is found to be ~ 1.3 rapidity units, independent of multiplicity or rapidity position within an event; finally, we find that the average size of the even-ΔQ gaps (which are mainly $\Delta Q = 0$) and the odd-ΔQ gaps (which are mainly $|\Delta Q| = 1$) are consistent with one another (with the possible exception of the end gap) when viewed as a function of rapidity-gap position within an event. These results, presenting direct evidence of limited charge exchange and an important universality in the dynamics regarding independence of multiplicity and gap position, should provide significant constraints on multiperipheral or other multiparticle production models.

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[3]Particles having no obvious kinks and with momentum greater than 1.5 GeV/c are assumed to be pions. Positive pions with $p < 1.5$ GeV/c are distinguished from protons by ionization. Misidentified protons with $p > 1.5$ GeV/c are shifted in y, but since $\langle n_{ch} \rangle = 8$ and most protons are slow, the background of misidentified protons is only $\simeq 3\%$.

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Pion Charge-Exchange Scattering at High Energies*

A. V. Barnes, D. J. Mellema, A. V. Tollestrup, and R. L. Walker
California Institute of Technology, Pasadena, California 91125

and

O. I. Dahl, R. A. Johnson, R. W. Kenney, and M. Pripstein
Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 21 April 1976)

We report on a study of pion charge-exchange scattering carried out at Fermilab in the energy range 20 to 200 GeV. The results can be described remarkably well by a simple Regge-pole model. The charge-exchange cross sections in the forward direction lead to a prediction for the difference in total cross sections of $\pi^- p$ and $\pi^+ p$ which is in satisfactory agreement with direct measurements of this difference in another experiment at Fermilab.

In the study of strong-interaction dynamics, two reactions have a special importance because of their simplicity from a theoretical point of view. These are the pion charge-exchange scattering,

$$\pi^- p \to \pi^0 n,$$

and the related reaction in which η mesons are produced,

$$\pi^- p \to \eta n.$$

In Regge theory, the amplitude of each of these reactions is dominated by a single Regge pole (the ρ pole in charge exchange and the A_2 in the η reaction) so that the experimental data have a relatively direct theoretical interpretation. In addition, the forward charge-exchange cross section provides a measure of the difference between $\pi^- p$ and $\pi^+ p$ total cross sections.

With these motivations we have carried out an experiment at Fermilab to study these reactions over a large beam energy range extending from

TABLE I. Differential cross sections in μb/GeV^2 and other results for $\pi^- p \to \pi^0 n$. These data have been corrected for the effects of experimental t resolution and finite bin widths. Only statistical errors and errors in t-dependent corrections are given for $d\sigma/dt$ but all systematic errors are included in the integral cross sections σ_I. The right-hand column, $\alpha^*(t)$, contains values of the trajectory obtained by fitting to the data for each value of t separately.

