configurations.

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[11]We have also observed a similar excitation of leaky modes at $\theta_i < \theta_{\text{critical}}$ corresponding to "virtual" modes within the air gap between the prism and metal which have $\tilde{k}_{2z}$ predominantly real.

[12]It is also obvious that similar effects will occur with excitation of SEM waves by finite-beam-width VEM waves in grating-coupling configurations.

"Grooving" of a Uniform Plasma with a Magnetic Field by an Unstable Wave

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Presented numerically is an apparently paradoxical nonlinear process that a two-stream instability is stabilized by inhomogenizing or "grooving" an initially homogeneous plasma, not by limiting the stream which is the direct cause of the instability. For a relatively wide and slow electron stream, the instability is shut off by reducing the speed to the threshold value (ion sound speed), as is naturally expected. For a narrow and fast stream, however, the instability is shown to be stabilized by the grooving effect.

From the standpoint of causality, one may say that an instability proceeds in general so as to relax its driving force. For instance, drift waves would be stabilized by flattening the density gradient which causes the instability. Recently, however, an apparently paradoxical stabilization mechanism has been proposed for a resistive two-stream (Hall current) instability.[1] This instability is widely known among ionospheric plasma physicists as the two-stream instability or "type I" irregularities.[2] The only driving force of this instability in a uniform plasma is the electron stream. It may therefore be natural that one considers the instability to be stabilized by self-limitation of the stream. In fact, it has been shown that an unstable wave itself can reduce the electron drift speed to the threshold (ion acoustic) speed.[3] The recent theory,[1] however, indicates that the two-stream instability can also be stabilized by "inhomogenizing" a uniform plasma. If this process, which does not take part in the linear instability mechanism, is indeed capable of stabilizing the two-stream instability, then one must recognize a strange fact that the nonlinear development of an instability is not necessarily connected by one causal chain but may happen to be governed by an apparently irrelevant causality.

Let us consider a magnetized, uniform plasma in which a Hall current (primarily carried by Hall electrons) is concentrated in a limited layer. I use a coordinate system (x, y, z), the z axis being directed antiparallel to the magnetic field $\vec{B}_0$, the y axis being directed parallel to the electron drift, and the x axis being directed so as to form a right-handed system.

I assume slightly collisional electrons ($\nu_e/\Omega_e \ll 1$) and heavily collisional ions ($\nu_i/\Omega_i \gg 1$), ν_e and ν_i being the collision frequencies of electrons and ions (against neutrals), respectively, and Ω_e Ω_i being the gyrofrequencies of electrons and ions, respectively. Accordingly, the motion of electrons is dominated by the electric (Hall) drift, while the motion of ions is almost completely controlled by collisions (hence no appreciable ion Hall drift is present). Furthermore, I assume a low-frequency electrostatic mode which propagates perpendicular to the magnetic field, so that the electron inertia is neglected in the equation of motion.

Under these conditions, the governing equations to be solved are given by

$$n_e\vec{v}_e = \frac{1}{B_0}\left(\frac{\nu_e}{\Omega_e}\right)n_e\nabla\varphi - n_e\frac{\nabla\varphi\times\vec{B}_0}{B_0^2} - \frac{\nu_e c_s^2}{2\Omega_e\Omega_i}\nabla n_e + \frac{c_s^2}{2\Omega_i B_0}\nabla n_e\times\vec{B}_0, \quad (1)$$

$$\frac{d\vec{v}_i}{dt} + \nu_i\vec{v}_i = -\frac{e}{m_i}(\nabla\varphi - \vec{v}_i\times\vec{B}_0) - \frac{c_s^2}{2n_i}\nabla n_i, \quad (2)$$

$$\frac{\partial n_e}{\partial t} + \nabla\cdot n_e\vec{v}_e = 0, \quad (3)$$

$$\frac{\partial n_i}{\partial t} + \nabla\cdot n_i\vec{v}_i = 0, \quad (4)$$

$$\nabla^2\varphi = \frac{e}{\epsilon_0}(n_e - n_i), \quad (5)$$

where the subscripts e and i refer to electrons and ions, respectively; n is the density, φ is the electric potential, and $\vec{v}$ is the velocity; c_s is the ion acoustic velocity (the ion and electron temperatures are assumed to be equal); ϵ_0 is the dielectric constant in the vacuum and e is the electronic charge.

