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Determination of the Limiting Temperature and Scaling in the Mean in *pp* Interactions*

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The transverse-momentum distribution of charged pions produced in 28.5-GeV/c pp interactions was studied. Scaling of the transverse-momentum distribution versus the "mean" variable $p_T/\langle p_T\rangle$ was observed, independent of charge multiplicity, missing mass, and energy, in the central region. From the scaling behavior of the pions, the limiting temperature was determined to be $T_0 = 117 \pm 4$ MeV.

Since the temperature (T) enters into hydrodynamical,[1] statistical-thermodynamic,[2,3] and gauge[4] models as an empirical constant, it constitutes one of the most important parameters in theories of multiparticle hadron physics. It has been conjectured by Hagedorn[2] that T has a limiting value T_0. It has been suggested by Weinberg[4] that symmetry in gauge theories may be restored at sufficiently high temperature (T_c) just as ferromagnetism vanishes at the Curie temperature. An important physical consequence of the hadronic phase transition within a unified gauge theory is that, for sufficiently high-energy interactions, parity and strangeness could be conserved even in neutrino-induced interactions. These theoretical speculations[5] have revived interest in a direct experimental determination of Hagedorn's limiting temperature T_0 since it sets an upper limit for T_c contained in other models. Up to the present, the lack of high-statistic experiments with momentum analysis of the produced particles and mass identification of the leading particle made it impossible to isolate the kinematic region where the momentum distribution of the produced particles, *restricted to small longitudinal momentum* (p_L), was independent of missing mass and multiplicity.[6] As a result no experimental procedure has been available to determine a unique value for T_0 to date.[7] (See, however, Erwin *et al.*[8])

This experiment was carried out at the Brookhaven National Laboratory alternating gradient synchrotron with the multiparticle Argo spectrometer system (MASS).[9] Using the high-momentum spectrometer (HMS) and the vertex spectrometer (VS), we studied the reaction

$$p_1 + p_2 \to p_3 + X, \tag{1}$$

at 28.5 GeV/c. The HMS was used to trigger upon, identify, and momentum-analyze a forward proton (p_3). The VS, which consisted of nine cylindrical wire spark chambers surrounding a liquid-hydrogen target in a 10-kG magnetic field, detected the remaining charged particles.[10] Spark association into tracks was performed by the track recognition code PITRACK.[9] Finally the events were fitted to a common vertex and classified according to prong number.

The first step in our analysis is to demonstrate the existence of a universal transverse-momentum (p_T) distribution, which is independent of

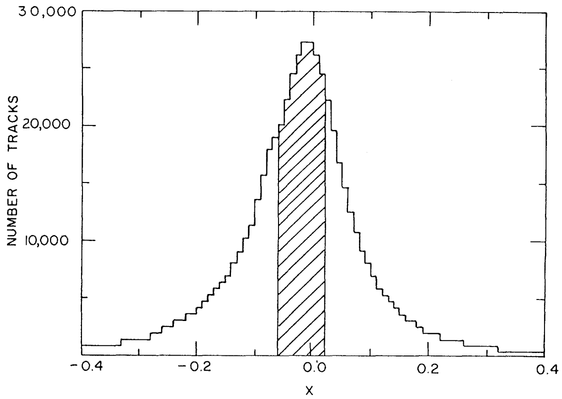

FIG. 1. The x distribution for the final-state charged particles in the reaction $p_1+p_2 \rightarrow p_3+X$. The trigger proton p_3 is excluded. The dashed region indicates the data used to obtain the scaling curves and limiting temperature.

