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Quantum Effects of Pseudoparticle Size*

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It is shown that the calculation of quantum effects associated with the classical four-dimensional pseudoparticle can be carried to completion in the one-loop, lowest-order-in-source approximation, with size effects calculated exactly. The result is finite, and leaves symmetry-breaking results intact. For the case of two flavors the theory reduces to a particularly simple form which suggests that the pseudoparticle acts as a source of zero-mass bosons which, in turn, are made of fermion pairs.

Recently, 't Hooft[1,2] has calculated the quantum effects associated with the classical pseudoparticle solution found by Belavin *et al.*[3] in non-Abelian gauge theories. This pseudoparticle was conjectured by Polyakov[4] to be the basis of the confinement mechanism of a color theory of strong interactions. Related vacuum tunneling effects have been stressed by Jackiw and Rebbi,[5] and by Callan, Dashen, and Gross as well.[6]

't Hooft's calculation of the generating funtional

$$W = {}_{\rm out}\langle 0|0\rangle_{\rm in} = \int \mathfrak{D}A\mathfrak{D}\bar{\psi}\mathfrak{D}\psi\mathfrak{D}\Phi \exp\{\int d^4x \mathfrak{L}\} \tag{1}$$

proceeds via a perturbation expansion about the classical pseudoparticle solution keeping lowest-order (quadratic) terms, i.e., calculating the one-loop contribution. The zero-energy modes are handled by the collective coordinate formalism. They involve the position and size of the classical pseudoparticle solutions. Apart from a gauge and ghost fixing term and the scalar multiplets, the Lagrangian L describes N_f-flavored color-doublet fermions coupled color-gauge-invariantly (D_μ is the covariant derivative) to the triplet of color-vector-meson fields,

$$\mathfrak{L} = -\tfrac{1}{4}G_{\mu\nu}{}^a G_{\mu\nu}{}^a - \bar{\psi}\gamma_\mu D_\mu\psi + \bar{\psi}_s J_{st}\psi_t. \tag{2}$$

Here a labels color isospin; and, s and t are fermion flavor indices, suppressed in all but the arbitrary source term J_{st}. 't Hooft's calculation in Ref. 2 stops short of a generating functional with the collective pseudoparticle-size coordinate ρ completely integrated; rather, he finds W for fixed ρ, obtains a corresponding effective Lagrangian (ρ fixed), and then integrates over ρ. In the unbroken-color-gauge-theory interpretation of the effective Lagrangian, he then gets an apparently intractable infrared divergence.

In this Letter we would like to point out that at least for $N_f = 2$ the generating functional can be computed exactly by performing the remaining ρ integration explicitly. Then a corresponding effective Lagrangian can be constructed with some interesting properties to be discussed in the conclusion.

The functional integration that gives the one-loop contribution proceeds as follows:

$$\begin{aligned} W[J] &= e^{S_{\rm cl}}\int \mathfrak{D}A_0\mathfrak{D}A_{\neq 0}\mathfrak{D}\Phi\mathfrak{D}\psi\mathfrak{D}\bar{\psi} \exp\{\int d^4x\mathfrak{L}(A_j\Phi) + \bar{\psi}(M+J)\psi\} \\ &= \int d^4x \int_0^\infty d\rho \tilde{f}(\rho) \int \mathfrak{D}\psi\mathfrak{D}\bar{\psi} \exp \int d^4x \bar{\psi}(M+J)\psi \\ &= \int d^4x \int_0^\infty d\rho \tilde{f}(\rho) \det[M(\rho,x)+J], \end{aligned} \tag{3}$$

where $M(\rho,x)$ is the differential operator obtained from (2) by expanding about the classical solution. The notation A_0 and $A_{\neq 0}$ indicates the separation of zero and nonzero eigenvalues in the Gaussian integration, the former being handled by the collective-coordinate procedure. Now

$$\det[M(\rho,x)+J]\approx \pi_i E_i(\rho)\pi_k E_k^0(\rho,J), \quad (4)$$

where E_i's are the eigenvalues of $M+J$, with $E_k^0(\rho,J=0)=0$. With this separation 't Hooft[2] gets (u are some fixed spinors)

$$W[J]=\int d^4x\int_0^\infty d\rho f(\rho)\det_{st}\left[\int d^4z\frac{u_\alpha{}^*J_{st}(z)u^\alpha}{[\rho^2+(x-z)^2]^3}\right]. \quad (5)$$

In the case of no scalar multiplets, 't Hooft gives

$$f(\rho)=Ke^{-8\pi/g^2}g^{-8}(\mu_0\rho)^{7(1+N_f)/3}\mu_0{}^{5-3N_f}, \quad (6)$$

$$K=2^{(14+N_f)}\pi^{(6-2N_f)}\exp[2N_f\alpha(\tfrac{1}{2})-\alpha(1)], \quad (7)$$

where the α are numbers given in Ref. 2 and μ_0 is the renormalization subtraction point.