Since the main purpose of this Letter is to examine whether or not a uniform plasma can be inhomogenized by an unstable wave, one may reasonably assume a single mode; hence I assume the density and potential waves to take the form of $f(x, y, t) = f_k(x, t)\exp(iky)$, $f_k(x, t)$ being a complex amplitude function, and k being the wave number of the single mode of concern. The neglect of higher harmonic modes may be justified *a posteriori* because (as can be seen, for instance, in Fig. 3) the quasilinear modification becomes of the order of $|n_k|$ instead of $|n_k|^2$.

Equations (1)–(5) are accordingly Fourier-expanded with respect to y; and only $\pm k$ and 0 modes are retained. With respect to x and t the equations are transformed to explicit difference equations with equal mesh intervals (the mesh number along x is 100).[4]

We assume a Poiseuille-like flow pattern as shown by the solid line in Fig. 1, for the initial electron drift profile along x (the initial electric field E_0 driving this flow has the same profile and is directed towards the positive x axis). Note in Fig. 1 that the velocity is normalized by the maximum drift velocity V_d and the ion acoustic speed c_s is taken to be $0.7V_d$.

Linear fluid theory[3] states that the two-stream instability occurs if the drift speed exceeds the ion acoustic speed. In the present flow model, therefore, it is expected that the instability would occur in the region of $-0.35L < x < 0.35L$ (L is the width of layer wherein the electron stream exists). The flow pattern is taken to be parabolic in this unstable region and linear in other regions.

The initial perturbation is taken to be $n_e = 0.003 \times\cos\pi x\exp i(20\pi y)$ and $n_i = 0.003\cos\pi x\exp i(20\pi y + \delta)$ where δ is an arbitrary real number which determines the initial potential perturbation through Eq. (5); in the calculation δ is chosen to be 0.01 but the final result this choice is insensitive to if it is reasonably small. Note that the density is normalized by the initial background electron (equal to ion) density N_0 which is assumed to be constant along x and y. The distance is normalized by L so that the wavelength λ is taken to be $0.1L$ and that the time is normalized by L/V_d. The normalized time step is 8×10^{-6}. The plasma parameters are set to be $\nu_e/\Omega_e = 10^{-3}$, $\nu_i/\Omega_i = 10$, $\Omega_i L/V_d = 6$, $\Omega_e/\Omega_i = 5\times10^4$, and $\omega_{pi}^2/\Omega_i^2 = 5\times10^3$.

Figure 2 shows the time evolution of the density

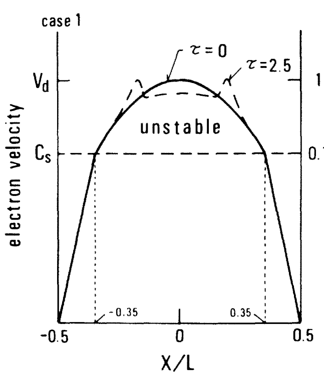

FIG. 1. Initial (solid line) and modified (dashed line) velocity profiles at $\tau = 2.5$ for $L = 10\lambda$.

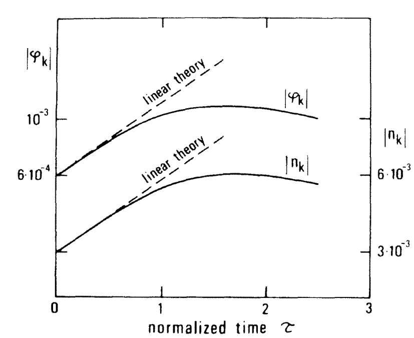

FIG. 2. Evolution of the potential wave amplitude $|\varphi_k|$ and the density wave amplitude $|n_k|$ at $x=0$ for $L = 10\lambda$.