multiplicity, missing mass, and beam momentum. For the 143 000 events of Reaction (1) with a charged multiplicity of at least four (four to ten prongs) the pion mass was assinged to all particles detected by the VS. Two-prong events and the trigger proton are excluded from the considerations to follow. The x distribution of the pions[11] is shown in Fig. 1. To obtain a universal curve[12] we studied the normalized distributions of the mean transverse-momentum variable $p_T/\langle p_T\rangle$ as a function of charged multiplicity. Small systematic deviations from a scaling behavior were observed. However, when the pions were restricted to the central region as indicated in Fig. 1, these deviations disappeared and scaling in the mean was observed.[13] The resulting universal scaling curve is shown in Fig. 2(a). Comparison of similar plots for positive and negative tracks taken separately indicated no discernible difference due to the occasional presence of an unidentified proton. We also studied the missing-mass dependence of scaling in the mean. In Fig. 2(b) we plot the normalized $p_T/\langle p_T\rangle$ distributions of central pions for four intervals of the missing mass to proton p_3, M_3, chosen to equalize the number of events. The resulting curve indicates an independence of missing mass. To show the independence of incident energy we fitted our universal curve to the form

$$\varphi_T=(a/E)(p_T/\langle p_T\rangle)^c \exp[-b p_T/\langle p_T\rangle], \qquad (2)$$

which was given in Svensson and Sollin.[14] We obtain $a=0.85\pm0.02$, $b=1.66\pm0.02$, and $c=1.28\pm0.02$ (χ^2 per degree of freedom $=\frac{152}{137}$), in reasonable agreement with the results at high energy.[12] Furthermore, recent intersecting storage rings (ISR) results have also been shown[15] to be consistent with scaling in the mean, thus proving the energy independence of the universal curve.

Since the scaling curve is independent of multiplicity, missing mass, and beam momentum up to ISR energies (76 GeV in the center-of-mass

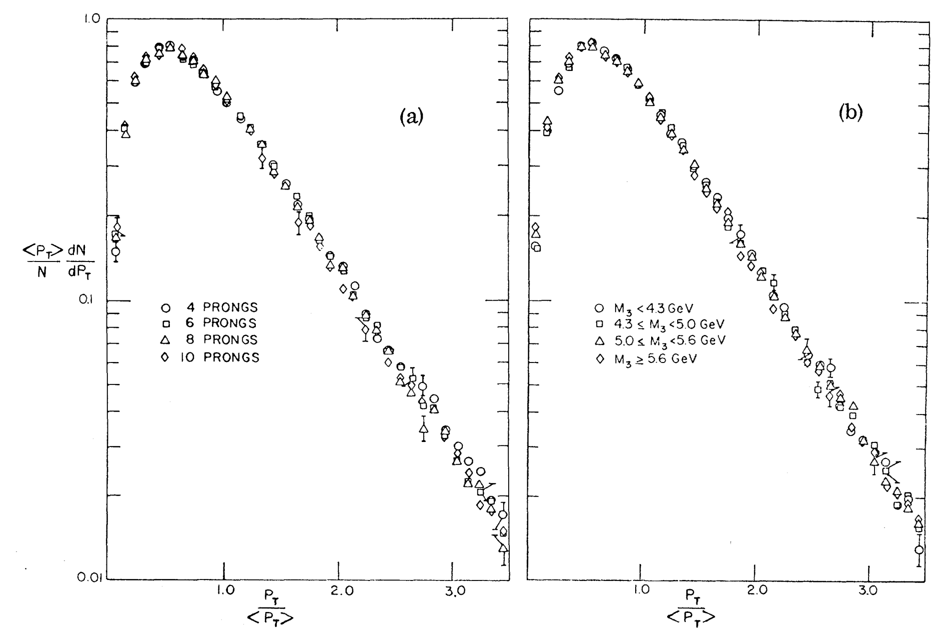

FIG. 2. Plot of $(\langle p_T\rangle/N)dN/dp_T$ vs $p_T/\langle p_T\rangle$ for the central particles in the reaction $p_1+p_2 \rightarrow p_3+X$. Some typical errors are indicated. (a) Charge multiplicites 4–10; (b) four intervals of missing mass.