To continue further it is profitable to use Fourier transform on the source $J_{st}(x)$. For notational simplicity we write the formulas, for the moment, only for a diagonal source J. We define

$$J_i(z_i)=\int d^4p_i\exp(ip_i\cdot z_i)\tilde{J}_i(p_i). \quad (8)$$

Then after translating $z_i=x+w_i$, we obtain

$$W[\tilde{J}]=K\frac{e^{-8\pi^2/g^2}}{g^8}\mu_0{}^{2(11-N_f)/3}\prod_{i=1}^{N_f}[\int d^4p_i u_\alpha{}^*\tilde{J}_i(p_i)u^\alpha](2\pi)^4\delta^4(\sum_{j=i}^{N_f}p_j)I, \quad (9)$$

where

$$I=\int_0^\infty d\rho\,\rho^{7(1+N_f)/3}\prod_{k=1}^{N_f}\int d^4\omega_k\frac{\exp(ip_k\cdot w_k)}{(\rho^2+w_k{}^2)^3}. \quad (10)$$

Now the angular integration over any w_k can be done and we get

$$\int d^4w_k\frac{\exp(ip_k\cdot w_k)}{(\rho^2+w_k{}^2)^3}=2\pi^2\int_0^\infty d|w_k||w_k|^3\frac{\sin(|p_k||w_k|)}{|p_k||w_k|(\rho^2+\omega_k{}^2)^3}. \quad (11)$$

From this we see that there will be no angular correlation between the N_f momenta apart from momentum conservation, i.e., I is a function of $|p_k|$ only. For $|p_k|>0$, I is a finite function bounded by, e.g.,

$$|I|<\left(\prod_{l=1}^{N_f}|p_e|^{-1}\right)^{(10+N_f)/3N_f}\frac{226N_f}{(10+N_f)(8N_f-10)}(2\pi)^{N_f}. \quad (12)$$

For the case $N_f=2$ we have $|p_1|=|p_2|$ and, therefore, by scaling the ρ, w_1, and w_2 integrals we get

$$W[\tilde{J}]=Ke^{-8\pi^2/g^2}g^{-8}4\pi^4\mu_0{}^6\int d^4p_1\int d^4p_2(2\pi)^4\delta^4(p_1+p_2)$$
$$\times[u_\alpha{}^*\tilde{J}_{11}(p_1)u^\alpha u_\alpha{}^*\tilde{J}_{22}(p_2)u^\alpha-u_\alpha{}^*\tilde{J}_{12}(p_1)u^\alpha u_\alpha{}^*\tilde{J}_{21}(p_2)u^\alpha]\,|p_1|^{-2}|p_2|^{-2}\cdot C, \quad (13)$$

where

$$C=\int_0^\infty ds\,s^7\left[\int_0^\infty dt\frac{t^2\sin t}{(t^2+s^2)^3}\right]^2 \quad (14)$$

is a finite positive constant.

It is important to compare and contrast our result with 't Hooft's. All the chiral properties which arise from the determinant structure involving right-handed spinors u (and left-handed spinors when the parity-reflected solution is added) remain the same. Thus, conclusions regarding breakdown of $U(N^f)\times U(N^f)$ symmetry persist. Behavior of the theory with regard to the renormalization-group subtraction point is also unaffected.

However, with pseudoparticle-size effects exactly incorporated, 't Hooft's simple interpretation of the amplitude in terms of fermions propagating to the pseudoparticle fails in an essential way, i.e., the momentum dependence and connections are changed. Rather, our final result for $N_f=2$ has the form

$$\int d^4p\frac{K(p)K(-p)}{p^4}. \quad (15)$$

The physical interpretation of the result (15) is obscure to us. Perhaps a full understanding of the Green's function singularity structure can only be obtained after summing the one-loop contributions of the other (multipseudoparticle) stationary points in the functional integral. Our result, by itself a single term in this sum, represents an unacceptable dipole singularity which, if it persists in the full theory, would appear to be a catastrophe.

It is interesting to note that the p^{-4} term may be the second term in an expansion $(p^2+m^2)^{-1}$ in orders of m^2. Such a term might appear in a theory of bound fermion-antifermion pairs, but there is as yet no clear indication of such a binding phenomenon in the non-Abelian gauge theory. If, however, this were the case, our result might be relevant to a calculation of hadron masses such as the η mass, for example. (It is interesting that we then get a nonzero contribution already from the one-loop single-pseudoparticle sector.)

But what, then, is the origin of the k^{-2} term? And is it really there at this level (one-loop), or must one extend the expansion not only in the winding number, but also to higher order in $\hbar$?

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SO(3)-Invariant Extended Supergravity*

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I construct the supergravity theory for a supersymmetry algebra whose spinor generators carry SO(3) internal symmetry. Spins $\frac{1}{2}$, 1, $\frac{3}{2}$, and 2 are unified in one gravitational multiplet. The method used should be applicable to theories based on other internal symmetry groups.

Supergravity field theories are closely related to irreducible representations[1] of global supersymmetry algebras which contain massless particles of maximum spin 2. The first locally supersymmetric field theory[2-4] corresponds to the representation with spin content $(2, \frac{3}{2})$, while a very recent construction[5] corresponds to the representation $(2, \frac{3}{2}, \frac{3}{2}, 1)$ of an extended supersymmetry algebra in which the Majorana spinor charges and spin-$\frac{3}{2}$ states transform as doublets under an O(2) [or U(1)] internal symmetry.

There are analogous algebras and irreducible representations[6] for larger internal symmetry groups, and lower spin fields would be unified with spin $\frac{3}{2}$ and spin 2 in one gravitational supermultiplet in the corresponding field theories. This unification offers hope of a more intimate connection between gravitation and other particle interactions. Arguments have been given that such field theories have finite one-loop scattering amplitudes[7] and that the leading two-loop divergences cancel.[8] These are sufficient reasons to undertake the construction of further-extended-supergravity theories.

The next simplest representation is based on the group SO(3), and I construct here the corresponding consistent field theory which has the spin content $(2, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 1, 1, 1, \frac{1}{2})$ involving triplets of Majorana spin-$\frac{3}{2}$ and real spin-1 fields and singlet graviton and spin-$\frac{1}{2}$ fields. There is a local supersymmetry involving a triplet of Majorana spinor parameters, while the internal symmetry is global. The method used here in which the internal symmetry is manifest at all stages should be applicable to further constructions of this type.

In first-order gravitational formalism, the