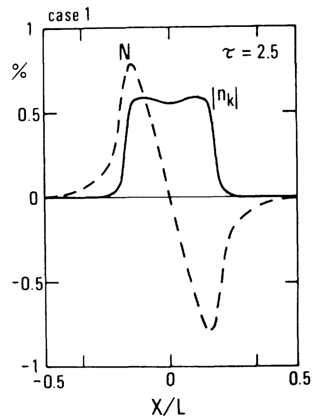

FIG. 3. "Grooved" density profile (dashed line) and the amplitude profile of the density wave (solid line) for $L = 10\lambda$.

wave amplitude $|n_k|$ and the potential wave amplitude $|\varphi_k|$ at $x = 0$. The dashed lines represent the theoretically predicted linear growth lines. From this figure one will find that the instability is certainly stabilized by some nonlinear mechanism and that the density wave amplitude has reached roughly 0.6% of the background density.

Shown in Fig. 3 are the wave amplitude profile $|n_k(x)|$ along the x axis at $\tau = 2.5$ (solid line) and the background density profile $N(x)$ at $\tau = 2.5$ (dashed line). One will find that the wave is concentrated in a well-defined layer (i.e., $-0.2L \lesssim x \lesssim 0.2L$) which is substantially narrower than the linearly unstable region (i.e., $-0.35L \lesssim x \lesssim 0.35L$).

A more important fact to be noted is that an initially homogeneous plasma is substantially inhomogenized as a result of the collective nonlinear transport across the magnetic field by an unstable wave[1]; one side of the plasma ($-0.5L < x < 0$) is grooved and the removed plasma is banked up on the other side. Judging from the symmetric profiles of the electron flow and wave amplitude, one may suspect the causality of producing such an asymmetric structure. However, the asymmetry comes from the geometrical asymmetry that the electric field causing the electron Hall motion directs toward the positive x axis; in other words, the space potential is positive in the negative x region and negative in the positive x region. By this asymmetric potential distribution, the plasma is digged on the negative potential side and piled on the positive side. One should note here that the density gradient resulting from this grooving effect is negative in the region where the wave exists. That is, the quasilinear wave interaction has proceeded in such a way that the Simon stability condition[5] (i.e., $E\,dN/dx < 0$) is satisfied. In this sense we may call the "grooving" process the nonlinear Simon dampling.

The electron velocity which is the direct cause of the instability, however, is not appreciably changed (see Fig. 1). Specifically, the velocity suffers a retardation of only 5% at the maximum velocity point ($x = 0$). Since the threshold speed (c_s) is 30% less than V_d, one can conclude that the 5% retardation is too small to be considered as the main stabilizing cause. Consequently, the numerical result can lead to conclusion that the two-stream instability can be stabilized by "grooving" a homogeneous plasma as a result of the collective nonlinear mass transport.

Nonlinear theory for a uniform flow[1] has given the saturation amplitude of the present instability as follows:

$$|n_k|_{\rm I}^2 \cong (\gamma/\omega)^2(\Omega_i/\nu_i)^2(lk/2\pi)^2 \qquad (6)$$

if the instability is stabilized by the "grooving" process alone, and

$$|n_k|_{\rm II}^2 = \tfrac{1}{2}(\Omega_i/\nu_i)^2(V_d - c_s)/c_s \qquad (7)$$

if stabilized by the velocity relaxation process alone, where l is the effective width of the unstable layer; $\omega \cong kV_d$ and $\gamma \cong (\nu_e/\Omega_i\Omega_e)(\omega^2 - c_s^2k^2)$. Substitution of the parameter values used in the calculation into (6) and (7) yields $|n_k|_{\rm I} = 0.75\%$ (l is chosen as $0.7L$) and $|n_k|_{\rm II} = 4.6\%$.

