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Causality Violation in Asymptotically Flat Space-Times*

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It is shown that a region containing closed timelike lines cannot evolve from regular initial data in a singularity-free asymptotically flat space-time. Furthermore, the causality assumption made in the black-hole uniqueness proofs is justified: It is demonstrated that no physically realistic nonsingular black hole can have a causality-violating exterior.

There are many solutions[1] to the Einstein equations which possess causal anomalies in the form of closed timelike lines (CTL). It is of interest to discover if our universe could have such lines. In particular, if the universe does not at present contain such lines, is it possible for human beings to manipulate matter so as to create them? I shall show in this paper that it is *not* possible to manufacture a CTL-containing region without the formation of naked singularities, provided normal matter is used in the construction attempt. More precisely, I shall show that a causality-violating region which is visible from infinity cannot evolve from regular initial data in an asymptotically flat, geodesically complete space-time. Furthermore, I shall demonstrate that in the generic case, it is not possible for causality violation to occur outside a nonsingular black hole which forms from regular initial data. This proof justifies the causality assumption made in the Israel-Carter-Robinson black-hole uniqueness theorems.[2]

Causality assumptions are also made in the Hawking-Penrose singularity theorems. I have proven several theorems which collectively show that causality violation is unlikely to prevent the formation of singularities. These results will be published elsewhere.[3]

My notation will be the same as that of Hawking and Ellis (HE).[4] (A proposition of HE—Proposition 4.5.12, say—will be denoted P 4.5.12.)

The notion of "regular initial data in asymptotically flat space-time" is made precise by the following *definition*: An asymptotically flat space-time (identical to a weakly asymptotically simple and empty space-time[5]) will be said to be partially asymptotically predictable from a partial Cauchy surface S if $\overline{D^+(S)} \cap \lambda \neq \phi$ and $\overline{D^-(S)} \cap \gamma \neq \phi$ for all generators λ of $\mathscr{I}^+$ and all generators γ of $\mathscr{I}$ (Closure means closure in $M \cap \partial M$.)

Imposing the condition of partial asymptotic predictability on a space-time serves two purposes: First, it makes cretain that the disjoint surfaces $\mathscr{I}^+$ and $\mathscr{I}^-$ are in the boundary of the *same* asymptotically flat region. (From the definition of weakly asymptotically simple and empty, it would be possible for $\mathscr{I}^+$ to be in one asymptotically flat region and $\mathscr{I}^-$ in another.) Second, it assures the existence of a partial Cauchy surface; the existence of such a surface is a minimum condition for the existence of regular initial data. Furthermore, the partial Cauchy surface is required to be "nice" in the sense that at least *some* of the structure of $\mathscr{I}^+$ and $\mathscr{I}^-$ can be predicted from initial data on S. If S were nor required to be "nice," then the breakdown of prediction could arise from the choice of partial Cauchy surface, and not from the formation of CTL or singularities. (An example of a "bad" partial Cauchy surface is given by Penrose,[6] Fig. 34.) Physically, the existence of a "nice" S means that the beginning of the causality-violating region is localized.

The following theorem shows that a time machine cannot be constructed without the formation of singularities; CTL cannot arise from regular initial data in an asymptotically flat, geodesically complete space-time.

Theorem 1.—An asymptotically flat space-time (M, g) cannot be null geodesically complete if the following conditions hold: (a) $R_{ab}K^aK^b \geq 0$ for all null vectors K^a. (b) The generic condition holds on (M, g). (c) (M, g) is partially asymptotically predictable from a partial Cauchy surface S. (d) The chronology condition is violated in

$$J^+(S) \cap J^-(\mathscr{I}^+).$$

[Note that condition (a) follows from the Einstein equations and the weak energy condition. Condition (d) says that CTL arise to the future of S and are visible from infinity.]

The *idea* behind the proof is quite simple; I first show that under the above conditions there exists a null geodesic which never leaves $H^+(S)$. This geodesic cannot be complete, for (a) and (b) imply that every complete null geodesic has a pair of conjugate points. The rigorous proof uses the following two propositions:

Proposition 1.—Let S be achronal and closed. Then if for some point $p \in S$ there is a neighborhood U such that $\mathrm{int}\tilde{D}^+(S) \cap U$ is empty, then $S \cap U = H^+(S) \cap U$.

