COMMENTS

Ferromagnetic Phase Transitions in Random Fields: The Breakdown of Scaling Laws*

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The scaling laws relating the critical exponents characterizing the ferromagnetic phase transition are shown to be modified in the presence of quenched random magnetic fields. The hyperscaling relation $d\nu = 2-\alpha$, for example, becomes $(d+\lambda_u)\nu = 2-\alpha$, where the index λ_u is negative and is related to the range of the ferromagnetic exchange interactions. This breakdown of hyperscaling results from a singular dependence of the thermodynamic functions on the scaling field associated with an irrelevant operator.

In a recent Letter,[1] Imry and Ma (IM) discussed the effect of quenched random magnetic fields on the ordered phase of ferromagnets with short-range exchange interactions. Arguing heuristically (but very convincingly) they concluded that for space dimensionality $d \leq 4$ the presence of such fields makes the spontaneously ordered phase of systems with continuous symmetry (i.e., number of spin components $n \geq 2$) unstable to the formation of domains. The magnetic moment of each domain is oriented along the direction of the local magnetic field, thereby lowering the energy of interaction with the field. For $d \leq 4$ this lowering more than compensates the increase in surface exchange energy which results from the partitioning. For Ising systems ($n=1$) the direction of magnetization cannot vary continuously across domain boundaries and the surface exchange energy cost is correspondingly more severe. The ferromagnetic state of Ising systems is therefore unstable to domain formation only when $d \leq 2$.

In addition to specifying these dimensionalities, $d_D(n)$, below which ferromagnetism is destroyed in n-component systems, IM predicted that the critical exponents characterizing the ferromagnetic transition in the presence of random magnetic fields have mean-field values for $d > d_c(n) \equiv 6$ but deviate from mean-field when $d_D(n) < d < d_c(n)$. Using standard renormalization-group[2] (RG) techniques they computed η and ν to first order in $\epsilon \equiv 6-d$. Earlier, Lacour-Gayet and Toulouse[3] studied the ideal Bose gas in the presence of a random source term, computing exact critical exponents of the Bose condensation at both constant volume and constant pressure. They found violations of the familiar scaling laws[4] relating critical exponents. Since the critical behavior of the ideal Bose gas at constant volume is identical[5] to that of the spherical model, which is in turn equivalent[6] to the $n=\infty$ limit of the n-component model, one concludes that random magnetic fields cause violations of scaling in the $n=\infty$ system. It is natural to ask whether such fields, presumably always present in real systems, precipitate a breakdown of scaling for finite values of n as well.

The purpose of this note is twofold: First, I point out that despite the existence of a nontrivial fixed point[2] of the RG transformation in $6-\epsilon$ dimensions, conventional scaling laws do indeed break down for all values of n. I derive a new set of scaling laws to replace the familiar ones. It is tempting, though fraught with the usual perils of large ϵ, to speculate that these relations hold generally for $d_D(n) < d < d_c(n)$. (Recall that this range includes $d=3$ when $n=1$.) Second, we note that the phenomenology of the ferromagnetic phase transition is complicated by the presence of random fields; there are two distinct spin-spin correlation functions that become long-ranged near T_c. One of these is unique to the random system. I use the RG to determine its scaling properties near criticality.

Following IM we consider the familiar[2] isotropic n-vector model with order parameter $\vec{s}(x)$ coupled to a random field $\vec{h}_0(x)$. The reduced Hamiltonian in momentum space takes the form

$$\beta\mathcal{H}_0 = \int_0^\Lambda \frac{d^dk}{(2\pi)^d} \left\{ \tfrac{1}{2}(k^\sigma + r_0)|\vec{s}(k)|^2 + u_0 \int_0^\Lambda \frac{d^dk_1}{(2\pi)^d}\frac{d^dk_2}{(2\pi)^d}[\vec{s}(k)\cdot\vec{s}(k_1)][\vec{s}(k_2)\cdot\vec{s}(-k-k_1-k_2)] - \vec{h}_0(k)\cdot\vec{s}(k) \right\}, \quad (1)$$