The nonlinear theory[1] therefore predicts that the dominant stabilization process (giving the lower saturation amplitude) be the "grooving" process for the present condition. The numerical result, therefore, is completely consistent with the theoretical prediction. A slight difference in saturation amplitudes (0.6% and 0.75%) may be due to the fact that in the theory l was taken to be the same as the linearly unstable range ($0.7L$); however, the effective range was substantially narrower (see Fig. 3).

For confirmation, a similar calculation has been performed for a different situation in which the layer width L is 100 times wider than the previous case (the time step is 8×10^{-8} in this case), keeping the other parameters the same. Figure 4 shows the electron velocity profiles at $\tau = 0$ and $\tau = 0.018$ (growth is stopped at about $\tau = 0.016$). From this figure one sees that the velocity profile is drastically changed and, in particular, that the maximum velocity (at $x = 0$) is reduced just to the threshold. The appearance of double maxima at $x \cong 0.35L$ and $-0.35L$ can be explained by the polarization charges accumulated near the bound-

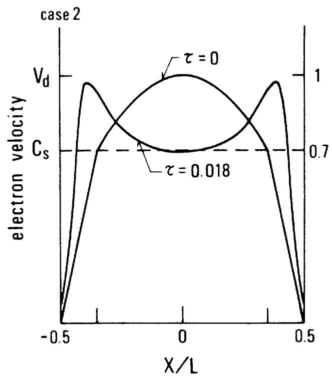

FIG. 4. Initial and retarded electron velocity profiles for $L = 10^3\lambda$.

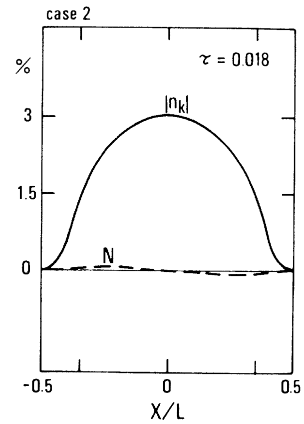

FIG. 5. Wave amplitude profile (solid line) and slightly modified density profile (dashed line) for $L = 10^3\lambda$ at $\tau = 0.018$.

aries of wave region.

Shown in Fig. 5 are the wave amplitude (solid line) and the background density profile (dashed line) at $\tau = 0.018$. This figure indicates that the grooving of the plasma is negligibly small. The theoretical saturation amplitudes for this case are given, from (6) and (7), as follows: $|n_k|_{\mathrm{I}} = 75\%$ and $|n_k|_{\mathrm{II}} = 4.6\%$, this indicating that the dominant stabilization process should be the velocity retardation. Thus the numerical result again is consistent with the theory. The difference in amplitudes (4.6% and 3%) may arise because a uniform flow with velocity V_d is assumed in the theory.

I have numerically demonstrated that an unstable wave can groove a *uniform* plasma as a result of collective mass transport, the instability thereby being stopped. This result, together with the previous one,[6] indicates that the collective transport can become more important than the wave-enhanced diffusion[7] in a magnetically confined plasma in which a single wave is excited.

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Insensitivity to Selection Rules of Two-Phonon Luminscence Spectra

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Analysis of newly detected luminescence lines in germanium indicates that two-phonon recombination processes in indirect-gap semiconductors are insensitive to selection-rule constraints, although such restrictions generally play a crucial role in the optical properties of semiconductors. The two-phonon replicas of electron-hole recombination lines are found to be constituted from well-defined groups of phonons whose momenta differ from points of high symmetry in the Brillouin zone.

Two sets of faint luminescence lines have been observed in photoexcited germanium at low temperatures. These sets arise from recombination, of carriers within the electron-hole liquid in one case and of excitons in the other, accompanied by the emission of a pair of phonons.[1-3] The new lines that we have detected are difficult to observe because they are over 10^3 times less in-