Proof: Consider a point q in $S \cap U$. If every future-directed timelike curve γ through q leaves $\tilde{D}^+(S)$ at q, then $\mathscr{I}^+(q) \cap \tilde{D}^+(S) = \phi$, so that $q \in H^+(S)$. Thus if $S \cap U \neq H^+(S) \cap U$, there exists a point $r \in S \cap U$ such that there is through r a future-directed timelike curve segment γ with nonzero length in $\tilde{D}^+(S) \cap U$. But since any timelike curve can intersect $H^+(S)$ at most once and S at most once, it follows that $\gamma \cap \tilde{D}^+(S)$ could consist of at most two points and this contradicts the nonzero length of $\gamma \cap \tilde{D}^+(S) \cap U$.

Proposition 2.—If (M, g) is partially asymptotically predictable from S, but $\mathscr{I}^+$ does not satisfy $\mathscr{I}^+ \cap J^+(S) \subset \overline{\tilde{D}^+(S)}$, then there exists a null geodesic generator η of $H^+(S)$ which has a future end point p on $\mathscr{I}^+$, such that $\eta \cap [M \cup \mathscr{I}^+] \subset M \cup \{p\}$.

Proof: Since $\mathscr{I}^+ \cap J^+(S) \not\subset \tilde{D}^+(S)$, there exists a point q on some generator λ of $\mathscr{I}^+$, such that $q \notin \overline{\tilde{D}^+(S)}$, but $q \in J^+(S)$. Since (M, g) is partially asymptotically predictable from S, there is a point $p \in \lambda$ to the past of q such that $p \in \partial\overline{\tilde{D}^+(S)} \cap \mathscr{I}^+$. [Since $p \in \overline{\tilde{D}^+(S)} \subset J^+(S)$ and $p, q \in \lambda$, we must have $p < q$; otherwise q would be in $\overline{\tilde{D}^+(S)}$.] Now p must be in either S or $H^+(S)$ (or both) since $\partial\tilde{D}^+(S) = H^+(S) \cup S$.

I now show that $p \notin S$. Suppose $p \in S$. Let t_i be a sequence of points in $J^+(S) \cap \lambda$ which converges to p. Let N be any normal neighborhood about p with compact closure. Then there exists some number j such that from each t_i with $i > j$ there is a past-directed timelike curve γ_i which intersects S at the point s_i and $\gamma_i \subset N$. [Such past-directed γ_i exist because every past-directed curve through a point $t_i \in \mathscr{I}^+$ enters M immediately after passing through t_i, since otherwise $\mathscr{I}^+$ could not be isometric to the $\mathscr{I}^+$ of the asymptotically simple and empty space-time associated with (M, g).] Since $\mathscr{I}^+$ is locally achronal, $s_i \in M$. That is, $s_i \notin \mathscr{I}^+$. Since $S \cap M$ is spacelike, it follows from Proposition 1 and the fact that $H^+(S)$ is null that $s_i \notin H^+(S)$, and that $\mathrm{int}\tilde{D}^+(S) \cap \gamma_i$ is nonempty. Since $\tilde{D}^+(S)$ is closed and $t_i \notin \tilde{D}^+(S)$ it follows that $\gamma_i \cap H^+(S)$ is nonempty for all $i > j$. Thus for every normal neighborhood N of p we have that $N \cap M \cap H^+(S)$ is nonempty. Since by P 6.5.3, $H^+(S)$ is generated by null geodesic segments, there must be a generator α of $H^+(S)$ which has an end point at p, and $\alpha \cap \mathscr{I}^+ = \{p\}$. But p cannot be the past end point of α since no past-directed null geodesic from M can intersect $\mathscr{I}^+$. Furthermore, p cannot be the future end point of α, since at p, α would leave $J^+(S)$ and enter $J^-(S)$. This is impossible since the space-time is time orientable. Thus we have a contradiction, which implies $p \notin S$.

