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Inclusive Reactions $p + n \to p + X$ and $\pi^+ + n \to p + X$ at 100 GeV/c

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(Received 24 February 1976; revised manuscript received 11 August 1976)

We have measured the inclusive cross section for the reactions $p+n\to p+X$ and $\pi^+ +n \to p+X$ at 100 GeV/c in the kinematic region $|t|< 1.0$ GeV^2. The data were obtained from an exposure of the Fermilab 30-in. deuterium-filled bubble chamber to a tagged positively charged beam. The differential cross sections for these reactions are observed to scale in the ratio of the pn and $\pi^+ n$ total cross sections and to be consistent with the predictions of a Reggeized one-pion-exchange model.

We present a study of the reactions

$$p+n\to p+X \quad (1)$$

and

$$\pi^+ +n\to p+X \quad (2)$$

from an analysis of 41 000 pictures of interactions in the Fermilab 30-in. deuterium-filled bubble chamber exposed to an unseparated beam of 100-GeV/c positive particles. A tagging system[1] allowed the identification of individual beam particles from their position in the bubble chamber. The film was scanned twice, with an efficiency of $(99\pm 1)\%$, for interactions with three or more outgoing charged particles. The tracks of the slow secondary particles (with projected laboratory momenta $\lesssim 1.5$ GeV/c) were measured and reconstructed in space; and protons with momenta less than 1.2 GeV/c were identified by their ionization in the bubble chamber. Inelastic one- and two-prong events, with all visible particles slow in the laboratory, were identified in a separate scan of 10 000 frames and processed as above. We assume the impulse approximation[2] to be valid and identify neutron-target events by the presence of a spectator proton with momentum less than 300 MeV/c.

We have previously argued that the invisible spectator proton events, which comprise about $\frac{2}{3}$ of our data on Reactions (1) and (2), may be interpreted as an unbiased sample of neutron-target interactions.[3] Accordingly, we calculate the cross section for Reactions (1) and (2) from the invisible spectator sample by normalizing the total number of invisible spectator events[4] to the inelastic cross sections[5,6] for pn and π^+n collisions, respectively. This method of normalization obviates the need to correct the data for Glauber screening and rescattering. We obtain a cross section of 5.7 ± 0.3 mb for Reaction (1) and 3.9 ± 0.3 mb for Reaction (2), for $|t|<1.0$ GeV^2, where t is the square of the four-momentum transfer from the neutron target to the slow proton. Our data constitute a 115 event/mb sample of Reaction (1) and an 82 event/mb sample of Reaction (2). The fraction of the total pn cross section which contributes to Reaction (1) is 0.15 ± 0.01, while 0.16 ± 0.01 of the π^+n total cross section contributes to Reaction (2), for $|t|<1.0$ GeV^2. The equality of these two fractions suggests that the fractional cross section for slow proton production from a neutron target is independent of the incident beam particle.

We display in Fig. 1(a) the distribution of the square of the missing mass (M^2) recoiling against the slow proton, or equivalently the square of the mass of the system X, from Reactions (1) and (2) for $|t|<1.0$ GeV. The uncertainty in the target momentum of the invisible spectator events leads to a resolution (full width at half-maximum) of 13 GeV^2 (a value large compared with the resolution resulting from measurement errors) independent of M^2 and is responsible for the negative M^2 data. The low M^2 peak attributed to diffractive fragmentation of the beam proton in the inelastic reaction[7]

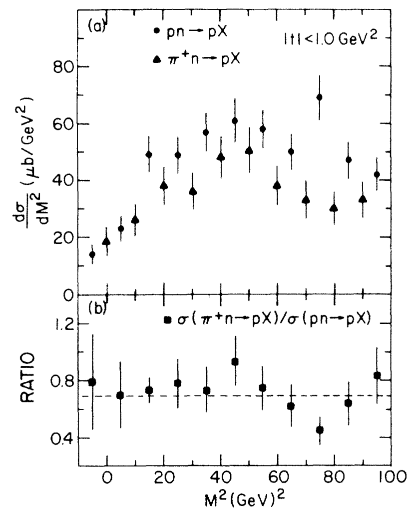

FIG. 1. (a) Distributions of the square of the invariant mass of the system X in the reactions $p+n\to p+X$ and $\pi^++n\to p+X$ for $|t|<1.0$ GeV^2. (b) Ratio of the distributions $[d\sigma(\pi^++n\to p+X)/dM^2]/[d\sigma(p+n\to p+X)/dM^2]$ as a function of M^2. The dashed line is the average value of this ratio.