$-t$ (GeV^2)	Bin Width (GeV^2)	Beam Momentum GeV						$\alpha^*(t)$
		20.8	40.8	64.4	100.7	150.2	199.3	
0.002	0.004	104 ± 5	50.6 ± 2.1	32.0 ± 1.3	20.3 ± 0.8	13.4 ± 0.5	10.4 ± 0.4	0.491 ± 0.012
0.006	0.004	105 ± 5	57.8 ± 2.3	35.9 ± 1.4	21.1 ± 0.8	14.0 ± 0.6	10.1 ± 0.4	0.474 ± 0.011
0.012	0.008	121 ± 5	55.2 ± 1.8	34.5 ± 1.1	20.8 ± 0.7	14.6 ± 0.5	11.0 ± 0.4	0.474 ± 0.010
0.020	0.008	120 ± 5	59.9 ± 1.9	36.8 ± 1.2	21.5 ± 0.7	14.8 ± 0.5	11.3 ± 0.4	0.471 ± 0.009
0.028	0.008	137 ± 5	62.1 ± 1.9	36.9 ± 1.2	22.6 ± 0.7	15.6 ± 0.5	11.3 ± 0.4	0.453 ± 0.009
0.040	0.016	134 ± 4	62.1 ± 1.6	37.0 ± 1.0	22.1 ± 0.6	14.5 ± 0.4	10.6 ± 0.3	0.439 ± 0.008
0.056	0.016	126 ± 4	59.0 ± 1.6	36.5 ± 1.0	20.5 ± 0.6	13.2 ± 0.4	9.3 ± 0.3	0.422 ± 0.008
0.072	0.016	116 ± 4	54.2 ± 1.5	30.7 ± 0.9	18.6 ± 0.5	11.9 ± 0.3	8.84 ± 0.26	0.427 ± 0.008
0.090	0.020	108 ± 3	48.0 ± 1.2	28.4 ± 0.8	16.1 ± 0.4	10.0 ± 0.3	6.86 ± 0.20	0.393 ± 0.007
0.110	0.020	92.1 ± 3	40.1 ± 1.1	22.9 ± 0.6	12.9 ± 0.4	8.2 ± 0.2	5.79 ± 0.18	0.387 ± 0.008
0.140	0.040	74.3 ± 2.1	31.4 ± 0.8	17.0 ± 0.4	9.5 ± 0.2	5.54 ± 0.15	3.88 ± 0.11	0.344 ± 0.007
0.180	0.040	50.9 ± 1.5	20.2 ± 0.5	10.6 ± 0.3	5.87 ± 0.17	3.54 ± 0.11	2.40 ± 0.08	0.326 ± 0.008
0.220	0.040	33.7 ± 1.2	13.1 ± 0.4	6.8 ± 0.2	3.64 ± 0.13	2.10 ± 0.08	1.36 ± 0.06	0.294 ± 0.010
0.260	0.040	22.0 ± 0.9	7.6 ± 0.3	4.20 ± 0.18	1.93 ± 0.09	1.15 ± 0.06	0.709 ± 0.040	0.248 ± 0.012
0.310	0.060	11.6 ± 0.5	4.24 ± 0.18	2.09 ± 0.10	0.99 ± 0.05	0.53 ± 0.03	0.311 ± 0.022	0.208 ± 0.013
0.370	0.060	4.8 ± 0.3	1.73 ± 0.11	0.75 ± 0.06	0.37 ± 0.03	0.204 ± 0.019	0.117 ± 0.013	0.184 ± 0.020
0.450	0.100	1.44 ± 0.13	0.50 ± 0.04	0.199 ± 0.023	0.088 ± 0.011	0.050 ± 0.007	0.026 ± 0.005	0.123 ± 0.031
0.550	0.100	0.44 ± 0.07	0.090 ± 0.018	0.058 ± 0.012	0.022 ± 0.006	0.0067± 0.0027	0.0071± 0.0026	0.040 ± 0.067
0.650	0.100	0.36 ± 0.06	0.103 ± 0.018	0.044 ± 0.010	0.015 ± 0.004	0.0067 ± 0.0024	0.0091 ± 0.0026	0.052 ± 0.064
0.750	0.100	0.51 ± 0.07	0.080 ± 0.016	0.041 ± 0.009	0.022 ± 0.005	0.0044 ± 0.0019	0.0055 ± 0.0019	-0.087 ± 0.071
0.900	0.200	0.50 ± 0.05	0.069 ± 0.011	0.028 ± 0.006	0.0155 ± 0.0031	0.0064 ± 0.0017	0.0034 ± 0.0011	-0.141 ± 0.064
1.100	0.200	0.30 ± 0.04	0.042 ± 0.008	0.018 ± 0.004	0.0041 ± 0.0016	0.0025 ± 0.0010	0.0013 ± 0.0007	-0.270 ± 0.080
1.300	0.200	0.16 ± 0.03	0.020 ± 0.006	0.005 ± 0.002	0.0023 ± 0.0012	0.0012 ± 0.0007	0.0003 ± 0.0003	-0.398 ± 0.128
$\frac{d\sigma}{dt}(t=0)$ from fit		100.3	49.8	30.9	19.51	12.88	9.61	
$\sigma_I(\mu b)=\int_{-1.5}^{0}\frac{d\sigma}{dt}\,dt$		22.6 ± 1.1	9.8 ± 0.5	5.61 ± 0.25	3.20 ± 0.14	2.02 ± 0.09	1.44 ± 0.06	
Number of Events		26,700	36,500	29,900	30,700	26,100	23,400	

20 to 200 GeV. A description of the apparatus and data-analysis procedures is presented in the following Letter[1] along with the results for the η reaction. Results for the π^0 reaction are presented here in Table I and in Figs. 1 and 2. Data on the production of other particles such as η' and ω will be reported later.