system), we conjecture that it remains unchanged in going to the asymptotic energy. In addition if we impose the condition that $\langle n\rangle/n \ll 1$ then $\langle p_T\rangle_n$ should become independent of n and the statistical-thermodynamic models apply. *If in these limits the limiting value of* $\langle p_T\rangle$ *is known,*[16] *the universal distribution curve can be converted into a limiting momentum distribution curve simply by multiplying the abscissa by* $\langle p_T\rangle$. It is instructive to study the relationship between scaling in the mean and the form of scaling suggested by Suranyi[17] and Koba *et al.*[18] According to their theoretical results the momentum distribution for a given multiplicity is of the form

$$\frac{E}{N_n}\frac{d^2N_n}{dp_L dp_T}=p_T g_{\rm SK}\left(\frac{2p_L}{\sqrt{s}}, p_T, \frac{\langle n\rangle}{n}\right), \tag{3}$$

where N_n is the number of tracks from events of multiplicity n, and $g_{\rm SK}$ is a universal function as suggested by Suranyi and Koba *et al.* The symbols $\sqrt{s}$, E, and $\langle n\rangle$ denote the total center-of-mass energy, the energy of the observed particle, and the average multiplicity at energy $\sqrt{s}$, respectively. The mathematical form of the experimentally observed scaling in the mean[12] is

$$\frac{1}{N_n}\frac{d^2N_n}{dp_L dp_T}=\frac{1}{\langle p_L\rangle_n\langle p_T\rangle_n}\varphi\left(\frac{p_L}{\langle p_L\rangle_n}, \frac{p_T}{\langle p_T\rangle_n}\right). \tag{4}$$

Comparing Eqs. (3) and (4) in the small-p_L and large-p_T limit ($p_T/E \to 1$) it follows that the Suranyi-Koba scaling function simplifies to[19]

$$g_{\rm SK}\left(\frac{2p_L}{\sqrt{s}}, p_T, \frac{\langle n\rangle}{n}\right)=\frac{\sqrt{s}}{\langle p_L\rangle_n\langle p_T\rangle_n}g_{\rm SK}'\left(\frac{2p_L}{\sqrt{s}f_L(\langle n\rangle/n)}, \frac{p_T}{f_T(\langle n\rangle/n)}\right), \tag{5}$$

where $f_L(\langle n\rangle/n)=\langle p_L\rangle_n/2\sqrt{s}$ and $f_T(\langle n\rangle/n)=\langle p_T\rangle_n$. We have domonstrated experimentally the existence of a universal $p_T/\langle p_T\rangle_n$ distribution curve[12] which is independent of energy, missing mass, and multiplicity in the central ($p_L \approx 0$) region. Thus the p_T distribution is uniquely determined by a single parameter $\langle p_T\rangle_n = f_T(\langle n\rangle/n)$ which in the central region is a function of $\langle n\rangle/n$ only.

Since the produced particles are predominantly pions, the distribution function of p_T in the large n limit should be[20] the Bose-Einstein distribution. Expanding $f_T(\langle n\rangle/n)$ in powers of $\langle n\rangle/n$ we obtain in the first order $f_T = a_0 + a_1(\langle n\rangle/n)$. Identifying $\langle p_T\rangle$ in the asymptotic small $\langle n\rangle/n$ limit with a_0, we fitted our universal curve in Fig.

