

to a pinch where the current is carried by a non-relativistic stream of electrons.

¹F. Winterberg, *Physics of High Energy Density* (Academic, New York, 1971), p. 397 ff.

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³J. M. Creedon, I. D. Smith, and D. S. Prono, *Phys. Rev. Lett.* **35**, 91 (1975).

⁴J. W. Poukey, J. R. Freeman, M. J. Clauser, and G. Yonas, *Phys. Rev. Lett.* **35**, 1806 (1975).

⁵H. Davitian and W. L. Gardner, *Plasma Phys.* **14**, 970 (1972).

⁶L. A. Artsimovich, *Controlled Thermonuclear Reactions* (Oliver and Boyd, Edinburgh and London, 1964), p. 54.

⁷F. Winterberg, *Nature (London)* **251**, 44 (1974), and *Plasma Phys.* **17**, 69 (1975).

⁸F. Winterberg, *Nucl. Fusion* **12**, 353 (1972).

COMMENTS

Mass Differences of Charmed Hadrons

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The mass difference $m(D^+) - m(D^0)$ of the charmed pseudoscalar meson doublet is estimated as about 6.7 MeV, as compared with the value 15 MeV given by De Rújula, Georgi, and Glashow. Mass differences are also estimated for charmed baryons.

The new particle at 1865 MeV discovered¹ at SPEAR may well be the long-sought bound state of a charmed quark c and an anti-up-quark \bar{u} , known among theorists² as the D^0 , or its antiparticle, the \bar{D}^0 . If this is the case, then the other member $D^+ \equiv c\bar{d}$ or $D^- \equiv \bar{c}d$ of the same isodoublet should also be found soon; and we can look forward to a measurement of the $D^+ - D^0$ mass splitting. This splitting will provide an interesting test of our ideas about the origin of isospin non-conservation. De Rújula, Georgi, and Glashow³ have estimated the mass difference between the D^+ and D^0 as about 15 MeV and have pointed out important consequences of such a large isospin splitting for the production rates of various charmed particles. In this Comment, we wish to suggest a slightly different method of calculating the $D^+ - D^0$ splitting, which leads to a rather different numerical result, about 6.7 MeV.

In any renormalization theory of strong interactions based on quarks and flavorless gauge bosons, the only sources of isospin breaking are the quark mass differences and ordinary one-photon exchange.⁴ However, it is not in general

obvious how to evaluate these two contributions in actual hadron states. De Rújula, Georgi, and Glashow³ employ a nonrelativistic atomic model: The mass difference within any isospin multiplet consists of a mass term, equal to the difference in the masses of the constituent quarks, plus a Coulomb term, equal to the difference of the products of the quark charges times a constant⁵ $\langle 1/r \rangle$. They determine the $u-d$ quark mass difference and $\langle 1/r \rangle$ from the $\pi^+ - \pi^0$ and $K^+ - K^0$ mass differences, and then use the same parameters to calculate the $D^+ - D^0$ mass difference, obtaining 13 MeV. Taking into account the fact that $\langle 1/r \rangle$ is likely to be a little larger for D mesons, their estimate is then increased to 15 MeV.

This nonrelativistic atomic model may be reasonable for the heavy pseudoscalar D mesons, and possibly even for the K mesons, but it seems to us unlikely that it could be applicable to the π mesons. On the other hand, there is a way that we can use the $\pi^+ - \pi^0$ mass difference to separate the mass and Coulomb parts of the $K^+ - K^0$ splitting. Dashen's theorem⁶ tells us that the photon part of the mass-squared splittings are the same