According to Regge theory, if the charge-exchange amplitude at high energies is dominated by the ρ pole, it is expected to have a simple power-law energy dependence at fixed t, giving a differential cross section of the form

$$d\sigma(s,t)/dt = \beta(t)\nu^{2\alpha(t)-2}, \qquad (1)$$

where $\nu = (s-u)/4M = \omega + t/4M$, with ω the π^- laboratory energy, and s, t, and u the usual energy and momentum transfer variables. The functions $\beta(t)$ and $\alpha(t)$ are not predicted by the theory but it has been found from previous data[2-4] that the ρ trajectory, $\alpha(t)$, is approximately linear in t,

the extrapolation of $\alpha(t)$ as determined for negative t passing nearly through the points corresponding to the observed spins and masses[2] of the ρ and g mesons.

Some deviation from the above simple power-law behavior might be expected because of possible Regge cuts and because the observation of asymmetry in charge-exchange scattering from polarized targets[5,6] indicates that the amplitudes must have other components besides the dominant pole, at least in the energy region below 10 GeV. Nevertheless, an excellent phenomenological description of our entire set of data is provided by the simple expression (1), with

$$\alpha(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2, \quad -1.4 < t \leqslant 0 \text{ GeV}^2;$$

$$\beta(t) = \beta_1 \exp(b_1 t) - t(t-t_0)^2(\beta_2 + \beta_3 t + \beta_4 t^2)\exp(b_2 t).$$

Values of the parameters found from a fit to our

data are

$$\alpha_0 = 0.481 \pm 0.004, \quad \alpha_1 = 0.928 \pm 0.034, \quad \alpha_2 = 0.205 \pm 0.055, \quad b_1 = 5.20 \pm 0.21, \quad b_2 = 6.16 \pm 0.61,$$

$$t_0 = -0.542 \pm 0.004, \quad \beta_1 = 2340 \pm 80, \quad \beta_2 = (2.34 \pm 0.10) \times 10^5, \quad \beta_3 = (-0.77 \pm 1.41) \times 10^5, \quad \beta_4 = (11.6 \pm 5.4) \times 10^5.$$

The units are such that t is in GeV^2, ν in GeV, and $d\sigma/dt$ in μb/GeV^2. The fit gives a χ^2 of 107 for 128 degrees of freedom.

Curves computed with the above parameters are shown in Figs. 1 and 2. The data points shown in these figures and also the cross sections given in Table I have been corrected for the effects of experimental t resolution and finite bin widths. This correction (or unfolding) has been made as follows: The fitting function (1) has been folded with the t resolution and integrated over the finite bin widths in order to obtain smeared cross sections to be compared with the measured data in minimizing χ^2. The ratios of unsmeared to smeared fitting cross sections are then used as the correction factors applied to the measured data. Separate fits at each energy were also made to investigate the energy dependence of the parameter t_0. The values of t_0 fluctuate by ± 0.02 GeV^2 but no systematic variation with energy is apparent.

The trajectory $\alpha(t)$ obtained from the overall fit is shown by the solid curve in Fig. 3, together with data points, $\alpha^*(t)$, obtained in the conventional manner by fitting to the data at each value of t separately, using the form (1). As may be seen in this figure, the observed values of $\alpha(t)$ in the t range from 0 to -0.3 GeV^2 fall remarkably close to a straight line drawn through the points corresponding to the ρ and g mesons.

Although the parametrization given above fits our data remarkably well, it agrees only qualitatively with previous data[2-4] as shown by an example at 5.9 GeV in Fig. 1. Effective trajectories obtained from the lower-energy data have had a significantly larger intercept α_0.

We next consider a comparison between our data and the difference in total cross sections for $\pi^- p$ and $\pi^+ p$ interactions, $\Delta\sigma = \sigma_{tot}(\pi^- p) - \sigma_{tot}(\pi^+ p)$. As is well known, isospin invariance and the optical theorem relate this difference to the imaginary part of the forward charge-exchange amplitude. Expressed in terms of cross sections, this relation is

$$(d\sigma/dt)_{t=0} = (\pi/p_{lab}^2)(1 + R^2)[\text{Im}A(0)]^2$$

$$= 25.54(1 + R^2)(\Delta\sigma)^2,$$

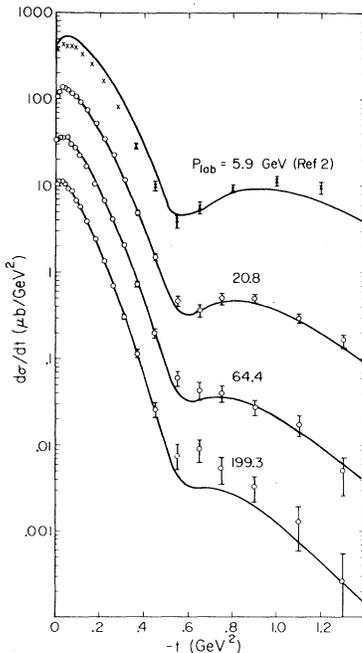where R is the ratio of real to imaginary parts of