$p+p\to p+X$ is observed to be absent from the neutron-target data. We plot the ratio

$$R=\frac{d\sigma(\pi^++n\to p+X)/dM^2}{d\sigma(p+n\to p+X)/dM^2}, \qquad (3)$$

as a function of M^2 in Fig. 1(b). The dashed line indicates the average value of this ratio. The ratio R in (3) is independent of M^2 within experimental errors.

Values of $s\,d\sigma/dt\,dM^2$ from Reaction (1) as a function of M^2 average over the three indicated t intervals are displayed in Fig. 2. The curves are discussed below. This reaction has previously been studied with data from the deuterium gas-

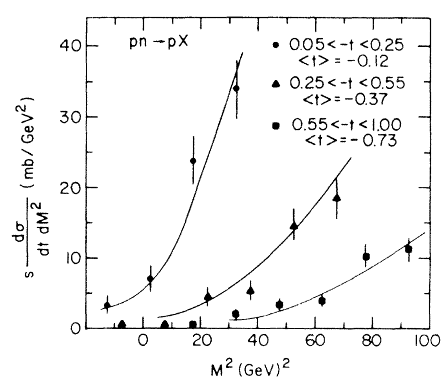

FIG. 2. Invariant cross section for the reactions $p+n\to p+X$ as a function of M^2, averaged over the indicated t intervals. The units of t are GeV^2. The curves are the predictions of Eq. (4).

jet target at Fermilab[8] in the restricted kinematic region $0.14<|t|<0.38$ GeV^2 and $0.07<M^2/s<0.20$. In contrast to the gas-jet experiment, the bubble chamber allows an event-by-event separation of pn interactions from both pp interactions and rescatterings in the deuteron. In addition, an intersecting storage ring (ISR) experiment[9] has investigated the reaction $p+p\to n+X$ over a wide range of t and M^2/s values. We expect the invariant cross sections for the reactions $p+n\to p+X$ and $p+p\to n+X$ to be equal in the target fragmentation region if the reactions are isospin invariant and beam-particle independent. Our invariant cross-section values are consistent with those of the gas-jet experiment[8] in the region of overlapping t and M^2/s and are a factor of 2 to 4 larger than those of the ISR experiment.[9]

In an exchange formalism, the isospin-one exchange particles which could mediate Reactions (1) and (2) are the π, ρ, and A_2. Bishari[10] and Field and Fox[11] have suggested that the dominant mechanism in high-energy charge-exchange reactions may be pion exchange. By extrapolation to the pion mass shell, Bishari parametrizes the pion-exchange contribution to the differential cross section for Reaction (1) as

$$s\frac{d\sigma(p+n\to p+X)}{dt\,dM^2}=\frac{1}{4\pi}\frac{g_{\pi np}^2}{4\pi}\frac{-t}{(t-m_\pi^2)^2}\left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)-\alpha_P(0)}\sigma_T^{\pi^-p}(M^2)\exp[b(t-m_\pi^2)], \tag{4}$$

where $g_{\pi np}^2/4\pi=2g_{\pi pp}^2/4\pi\approx 29.0$ is the on-mass-shell coupling, $\sigma_T^{\pi^-p}(M^2)$ is the total π^-p cross section at center-of-mass energy $\sqrt{s}=M$, and b is a parameter to account for possible off-mass-shell corrections. The pion Regge trajectory function is $\alpha_\pi(t)=t$, $\alpha_P(0)=1$, and m_π is the pion mass. Equation (4) is derived for t and M^2/s small, and M large. In the tripel-Regge formalism, Eq. (4) is the sum of the contributions of the $\pi\pi P$ and $\pi\pi R$ terms. Field and Fox include these terms in an analysis of the reaction $p+p\to p+X$ where they fit the various triple-Regge couplings. Their solutions favor $b\approx 0$. Accordingly, we compare Reaction (1) with the prediction of Eq. (4) with $b=0$, and use a parametrization of the π^-p data [12] for $\sigma_T^{\pi^-p}(M^2)$.