for the K and π . Furthermore, the u - d quark mass difference does not contribute to the $\pi^+-\pi^0$ mass splittings. It follows that the "photon exchange" part of the K -mass splitting is $[m^2(\pi^+) - m^2(\pi^0)]/2m(K)$, or 1.27 MeV. (De Rújula, Georgi, and Glashow³ get 3 MeV.) If we interpret this to be due to the Coulomb interaction in a nonrelativistic quark model, then $\langle 1/r \rangle$ would be 520 MeV, and the observed K^+-K^0 mass difference would require a quark mass difference $m(u) - m(d)$ of -5.27 MeV. However, the photon exchange contributes not only to the Coulomb force between the quarks but also to the quark masses themselves. Using a phenomenological Lagrangian with zero quark masses to carry out calculations in the chiral $SU(3) \otimes SU(3)$ limit, we find that the diagrams which contribute to the square of the mass of the K 's consist of a $K^0 - d\bar{s} - K^0$ or $K^+ - u\bar{s} - K^+$ quark loop, with the photon exchanged either (a) between opposite sides of the quark loop, or (b) on the same side of the quark loop, or (c) from one side of the quark loop to a separate charmed quark bubble. (Gluons and quark bubbles are inserted in diagrams of each type in all possible ways.) We will neglect diagrams of type (c) because the charmed quark bubble must be connected to the s and u or d quark lines by at least three gluon lines and the gluon-quark coupling is relatively weak at energies of the order of the charmed-quark mass. Denoting the sum of the other two classes of diagrams by A and B , respectively, we easily see that $A(K^+) = -2A(K^0)$ and $B(K^+) = 5B(K^0)/2$. But $A(K^0) + B(K^0)$ must vanish,⁶ so that the "Coulomb" part $A(K^+) - A(K^0)$ of the K^+-K^0 mass-squared difference is $\frac{2}{3}$ of the total photon exchange contribution $A(K^+) + B(K^+) - A(K^0) - B(K^0)$, and $[m(K^+) - m(K^0)]_{\text{Coul}} = 0.85$ MeV. Equating this to $\frac{1}{3}\alpha\langle 1/r \rangle$ gives

$$\langle 1/r \rangle \approx 350 \text{ MeV}. \quad (1)$$

The u - d quark mass difference can also be estimated from the observed K^+-K^0 mass difference as

$$\begin{aligned} m(u) - m(d) &\approx [m(K^+) - m(K^0)]_{\text{obs}} \\ &\quad - [m(K^+) - m(K^0)]_{\text{Coul}} \\ &\approx -4.84 \text{ MeV}. \end{aligned} \quad (2)$$

We can now calculate the D^+-D^0 mass splitting, using the formula

$$m(D^+) - m(D^0) = m(d) - m(u) + \frac{2}{3}\alpha\langle 1/r \rangle. \quad (3)$$

Taking the same values (1) and (2) for the param-

eters in Eq. (3), we find a mass splitting of 6.5 MeV.

It is interesting to compare the value (2) for $\langle 1/r \rangle$ with the value expected in a nonrelativistic potential model. Schnitzer⁷ has estimated that for a linear potential $V(r) = ar$, the ground-state expectation value of $1/r$ is

$$\langle 1/r \rangle \approx (32\mu a/3\pi^2)^{1/3}, \quad (4)$$

where μ is the reduced mass. For the force constant a , we will use the estimate of Kang and Schnitzer,⁸ $a \approx 0.3 \text{ GeV}^2$. If we adopt masses of 340 and 540 MeV for the masses of the u and s quarks, the reduced mass for a K meson is $\mu \approx 210$ MeV, and Eq. (4) gives a value of 410 MeV for $\langle 1/r \rangle$. Our value (1) is 15% less. On the other hand, the use by De Rújula, Georgi and Glashow³ of the $\pi^+-\pi^0$ mass difference to estimate $\langle 1/r \rangle$ gives a value $\langle 1/r \rangle \approx 1230$ MeV, three times greater than the value given by Eq. (4).

For a charmed-quark mass of 1500 MeV, the reduced mass for a D meson is 30% larger than for a K meson, so that (4) suggests that $\langle 1/r \rangle$ should be 10% larger for D mesons than for K mesons. Taking this into account in the Coulomb term of Eq. (3) gives a D^+-D^0 mass difference of 6.7 MeV. The D^* mass splitting should be comparable.