where $\sigma<2$ describes long-range exchange couplings[7] which die off as $r^{-d-\sigma}$ in position space, $\sigma=2$ describes short-range couplings, and the $h_0^\alpha(k)$ ($\alpha=1,\dots,n$) are Gaussian random variables with short-range spatial correlations[8]: $[h_0^\alpha(k)]_{av}=0$, $[h_0^\alpha(k)h_0^\beta(k')]_{av}=\delta_{\alpha\beta}\delta^d(k+k')f_0(k)$, $f_0(0)\equiv h_0^2$.

In the presence of $\vec{h}_0(k)$ the quantity $[\langle\vec{s}(x)\rangle\cdot\langle\vec{s}(x')\rangle]_{av}$ is nonzero even in phases where $[\langle\vec{s}(x)\rangle]_{av}$ vanishes. There are therefore two distinct correlation functions to consider:

$$G^\alpha(x,x')\equiv[\langle s^\alpha(x)s^\alpha(x')\rangle]_{av}-[\langle s^\alpha(x)\rangle\langle s^\alpha(x')\rangle]_{av}, \tag{2a}$$

is the analog of the usual correlation function studied in pure ferromagnets; its spatial dependence near T_c is characterized by the critical exponents ν and η. The function

$$C^\alpha(x,x')\equiv[\langle s^\alpha(x)\rangle\langle s^\alpha(x')\rangle]_{av}-[\langle s^\alpha(x)\rangle]_{av}[\langle s^\alpha(x')\rangle]_{av} \tag{2b}$$

on the other hand is peculiar to the random problem.[9] Both, as we shall see, become long ranged near the ferromagnetic transition; the extra "susceptibility" $C(k=0)$ diverges very strongly at T_c.

The RG methods of Harris and Lubensky[10] are readily applied to the disordered system described by (1). As in pure systems, s^6, s^8, and all higher-order interactions are generated by the RG transformation.[2] The random field induces randomness in the coefficients of all such terms under the action of the RG. One writes a recursion relation[10] for the joint probability distribution $P_l(h_l,r_l,u_l,\dots)$ of these random couplings after l RG iterations. The various cumulants of P_l are the analogs for random systems of the ordinary (l-times iterated) coupling constants of pure systems. When $d=3\sigma-\epsilon$ only the three quantities r_l, u_l, and h_l^2 defined by

$$[r_l(kk')]_{av}=\delta^d(k+k')r_l,\quad [u_l(kk'k''k''')]_{av}=\delta^d(k+k'+k''+k''')u_l,$$

and

$$[h_l^\alpha(k)h_l^\beta(k')]_{av}=\delta_{\alpha\beta}\delta^d(k+k')f_l(k),\quad f_l(0)\equiv h_l^2,$$

are "relevant."[11] It is most convenient to use u_l, r_l, and $w_l\equiv u_lh_l^2$ as independent variables. In terms of this set the recursion relations to $O(\epsilon)$ are

$$u_{l+1}=b^{-\sigma+\epsilon-2\eta'}u_l[1-8(n+8)K_dw_l\ln b], \tag{3a}$$

$$r_{l+1}=b^{\sigma-\eta'}\{r_l+4(n+2)(K_dw_l/\sigma)[\Lambda^\sigma-(\Lambda/b)^\sigma-2r_l\sigma\ln b]\}, \tag{3b}$$

$$w_{l+1}=b^{\epsilon-3\eta'}w_l[1-8(n+8)K_dw_l\ln b], \tag{3c}$$

where $(2\pi)^dK_d$ is the surface area of the d-dimensional unit hypersphere and $\eta'\equiv\sigma-2+\eta=O(\epsilon^2)$.[7] These equations evidently have, for $\epsilon>0$, a stable fixed point with w^* and r^* of $O(\epsilon)$, $u^*=0$. The vanishing of u^* is a consequence of the "irrelevance"[11] of s^4 terms for $d>2\sigma$ and results in the failure of the usual scaling laws. To understand the mechanism of this breakdown, consider the schematic RG expression[12] for the singular part of the random-averaged free energy,

$$F(u_0,\delta w_0,\delta r_0)=b^{-d}F(u_0b^{\lambda_u},\delta w_0b^{\lambda_w},\delta r_0b^{\lambda_T}). \tag{4}$$

