratory energy $E \leq 40$ MeV) should not be taken seriously because the various calculations either treat binding very poorly or else neglect the evaporation of protons from the target. Some measure of the numerical accuracy of the calculations can be determined from the fluctuations in the histograms. The inaccuracy in the fluid-dynamics calculation arises from the limited number of computational particles used, as well as from other finite-difference errors in the solutions of the equations of motion. The inaccuracy in the other three calculations is statistical in nature, arising from the limited number of initial conditions used. The contributions from ejected particles heavier than protons are excluded from the experimental results, but are included in all of the theoretical calculations. This does not affect the discussion below.

All of the calculations reproduce the general decrease in the experimental cross sections when going from forward to backward angles. However, some of the details of the cross section are significantly incorrect in all of the theoretical approaches. At 30° the values calculated from fluid dynamics lie below the experimental ones, whereas at 120° they lie above. At all angles the values calculated from classical hard spheres and from the first version of an intranuclear cascade lie mostly below the experimental ones. At forward angles the values calculated from the second version of an intranuclear cascade are in approximate agreement with the experimental ones, whereas at backward angles the calculated values lie substantially above the experimental ones.

The character of the discrepancy between fluid dynamics and experiment suggests that in collisions at energies per nucleon ≳ 250 MeV, heavy nuclei are partially transparent to each other. In other words, upon impact the target matter and projectile matter interpenetrate somewhat, without obeying one equation of state at all points in space. Such interpenetration is the result of a nonzero mean free path for nucleon-nucleon interactions and a nonzero momentum decay length in the forward direction.[2]

The large differences between the results calculated with the two versions of an intranuclear cascade arise from a combination of effects. These include different treatments of projectile Fermi motion, binding, and evaporation in the two versions, as well as different treatments of the cascades themselves. In the second version, the simultaneous evolution of all the nucleons in both the target and projectile permits the nuclear matter to become more compressed than in the first version. The simultaneous evolution also increases the chances of high-energy nucleon-nucleon collisions. The net result is that in the second version the number of outgoing nucleons is substantially larger than in the first version, especially at backward angles.

We show in Fig. 2 the calculated angular distributions of the outgoing protons for four laboratory proton energies ranging from 30 to 150 MeV. At the lower energies fluid dynamics predicts a small, very broad peak at about 50°, whereas at the higher energies it predicts a somewhat sharper peak at about 120°. This is to be contrasted with the sharp peaks predicted for both forward and backward angles in other fluid-dynamics approaches based on a small projectile hitting an infinitely large target.[1]

Although the calculated curves shown in Figs. 1 and 2 are somewhat similar to each other, this similarity is largely the result of an integration over impact parameter. Figure 3 shows that for head-on collisions the predictions of fluid dynamics are substantially different from those of the other approaches. In particular, at forward angles fluid dynamics yields only particles of very

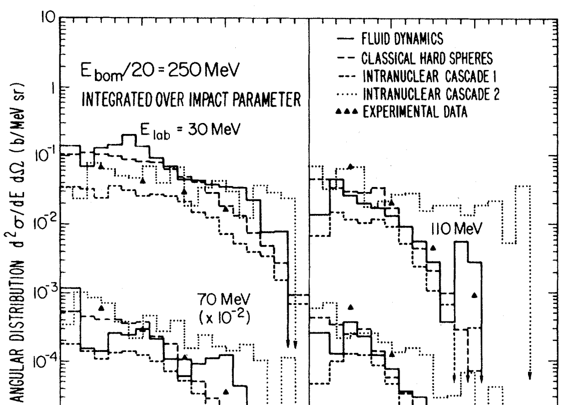

FIG. 2. Calculated and experimental (Ref. 14) proton angular distributions. Energy bins of 20 MeV width are used for the calculated curves.

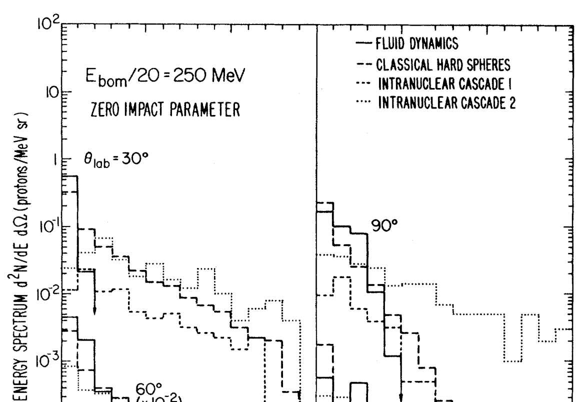

FIG. 3. Calculated proton energy spectra for head-on collisions. Angular bins of 10° width are used.

low energy. Experiments concentrating on nearly head-on collisions are therefore of great interest.

In conclusion, the present macroscopic and microscopic calculations reproduce some experimental features of high-energy $^{20}Ne + ^{238}U$ collisions, but there are some major discrepancies. The removal of these discrepancies will probably require some significant modifications of the present theoretical approaches. These include the incorporation of transparency in the fluid-dynamics calculations, the use of more realistic two-nucleon potentials in the classical many-body calculations, and an improved treatment of binding and evaporation in the intranuclear-cascade calculations.

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