FIG. 1. Differential cross sections at 20.8, 64.4, and 199.3 GeV from this experiment, and at 5.9 GeV from Ref. 2. The curves are the result of a fit described in the text.
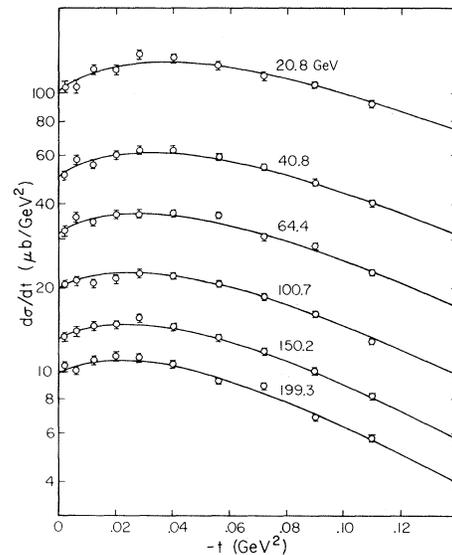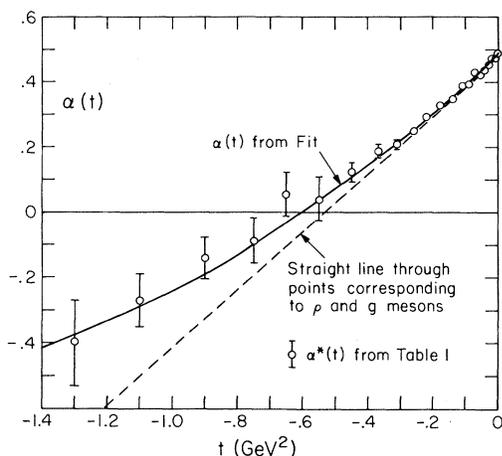

FIG. 2. Differential cross sections in the region of small t. The curves are the result of a fit described in the text.
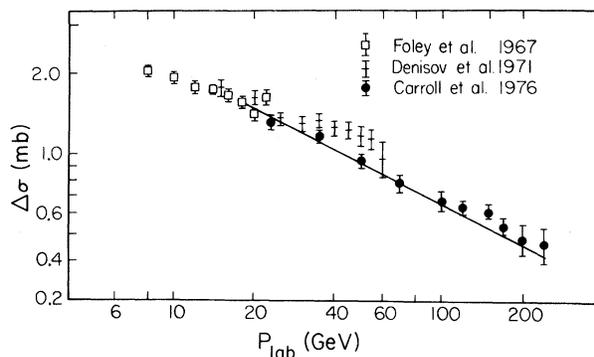
FIG. 3. The effective trajectory $\alpha(t)$.

FIG. 4. Data from three experiments (Refs. 7–9) on the difference $\Delta\sigma$ of π^-p and π^+p total cross sections, compared to the prediction (solid line) from our data on charge-exchange scattering. This prediction is obtained from the parameters of the fit described in the text.

the forward charge-exchange amplitude $A(0)$, p_{lab} is the incident beam momentum in GeV, and $d\sigma/dt$ is the charge-exchange cross section in $\mu b/GeV^2$ if $\Delta\sigma$ is in millibarns. Furthermore, if we assume that the observed power-law dependence of $(d\sigma/dt)_{t=0}$ extends to higher energies, dispersion relations give a relation between the ratio R and the exponent $\alpha(0)$: $R = \tan\frac{1}{2}\pi\alpha(0)$.

We are thus able to predict the difference $\Delta\sigma$ from the charge-exchange data. The result is shown in Fig. 4 where it is compared with direct measurements of this difference in three experiments.[7-9] Our prediction agrees very well with the results of Carroll et al.[9] obtained at Fermilab in the energy range 23 to 240 GeV.

Stevens.

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