Here δw_0 and δr_0 are the deviations (assumed infinitesimal) of w_0 and r_0 from their fixed-point values, u_0 is likewise taken infinitesimal, and the λ's are eigenvalues of the RG recursion relations linearized about the fixed point.[12] It follows readily from (3) that λ_T is positive while λ_u and λ_w are both negative for small ϵ. Equation (4) is schematic in that we have treated u_0, δw_0, and δr_0 as "scaling fields"[13] of the linearized RG. The true scaling fields are linear combinations of u_0, δw_0, δr_0, and the infinite number of additional variables (here suppressed) which are generated[2] by the RG transformation. It is easy to show, however, that more careful treatment of these effects does not alter the results we shall obtain from (4). Let us therefore proceed as if this equation were literally correct. Setting w_0 equal to w^* (or $\delta w_0=0$) and identifying[2] δr_0 with the reduced temperature $t\equiv(T-T_c)/T_c$ and the thermal eigenvalue λ_T with the inverse of the critical exponent ν (Riedel[13]), we have $F(u_0,t)=t^{\nu d}f(u_0t^{-\nu\lambda_u})$ for some function f. If $f(x)$ were to approach a constant as $x\to0$ then $F\sim t^{\nu d}$ and the usual scaling law $d\nu=2-\alpha$ would obtain. To see that this does *not* occur here let us return to the Hamiltonian (1) and imagine constructing a brute-force graphical perturbation expansion for F as a double pow-

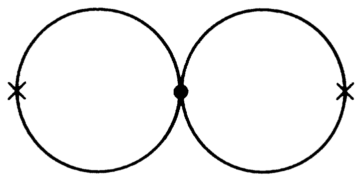

FIG. 1. Typical graph for random-averaged free energy F. Each cross represents a factor of h_0^2.

er series in u_0 and h_0^2. A typical term in this series is shown in Fig. 1. This term behaves like $u_0 h_0^4$; expressed in terms of the proper variables u_0 and w_0 it goes like $u_0^{-1} w_0^2$. For infinitesimal u_0 the leading higher terms in the series behave like u_0^{-1} as well. I conclude that $f(x) \to x^{-1}$ as $x \to 0$, whereupon $F(u_0, t) \sim t^{\nu(d+\lambda_u)}$ for small t. The modified scaling law $2-\alpha = \nu(d+\lambda_u)$ results; that is, the factor d in the usual relation is replaced by $d+\lambda_u$.[14] Proceeding similarly one can show that the other conventional scaling laws are altered in identical fashion: The *only* effect of the random field is to convert every factor of d in these laws to $d+\lambda_u$.[15] All exponents characterizing the thermodynamics and the correlation function $G(x, x')$ near criticality can thus be determined in terms of three indices, say ν, η, and λ_u.

Wilson's Feynman-graph expansion technique[16,2] can be straightforwardly employed to compute η, ν, and λ_u as power series in ϵ. To first order one obtains[17] $\eta = 2-\sigma$, $\sigma\nu = 1+(n+2)\epsilon/2(n+8)$, and $\lambda_u = -\sigma$. For the special case of short-ranged interactions ($\sigma = 2$) I have calculated η and λ_u to $O(\epsilon^3)$ and found that λ_u is exactly -2 to this order. When $n=\infty$, moreover, $\lambda_u = -\sigma$ for all d between 2σ and 3σ.[15] I speculate on this basis that λ_u is identically equal to $-\sigma$ for *all* values of n and d for which a ferromagnetic transition occurs.[18]