The same simple nonrelativistic approach can be applied to the baryons. The Σ and Ξ mass differences can be moderately well fitted with $m(d) - m(u) \approx 4.5$ MeV and with a common value $\langle 1/r \rangle \approx 240$ MeV for all quark pairs. Using these parameters for the charm-one baryons described in Ref. 2 gives the mass splittings very roughly as

$$\begin{aligned} m(C_1^{*+}) - m(C_1^+) &= m(u) - m(d) + \frac{4}{3}\alpha\langle 1/r \rangle \\ &\approx -2 \text{ MeV}, \\ m(C_1^+) - m(C_1^0) &= m(S^+) - m(S^0) = m(A^+) - m(A^0) \\ &= m(u) - m(d) + \frac{1}{3}\alpha\langle 1/r \rangle \\ &\approx -4 \text{ MeV}. \end{aligned}$$

These are only very rough estimates, because for baryons there is a partial cancelation between the Coulomb and quark mass contributions.

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Note added.—Since completion of this paper, I. Peruzzi *et al.* [Phys. Rev. Lett. **37**, 569 (1976)] have reported strong evidence for \overline{D}^+ at 1876 ± 15 MeV. As expected, this is heavier than $D^0(1865)$, but the mass difference is too imprecisely known to decide the issue that we have addressed. The problem of calculating mass differences of the charmed hadrons has also been considered by D. B. Lichtenberg [Phys. Rev. D **12**, 3760 (1975)] and H. Fritzsche [CERN Report No. TH 2191 CERN (to be published)].

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²M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. **47**, 277 (1975).

³A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. **37**, 398 (1976). Also see K. Lane and E. Eichten, Phys. Rev. Lett. **37**, 477 (1976).

⁴S. Weinberg, Phys. Rev. Lett. **31**, 494 (1973), and Phys. Rev. D **6**, 4482 (1973). Note that the quark mass difference is not an effect of electromagnetism alone;

in unified gauge theories it receives contributions of order αm also from the weak interactions.

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⁶R. Dashen, Phys. Rev. **183**, 1245 (1969). Corrections to this theorem are discussed by P. Langacker and H. Pagels, Phys. Rev. D **8**, 4620 (1973).

⁷H. J. Schnitzer, Phys. Rev. D **13**, 74 (1976). Schnitzer's analytic approximation to $\langle 1/r \rangle$ has been found to be accurate for a linear potential to within 2% in a computer calculation by J. Borenstein, Ph.D. thesis, Harvard University, 1975 (unpublished). In order to apply Schnitzer's result to bound states of unequal mass quarks, replace the quark mass in his Eqs. (3.6) and (3.20) with twice the reduced mass.

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Probing Giant Magnetization Clouds with Polarized Neutrons

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A method of analyzing the diffuse scattering of polarized neutrons from ferromagnetic alloys close to the critical concentration is outlined. Close agreement is found between the earlier work of Hicks *et al.* on giant magnetization clouds in Ni-Cu and this analysis of the new polarized neutron results assuming that each magnetization cloud is "seeded" by a nickel atom with eleven or more first neighbors.

Recent polarized neutron measurements on Ni-Cu alloys close to the critical concentration for ferromagnetism by Radhakrishna *et al.*¹ and Medina and Cable² have rekindled interest in the spatial distribution of magnetization in this and similar alloy systems. The original work, just on the ferromagnetic alloys, was done³ using unpolarized neutrons by switching a magnetic field along the scattering vector to isolate the magnetic scattering. In this work the authors found that they could describe their scattering results by assuming a random array of giant magnetization clouds of $8-10\mu_B$ but in concentrations ranging from 0.2% to 0.7% in alloys containing 46 to 50

at.% of nickel. The importance of this work was that it modeled the spatial fluctuations close to a new critical point (the concentration critical point). In one case¹ the new polarized neutron work is interpreted in terms of a linear superposition of moment defects around each copper atom and the giant magnetization cloud model is dismissed. This is surprising in view of the fact that the bulk moment is not even nearly proportional to the nickel concentration in this region and that a strict reading of the linear interpretation would have groups of copper atoms driving negative moments of nickel atoms in their vicinity. In the other case² no interpretation of the