The curves in Fig. 2. are the predictions of Eq. (4), which we have modified to include the effects of the invisible spectator events.[13] We observe substantial agreement with the predications of the Reggeized pion-exchange model, as was also found in Ref. 8. The data do not discriminate against a model incorporating ρ/A_2 exchange if b in Eq. (4) is allowed to be nonzero. However, we find it remarkable that the pion-exchange model is sufficient to describe the data over the full range of t and M^2 values accessible in this experiment, including data in the large $|t|$ and M^2/s regions where *a priori* we may not expect the model to be valid.

We may expect Reaction (2) also to be described by the one-pion-exchange model. The pion exchange contribution to the differential cross section for Reaction (2) is parametrized by replacing $\sigma_T^{\pi^-p}(M^2)$ in Eq. (4) with total $\pi^-\pi^+$ cross section. Since there are no directly measured values of the $\pi^-\pi^+$ cross section, we can use our data to deduce the $\pi^-\pi^+$ cross section, as a function of M^2, from the pion-exchange-model relationship

$$\sigma_T^{\pi^-\pi^+}(M^2)=s\frac{d\sigma(\pi^++n\to p+X)}{dt\,dM^2}\left[\frac{1}{4\pi}\frac{g_{\pi np}^2}{4\pi}\frac{-t}{(t-m_\pi^2)}\left(\frac{s}{M^2}\right)^{2t-1}\right]^{-1}. \tag{5}$$

We plot the quantity given by Eq. (5)[14] in Fig. 3, averaged over the same t intervals as in Fig. 2. We include in Fig. 3 low energy values of $\sigma_T^{\pi^-\pi^+}(M)$ from an analysis of 25-GeV/c π^-p data.[15]

An alternative method of deducing the $\pi^-\pi^+$ cross section utilizes only the factorization properties of the Regge formalism. If we integrate over our full range of t, the ratio of total cross sections $\sigma_T^{\pi^-\pi^+}(M^2)/\sigma_T^{\pi^-p}(M^2)$ may be equated with the ratio R of Eq. (3) [Fig. 1(b)]. Our average value $\bar{R}=0.68\pm0.06$ is consistent with the factorization[16] prediction[12,17] $\sigma_T(\pi^-\pi^+)/\sigma_T(\pi^-p)=\sigma_T(\pi^+p)/\sigma_T(pp)\approx 0.62$ for $s\gtrsim 20$ GeV^2. The large $|t|$ region may be excluded from this comparison by simply ignoring the large M^2 data in Fig. 1(b), since high $|t|$ events contribute appreciably only at high M^2 (see Fig. 2). The same remark applies to data shown in Fig. 3. The result above may be compared with a bubble-chamber experiment[18] where the distribution of Δ^{++} decay angles in the reactions $\pi^++p\to\Delta^{++}+X$ and $p+p\to\Delta^{++}+X$

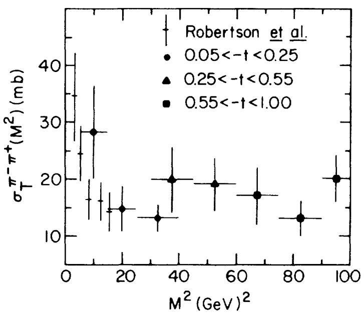

FIG. 3. $\sigma_T^{\pi^-\pi^+}(M^2)$ computed with Eq. (5) from the $\pi^+ + n \to p + X$ data as a function of $s = M^2$, averaged over the indicated t intervals. The low M^2 data points are from Ref. 15.

were observed to be compatible with the one-pion-exchange model, and where the average value of this ratio was found to be 0.84 ± 0.11 for $|t| < 0.88$ GeV^2.

We thank the members of the Neutrino Laboratory at Fermilab, the Proportional Hybrid System Consortium, and our scanning and measuring staffs for their help and cooperation on this experiment. We thank R. D. Field, P. Hoyer, and J. M. Wang for valuable discussions.

*On leave of absence from CERN, Geneva, Switzerland.

†Research supported by the National Science Foundation.

‡Research supported by the U. S. Energy Research and Development Administration.

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[13]We modify Eq. (4) with a Monte Carlo calculation in which we assume that the neutron target is described by the Hulthén wave function, and that target momenta less than 100 MeV/c result in invisible spectator events for which we compute t and M^2 assuming the target to be at rest.

[14]We plot the invariant cross section $s\, d\sigma/dt dM^2$ for Reaction (2) in Fig. 3, weighting each event by the inverse of the denominator of Eq. (5) before binning in t and M^2.

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