It remains to compute the exponents describing the behavior of the correlation function $C(x, x')$ near T_c. For this purpose it is convenient to write the Fourier transform $C(k,t)$ as $[G(k,t)]^2 \times D(k,t)$ and consider the graphical perturbation series for the quantity D. One finds that D, like F, behaves as u_0^{-1} for small u_0, whence it follows via the usual RG route that $C(k,0) \sim k^{-2+\bar{\eta}}$ and $C(0,t) \sim t^{-\bar{\gamma}}$ with $\bar{\eta} = \eta + \lambda_u$ and $\bar{\gamma} = \nu(2-\bar{\eta})$. Thus are the new critical exponents associated with C simply related to the old ones; knowledge of ν, η, and λ_u suffices to determine all relevant critical indices.

Only in $6-\epsilon$ dimensions with ϵ infinitesimal can I display a legitimate fixed point of the recursion relations (3) and predict a random-field-induced modification of scaling laws with some assurance. I am postulating that this breakdown of scaling persists in lower dimensionalities and that λ_u remains equal to $-\sigma$ for all d, but this is pure speculation based only on calculations to $O(\epsilon^3)$ and the exact $n=\infty$ results. Since Ising systems with short-range forces in random fields are expected[1] to undergo ferromagnetic transitions down to $d=2$, the possibility of scaling violations in the $d=3$ Ising model is an intriguing consequence of this speculation.[19]

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[4]See, e.g., L. P. Kadanoff, Physics (L.I. City, N.Y.) 2, 263 (1966).

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[6]H. E. Stanley, Phys. Rev. 176, 718 (1968).

[7]M. E. Fisher, S. Ma, and B. G. Nickel, Phys. Rev. Lett. 29, 917 (1972). The values of $d_d(n)$ and $d_c(n)$ for arbitrary $\sigma < 2$ are $d_c^{\sigma}(n) = 3\sigma$; $d_D^{\sigma}(n \geq 2) = 2\sigma$; $d_D^{\sigma}(n=1) = \min(2\sigma, 2)$.

[8]Angular brackets denote a thermodynamic average taken for one configuration of the random field. The bracket $[\,]_{\rm av}$ denotes an "imputity" average over all possible configurations of the field.

[9]In systems with random exchange bonds but no random fields, $C(x, x')$ is only nonzero below T_c or in the presence of a uniform field. See G. Grinstein, S. Ma, and G. F. Mazenko, unpublished. Note that $C(x, x)$ is the order parameter appropriate to spin-glasses. See S. F. Edwards and P. W. Anderson, J. Phys. F 5, 965 (1975).

[10]A. B. Harris and T. C. Lubensky, Phys. Rev. Lett. 33, 1540 (1974).

[11]For a discussion of relevant and irrelevant variables see, e.g., F. J. Wegner, Phys. Rev. B 5, 4529 (1972). It is to be understood that r_l and u_l are defined in the limit where all k's $\to 0$.

[12]See, e.g., Wegner, Ref. 11.

[13]For a definition and discussion of scaling fields see

E. K. Riedel, Phys. Rev. Lett. 28, 675 (1972); E. K. Riedel and F. J. Wegner, Phys. Rev. Lett. 29, 349 (1972).

[14]That hyperscaling breaks down whenever the free energy behaves like a nonzero power of the scaling field (in this case u_0) associated with an irrelevant operator was first pointed out by M. E. Fisher, in *Renormalization Group in Critical Phenomena and Quantum Fields: Proceedings of a Conference*, edited by J. D. Gunton and M. S. Green (Temple Univ., Philadelphia, Pa., 1974), p. 66. The irrelevant operator is sometimes referred to as a "dangerous irrelevent variable."