Thus $p \in H^+(S)$, and since $p \in \tilde{D}^+(S) - S$, every past-directed timelike curve from p is in $\mathrm{int}\tilde{D}^+(S)$ for some nonzero proper-time length before intersecting S. One can introduce normal coordinates (x^1, x^2, x^3, and x^4) in a normal neighborhood N about p with $\partial/\partial x^4$ timelike such that the curves $\{x^i = \text{const } (i = 1, 2, 3)\}$ intersect both $\mathscr{I}^+(p) \cap N$ and $\mathscr{I}^-(p) \cap N$. (We consider N to be a subset of $\tilde{M}$.) Details of the construction of such coordinates in some N can be found in Ref. 6, p. 23. Since for some N we have $\mathscr{I}^-(p) \cap N \subset \tilde{D}^+(S) \cap M$ and $\mathscr{I}^+(p) \cap \tilde{D}^+(S) = \phi$, each curve $x^i = \text{const}$ must intersect $H^+(S)$ once and only once. Consider a sequence of points t_j in $J^+(p) \cap \lambda \cap N$ which converges to p. Then for each t_j there will be a curve λ_j of the family $\{x^i = \text{const}\}$ which passes through t_j. We can choose N such that all curves $\{x^i = \text{const}\}$ intersect $\mathscr{I}^+$ at most once. Thus each λ_j will intersect $H^+(S)$ in M, and so for every neighborhood U of p, $H^+(S)\cap M\cap U$ is nonempty. By P 6.5.3, $H^+(S)$ is generated by null geodesic segments without past end points or with past end points only at edge S. Thus p must be the future end point of a generator η of $H^+(S)$ with $\eta\cap N\cap \mathscr{I}^+ =\{p\}$ since if $\eta\cap N\cap\mathscr{I}^+$ consisted of more than one point $\eta\cap N$ would lie in $\mathscr{I}^+$. Furthermore, η must have no past end point in $M\cup\mathscr{I}^+$ since such an end point would lie only on edge $S\subset\mathscr{I}^+$ and no null geodesic segment from M can intersect $\mathscr{I}^+$ in the past direction. Thus η is a null geodesic generator of $H^+(S)$ with future end point p on $\mathscr{I}^+$, such that $\eta\cap[M\cup\mathscr{I}^+]\subset M\cup\{p\}$.

Proof of Theorem 1.—First of all, note that it is not possible for the chronology condition to be violated at a point on $\dot{J}^-(\mathscr{I}^+)\cap J^+(S)$ but not in $\mathrm{int}[J^-(\mathscr{I}^+)]\cap J^+(S)$, since this would contradict the achronality of $\dot{J}^-(\mathscr{I}^+)$. Thus chronology must be violated in $\mathrm{int}J^-(\mathscr{I}^+)$, which implies that $\mathscr{I}^+\cap J^+(S)$ is *not* a subset of $\tilde{D}^+(S)$. Since (M,g) is partially asymptotically predictable from S, Proposition 2 implies that there is a null geodesic generator η of $H^+(S)$ which has a future end point on $\mathscr{I}^+$, such that $\eta\cap\mathscr{I}^+$ consists only of that end point. Pick a point p on η in M. If v is an affine parameter on η with $\eta(v_0)=p$, then $v=+\infty$ when η reaches $\mathscr{I}^+$, because the affine parameter v in the metric g is related to an affine parameter $\tilde{v}$ in the metric $\tilde{g}$ by $dv/d\tilde{v}=\Omega^{-2}$. As $\Omega=0$ at ∂M, $\int dv$ diverges (HE, p. 222). Thus η is future complete. If η were also past complete, then by assumptions (a), (b), and P 4.4.5, η would contain a pair of conjugate points. However, by P 6.5.3, η can be continued into the past indefinitely without leaving $H^+(S)$, and so by P 4.5.12, $H^+(S)$ could not be achronal if it possessed a pair of conjugate points. But $H^+(S)$ *must* be achronal; hence η cannot be past complete. This completes the proof.

The previous theorem can be generalized to demonstrate the uniqueness of Kerr black holes. In the proofs showing uniqueness, it is *assumed* that the causality condition holds in the region exterior to the black hole and also on its surface. I will *prove* that the causality condition holds in these regions for any physically realistic black hole formed from regular initial data, provided that cosmic censorship holds (this phrase is defined below). [I will use the term "black hole" to refer to an asymptotically flat solution of the Einstein equations in which there is an event horizon $\dot{J}^-(\mathscr{I}^+)$. This is different from the HE definition (p. 315), for in that case, a black hole could not exist *by definition* in a region with causality violation, since the HE definition required future asymptotic predictability. The following theorem essentially shows that any black hole in my more general sense would possess the future asymptotic predictability required by the HE definition.]

Theorem 2.—If a black hole forms in the future of a partial Cauchy surface S from which the space-time is partially asymptotically predictable, then the causality condition holds in $\overline{J^-(\mathscr{I}^+)}\cap J^+(S)$ provided that the following are true: (a) The space-time is generic. (b) $R_{ab}K^aK^b\geq 0$ for all null vectors K^a. (c) The cosmic censorship hypothesis holds; i.e., all causal geodesics in $\overline{J^-(\mathscr{I}^+)}\cap J^+(S)$ are complete.[7]

In order to prove Theorem 2, we will need a third proposition.