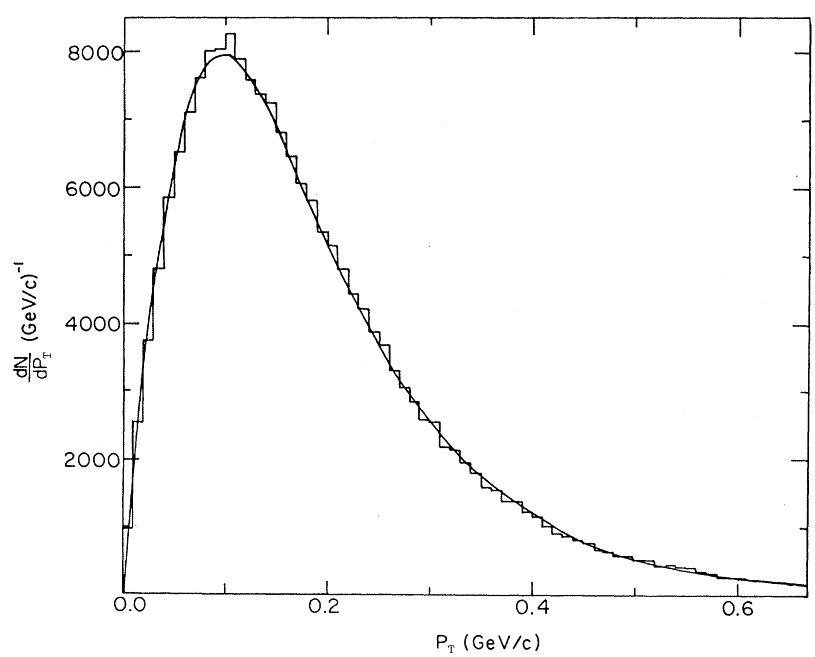

FIG. 3. The transverse-momentum distribution for central particles in the asymptotic large-multiplicity limit in the reaction $p_1+p_2 \to p_3+X$. The curve is a fit to the Bose-Einstein statistical distribution.

2(a), after multiplying the abscissa by a_0, to the Bose-Einstein thermodynamical formula:

$$\frac{1}{N}\frac{dN}{dp_T}=\frac{(p_T/E)^{\alpha}}{e^{(p_T^2+\mu^2)^{1/2}/T_0}-1}\quad (p_L\approx 0), \tag{6}$$

where μ is the pion mass and T_0 is the limiting temperature. The parameters varied were α, T_0, and a_0. Our fit gave the following values for these parameters $\langle p_T\rangle = a_0 = 0.188 \pm 0.005$ GeV/c, $T_0 = 0.117 \pm 0.004$ GeV, and $\alpha = 0.90 \pm 0.02$. The p_T distribution corresponding to this value of $\langle p_T\rangle = a_0$ and the fitted curve are shown in Fig. 3. The p_T distribution gives a good fit to the Bose-Einstein formula with χ^2 per degree of freedom $=\frac{165}{130}$.

In conclusion, we have observed the onset of scaling in the mean at low energies in the central region. We used this experimental fact and the limiting value of $\langle p_T\rangle_n$ at small $\langle n\rangle/n$ to obtain the limiting temperature T_0 of Hagedorn. Our results give $T_0 = 117 \pm 4$ MeV, which is lower than Hagedorn's value of $T_0^H \approx 160$ MeV derived from the hadronic mass spectrum but it is in the 120 MeV $< T_0^H <$ 160 MeV range he obtained from the average transverse momentum of negative pions produced in pp interactions.

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[10]The wire spark chambers of the VS subtended a solid angle such that 89% of all charged particles were detected.

[11]The excess of tracks at negative x is due to the trigger requirements imposed on the forward proton p_3.

[12]F. T. Dao *et al.*, Phys. Rev. Lett. 33, 389 (1974).

[13]If a severe x cut is not made, the scaling holds only approximately and to the same degree for the $p_T/\langle p_T\rangle$ and $p_L/\langle p_L\rangle$ distributions.

[14]B. E. Y. Svensson and L. Sollin, Phys. Rev. Lett. 34, 1199 (1975). In order to compare our curve with the results of Ref. 12 we fitted the scaling curve to Eq. (2). It has an E^{-1} factor (E is the energy of the pion) which is not present in the expression given in Ref. 12. Its inclusion or omission does not affect substantially the quality of the fit in the p_T range under consideration. As was pointed out by Svensson and Sollin, theoretically one expects E^{-1} to be present and it ensures consistency with Eq. (6).

[15]W. Ernst and I. Schmitt, Lett. Nuovo Cimento 16, 39 (1976).

[16]In our analysis we assume that limiting temperature means limiting $\langle p_T\rangle$. Experimental tests of this hypothesis require not only high energy but also high multiplicity of mesons produced in the central region.

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[19]Contrary to the assumptions of Ref. 14, $\langle p_T\rangle$ and $\langle p_L\rangle$ are correlated for nonzero p_L.

[20]One of the main postulates of thermodynamical models is that in the first approximation the longitudinal motion is hydrodynamical while the transverse motion is thermal in nature in statistical equilibrium; thus the Bose-Einstein distribution applies.

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