[15]In particular, $\gamma=\nu(2-\eta)$ still holds. This result is in apparent disagreement with the exact $n=\infty$ calculations performed in Ref. 3 (see, e.g., the third line of Table III of that reference and note that $\theta=0$ corresponds to the situation we are describing here). The reason for this discrepancy is that in Ref. 3 the correlation function considered is (in my notation $[\langle s^\alpha(x) \times s^\alpha(x')\rangle]_{\rm av}$; the exponents associated with this function do not obey scaling. However, if one computes for $n=\infty$ the exact exponents associated with the true "connected" correlation function, $G^\alpha(x,x')$, defined in (2a), one finds (for $\theta=0$ and $2\sigma<d<3\sigma$) $\alpha=(d-3\sigma)/(d-2\sigma)$, $\beta=\frac{1}{2}$, $\gamma=\sigma/(d-2\sigma)$, $\delta=d/(d-2\sigma)$, $\eta=2-\sigma$, and $\nu=(d-2\sigma)^{-1}$. It is trivial to verifty that these exponents do obey my modified scaling laws with $\lambda_u=-\sigma$.

[16]K. G. Wilson, Phys. Rev. Lett. 28, 548 (1972).

[17]Imry and Ma (Ref. 1) obtained these results for η and ν in the case $\sigma=2$.

[18]We have thus far been unable to construct a proof to all orders in ϵ.

[19]It is worth emphasizing the relationship between the present result and the situation in pure ferromagnets above $d=2\sigma$. Fisher (Ref. 14) has pointed out that the presence of a "dangerous irrelevant variable" in pure magnets above $d=2\sigma$ causes hyperscaling to break down in a rather trivial manner: The Gaussian fixed point is stable, λ_u is identically equal to $2\sigma-d$, and α is zero for all $d\geq 2\sigma$. [See also S. Ma, *Modern Theory of Critical Phenomena* (Benjamin, Reading, Mass., 1976).] The magnet in a random field above $d=2\sigma$ provides a nontrivial realization of this same breakdown mechanism: The stable fixed point is not simply Gaussian and I believe that λ_u is $-\sigma$ instead of $2\sigma-d$. I further postulate that $\lambda_u=-\sigma$ holds even below $d=2\sigma$ for Ising systems in random fields, but there is no hard evidence to support this speculation.

Cumulative Enhancement of J/ψ Production in Hadron-Nucleus Collisions*

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Inclusive J/ψ production from nuclear targets is discussed in a model that describes well the multiplicity and momentum distributions of particles produced in high-energy hadron-nucleus collisions. At incident energies below 30 GeV, cumulative effects (via energy rescaling) lead to an A dependence of the cross section for $p+A\to J/\psi+X$ that is much stronger than the commonly assumed A or $A^{2/3}$ dependence of this cross section.

The cross section for inclusive production of J/ψ in hadron-induced reactions shows a dramatic increase with incident laboratory momenta from $p_{\rm lab}=20$ GeV/c to $p_{\rm lab}=1500$ GeV/c, resembling a threshold phenomenon.[1] Experimental data have been obtained mainly from nuclear targets.[2] Cross sections for nucleons have been extracted by dividing the nuclear cross section by either A or $A^{2/3}$, where A is the atomic number of the target nucleus. Here we show that such a procedure may lead to an overestimate of J/ψ production in pp collisions and to an underestimate of J/ψ production off heavy nuclei, mainly at energies below 30 GeV. Verification of our predictions for $p+A\to J/\psi+X$ has practical consequences for the production of new particles.

According to the coherent tube model[3,4] (CTM) the interaction of a hadron with a target nucleus results from its simultaneous collision with the tube of nucleons of cross section σ that lie along its path in the target nucleus.[5] Thus if there are i nucleons within the tube, the cumulative square of the center-of-mass energy, s_i, is approximately given by

$$s_i=2imp_{\rm lab}, \tag{1}$$

where m is the nucleon mass and $p_{\rm lab}$ is the laboratory momentum of the incident hadron. Assuming that in the respective center-of-mass systems the hadron-tube collision resembles a hadron-nu-