Proposition 3.—Let $\lambda(t)$ be a complete, closed, null geodesic. For any point q of λ, we will have $q\ll q$, provided that (a) the space-time containing λ is generic and (b) $R_{ab}K^aK^b\geq 0$ for all null vectors K^a.

Proof: Since λ is complete, then by conditions (a), (b), and P 4.4.5, it must contain a pair of conjugate points. Furthermore, there will exist an affine parameter value t_0 at q for which the conjugate points lie in the future of t_0, say at t_2 and t_1, with $t_2>t_1>t_0$. Also, there exists another affine parameter value t_3 for which $\lambda(t_3)=q$ and $t_3>t_2$. By P 4.5.12, we have $\lambda(t_1)\ll\lambda(t_2)$. Since λ is null we have $q=\lambda(t_0)<\lambda(t_1)$ and $\lambda(t_2)<\lambda(t_3)=q$. Thus

$$q=\lambda(t_0)<\lambda(t_1)\ll\lambda(t_2)<\lambda(t_3)=q,$$

or $q\ll q$.

Proof of Theorem 2.—Suppose that the causality condition were violated at a point $p\in\dot{J}^-(\mathscr{I}^+)\cap J^+(S)$ but not in $\mathrm{int}[J^-(\mathscr{I}^+)]\cap J^+(S)$. Then there would exist a closed null geodesic λ through p, since we cannot have $p\ll p$, by the achronality of $\dot{J}^-(\mathscr{I}^+)$. By assumption (c), λ must be complete, and so by assumptions (a) and (b) together with Proposition 3, we have $p\ll p$, thus contradicting the achronality of $\dot{J}^-(\mathscr{I}^+)$. Hence, if causality is violated in $\overline{J^-(\mathscr{I}^+)}\cap J^+(S)$, it must be violated in $\mathrm{int}[J^-(\mathscr{I}^+)]\cap J^+(S)$. Furthermore, by assumption (a) and (b) and Proposition 3, the chronology condition must be violated in $\mathrm{int}[J^-(\mathscr{I}^+)]\cap J^+(S)$ if the causality condition is. But if the chronology condition were violated in this region, the proof of Theorem 1 implies that some null geodesic in $J^-(\mathscr{I}^+)\cap J^+(S)$ must be incomplete, contradicting assumption (c). Hence, the causality condition must hold in $\overline{J^-(\mathscr{I}^+)}\cap J^+(S)$.

Scholium.—The preceding theorem does not really *prove* that the causality condition holds in the situation of the Kerr black-hole uniqueness proofs, since in these proofs very strong symmetry requirements are made, which are inconsistent with the generic condition: The generators of the Kerr event horizon, for example, never feel tidal forces. However, the black-hole uniqueness theorems are frequently used to argue that the *final* black-hole state is of Kerr type; the symmetry requirements are then not expected to hold exactly for *all* time, just approximately in the limit of a long time after the black hole formation. In this actual physical case the generic condition *does* hold, for the reasons given on page 101 of HE. Thus Theorem 2 tells us that although there *may* be nonsingular causality-violating black-hole solutions, they would have to satisfy strong symmetry requirements *exactly* over their entire history. In other words, the existence of CTL would be an unstable property of black holes. Hence, no physically realistic, causality-violating, nonsingular black-hole solutions exist.

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Observation of a Narrow Antibaryon State at 2.26 GeV/c^2 *

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We report evidence, from a study of multihadron final states produced in the wide-band photon beam at Fermilab, for the production of a new antibaryon state which decays into $\bar{\Lambda}\pi^-\pi^-\pi^+$. The mass of this state is 2.26 ± 0.01 GeV/c^2 and its decay width is less than 75 MeV/c^2. We also report evidence of a state of higher mass (~ 2.5 GeV/c^2) which decays into the state at 2.26 GeV/c^2.

This Letter is based on the study of multihadron final states produced in the wide-band photon beam at Fermilab. We report on the observation of a narrow peak near 2.26 GeV/c^2 in the invariant-mass spectrum of the $\bar{\Lambda}\pi^-\pi^-\pi^+$ final state. We do not observe a significant peak near 2.26 GeV/c^2 in the $\bar{\Lambda}\pi^+\pi^+\pi^-$ final state. We also observe evidence for a state of higher mass near 2.5 GeV/c^2 which then decays into $\bar{\Lambda}\pi^-\pi^-\pi^+$ (2.26 GeV/c^2) and π^+.

Our results are based on studies of multihadron data taken between September and December